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Slovenian Annuity Tables

Ljubljana, 2012

Aleš Ahčan, Darko Medved, Ermanno Pitacco, Jože Sambt, Robert Sraka: Slovenian Annuity Tables
Faculty of Economics – scientific monograph Code: AMP12ZM112

Publisher: Faculty of Economics,
Publishing Office
First Edition: 50 copies
Printed and Bound by: Copis, Ljubljana, Dunajska 158
Design by: Robert Ilovar
Reviewers: dr. Aleš Berk Skok
dr. Steven Vanduffel
Proofread by: Shanna Pafford Groznik

CIP – Kataložni zapis o publikaciji
Narodna univerzitetna knjižnica, Ljubljana

336.77
347.464

Slovenian Annuity Tables / Aleš Ahčan [et al.]. - 1. natis. - Ljubljana : Ekonomska fakulteta, 2012

ISBN 978-961-240-237-2

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262727936

SLOVENIAN ANNUITY TABLES

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This project was financially supported by the Slovenian Association of Insurers.

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1 INTRODUCTION

Valuing technical provisions of life annuities depends mainly on projected demographic trends. A life annuity is a specific insurance contract in which one party (an insurance company), in exchange for payment of a premium, guarantees a series of payments until the death of the other party (the insured person). The projection of future mortality improvements has significant effects on pricing and reserving for life annuities. As such, annuities are associated with longevity risk, in that decreasing mortality rates of the insured population lead to an increase in the number of annuity payments.

There are no official projected annuity tables for annuity owners in Slovenia. To value life annuities, insurance companies in Slovenia use annuity tables that are based on the mortality profile of populations in foreign countries. The Slovenian Insurance Supervision Agency has set the German annuity tables DAV 1994 R as the minimum standard. This means that insurance companies value their liabilities using DAV 1994 R annuity tables; however, they can use other tables, as long as those tables produce higher technical provisions than the DAV 1994 R. The result, though, is that the industry standard is to use the DAV 1994 R tables for pricing and reserving and, in turn, mortality statistics from 1994 on the insured in Germany are used to value liabilities for annuities and pensions in Slovenia.

The DAV 1994 R tables were used in the German insurance industry until the DAV 2004 R tables were introduced in 2005. The replacement resulted in a 10–20% increase in premiums for deferred annuities in Germany, depending on the insured's age and sex. This substantial increase in premium rates raised an important question for the Slovenian insurance industry: Are the DAV 1994 R tables still sufficient or even appropriate for measuring the best estimate of liabilities from annuities and pensions in Slovenia?

To answer this question, the international working group on mortality was established in 2010 to develop the first annuity mortality tables for the Slovenian market. This monograph presents the results of the group's work. The work of the group was financially supported by the Slovenian Association of Insurers.

The structure of this monograph is as follows. Chapter 2 and 3 are devoted to the basic notation, data specification and calibration. In Chapter 4 we present the main features of the LC, Poisson log-bilinear and APC methods for projecting mortality. Chapter 5 reports results for a simple exponential extrapolation. In Chapter 6 an attempt is made to project cause-specific mortality. In Chapter 7 we apply the stochastic methods to Slovenian data and present the results. In this chapter we also explain in detail how projections of kappa are calculated. We conclude this section with back-testing. In Chapter 8 we construct the first Slovenian gender-specific annuity tables and propose how unisex annuity tables can be constructed. Chapter 9 outlines our conclusions.

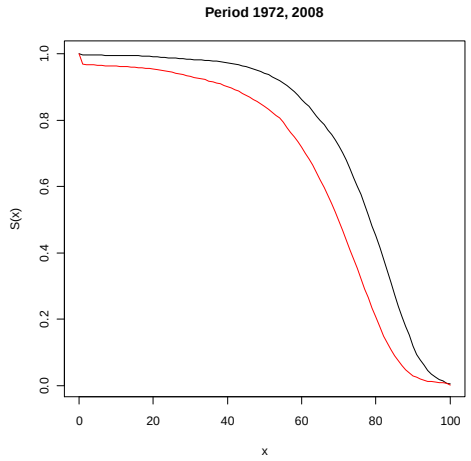
2 BASIC NOTATION

Let us start by defining the basic notation and terminology that will be used throughout this document. If we denote with T_0 the random lifetime of a new-born, then we define survival function $S(x), x \geq 0$ with

$$S(x) = P[T_0 > x] \tag{2.1}$$

Figure 2-1 illustrates the typical behaviour of survival function $S(x)$, depending on age (x). The shifting of the survival function to the right is called a rectangularization. The red line denotes the year 1972, while the black line represents the year 2008.

Figure 2-1 Survival function



At limiting age ω we have $S(\omega) = 0$. The probability of a person aged x surviving for the next h years is calculated from:

$$_h p_x = \frac{S(x+h)}{S(x)} = P[T_x > h] \tag{2.2}$$

If we denote with $f_0(t)$ the probability density function (pdf) and with $F_0(t)$ the distribution function of T_0, we define

$$_t q_x = F_x(t) = \frac{F_0(x+t) - F_0(x)}{S(x)} \tag{2.3}$$

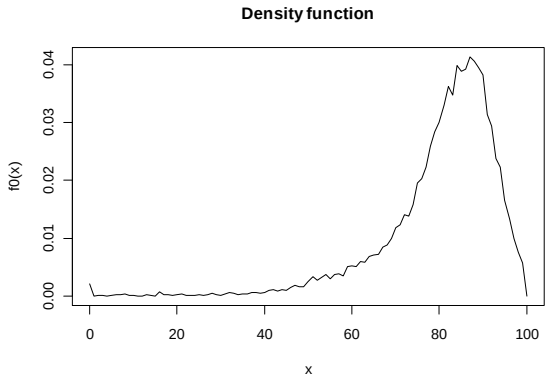
We will often need a force of mortality μ_x, which is defined as

$$\mu_x = \lim_{t \to 0} \frac{_t q_x}{t} = -\frac{d}{dx} \ln S(x) \tag{2.4}$$

It represents the instantaneous rate of mortality at a given age x. The concept is usually referred to as a failure rate or hazard function. From the definition it is obvious that $f_x(t) = {}_t p_x \mu_{x+t}$.

Figure 2-2 presents the probability density function for the year 2008 for a new-born.

Figure 2-2 Density function for year 2008



The behaviour of the force of mortality over the interval (x, x + 1) can be described by the central death rate at age x, which is denoted m_x and defined as

$$m_x = \frac{\int_0^1 S(x+u)\mu_{x+u}du}{\int_0^1 S(x+u)du} = \frac{S(x)-S(x+1)}{\int_0^1 S(x+u)du} \approx \frac{S(x)-S(x+1)}{\frac{1}{2}\left(S(x)+S(x+1)\right)} \tag{2.5}$$

For non-integer ages $x+t, \quad 0<t<1,$ we will assume a constant force of mortality in our projections:

$$\mu_{x+t} = \mu_x \tag{2.6}$$

This assumption, consisting in a piece-wise constant force of mortality, is frequently adopted in actuarial calculations. Equation (2.6) has important consequences, namely

$$m_x = \mu_x \tag{2.7}$$

We define life expectancy at birth as

$$\overline{e}_0 = \int_0^{\omega} t f_0(t)dt \tag{2.8}$$

The expected lifetime is often used to compare mortality in various populations and time periods. In our calculations we will often use curtate expectation of life (life expectation of whole survived years), which is defined as follows

$$e_x = \sum_{k=1}^{\omega-x} {}_k p_x \tag{2.9}$$

We will use an approximation of $\overline{e}_x$ as $e_x + \frac{1}{2}$.

Figure 2-3 presents the development of life expectancy at birth for males and females for the period from 1971 to 2008 using the approximation described above.

Figure 2-3 Life expectancy at birth

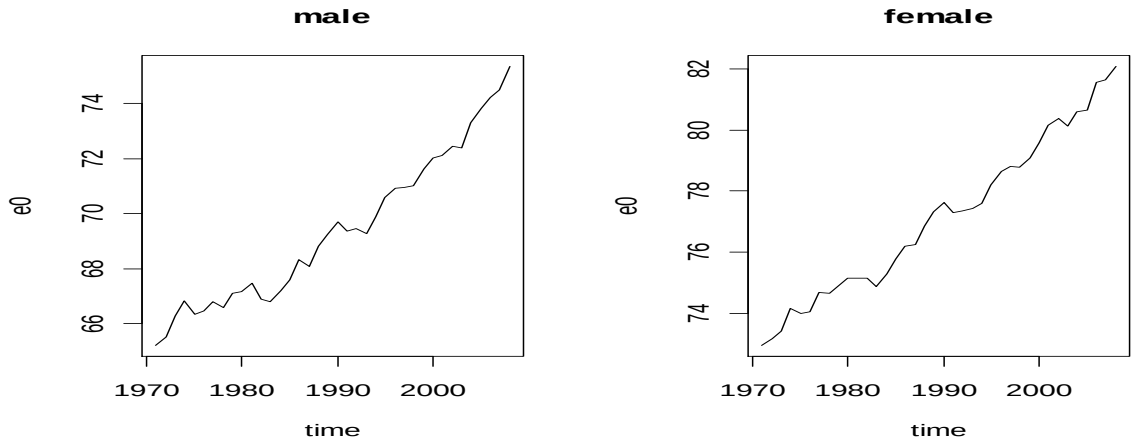


Figure 2-3 shows that life expectancy at birth has increased by 10 years from 1971 to 2008 for both genders. Life expectancy at birth in 2008 was 82.34 years for females and 75.38 years for males. The steep curve suggests that life expectancy will most likely also substantially increase in the future, which is essential information for annuity providers. The question arises of which model we should use for Slovenia to accurately prognosticate the future development of the life curve of the Slovenian population.

Life expectancy statistics is very useful as an overall measure of mortality, and can be interpreted easily. Nevertheless, it is important to differentiate between period life expectancy and cohort life expectancy. Period life expectancies are calculated using mortality rates for a given period (say 2008, 2010 etc.). In this respect, period life expectancy does not allow for future changes in mortality since it rests on past observations.

Cohort life expectancies are calculated using a cohort life table, which allows for known or projected changes in mortality at later ages. This is why we will construct a cohort life table to project future mortality.

3 MORTALITY DATA

One of the most important parts of accurately forecasting mortality is to collect appropriate statistics. The quality of forecasting very much depends on the length of time series and the data quality. Since mortality data usually exhibits some irregularities, different methods are used to interpolate and extrapolate statistical data. This chapter explains how mortality data are properly prepared for forecasting.

3.1 Gathering Slovenian population mortality data

3.1.1 Population data

The Statistical Office of the Republic of Slovenia provided population data. Data are available for the time span from 1971 to 2008, for each age and separated for men and women. During that period, some methodological changes were introduced:

- in the middle of 1995 the definition of population changed and recently, at the beginning of 2008, it changed again; and
- after 1985 the Central Population Register is used as a data source (CRP – *centralni register prebivalstva*), while for the years 1971 to 1985 estimates based on census data are used. The estimates were made by the Slovenian Statistical Office (being part of the Yugoslav statistical system at that time). Further, data for broader age groups (5-year age groups and the age group 85+) were subsequently distributed to 1-year age groups; the Statistical Office of the Republic of Slovenia made the estimations.

According to the definition of population that was valid from mid-1995 to 1 January 2008, the criteria for defining population was “usual residence”, which could be permanent or temporary residence in Slovenia. The key criterion for determining “usual residence” was a three-month residence period – at this address (according to actual, i.e. already realised, or intended residing at this address). In 2008 the three-month criterion was turned into an one-year residence period.

These changes in the methodology could affect population projections, but should not affect the mortality projections for annuity calculations.

3.1.2 Mortality data

Population morality data were provided by the Statistical Office of the Republic of Slovenia as official data on the population of Slovenia. Data were provided for the time span from 1971 to 2008, for each age and separated for men and women. For the 1971–1980 period, there are some minor discrepancies between the data used and the official cumulative data for the same years, especially for the years 1972 and 1973. The discrepancies are in the range of 10 persons in total, which is negligible.

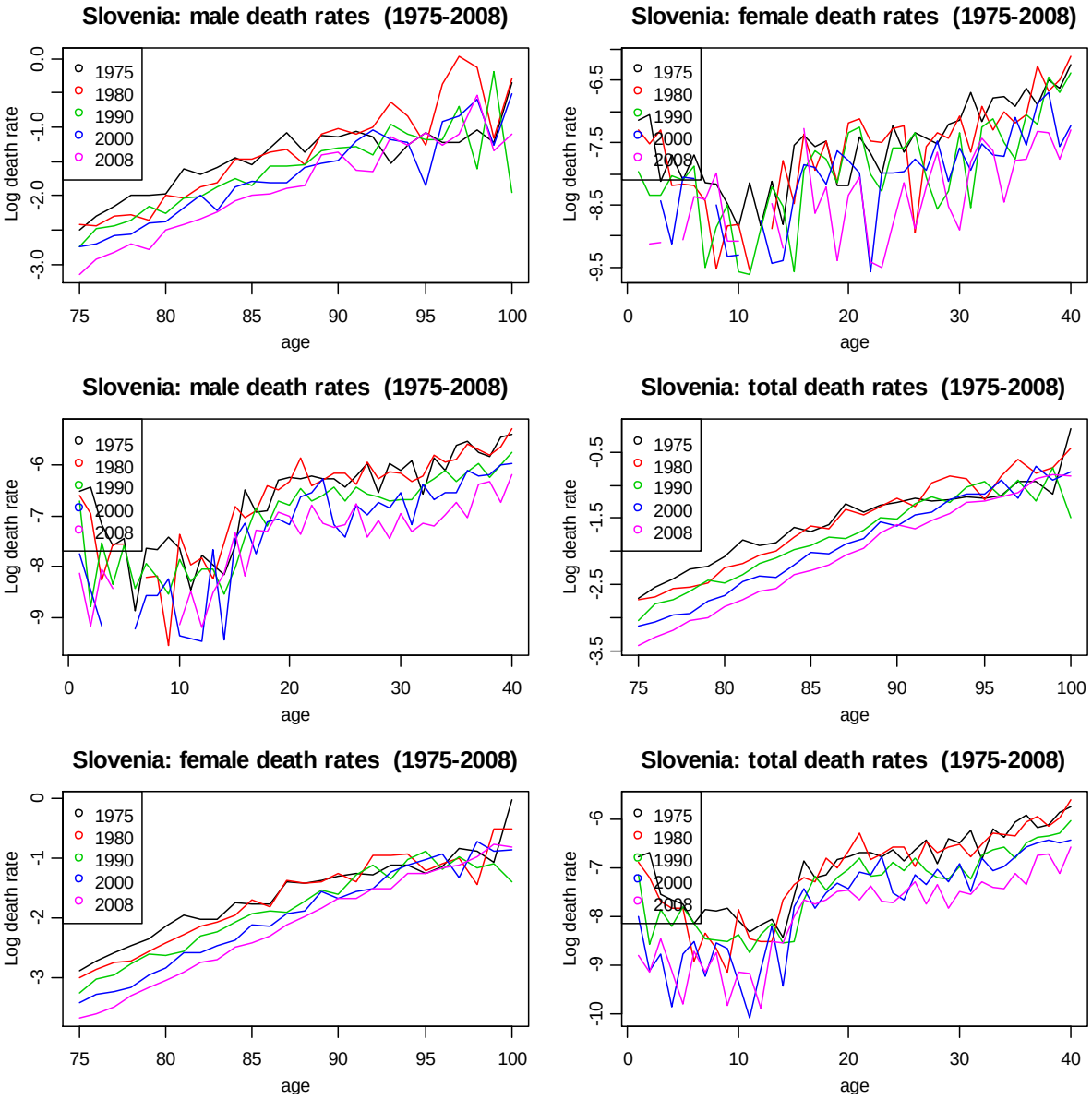
3.1.3 Mortality by cause

Data for mortality by cause analysis were obtained from the Institute of Public Health of the Republic of Slovenia (IVZ – *Inštitut za varovanje zdravja*) database. Data were provided for the time span from 1971 to 2008, for five-year age groups, separately for men and women. We checked the total population mortality data grouped in the five-year groups and compared these totals with the totals of mortality by cause. The comparison only revealed minor discrepancies, mostly for the years 1971 to 1980. A bigger discrepancy (57 with men and 59 with women) was found for the year 1982.

3.2 Central death rate

Let us denote with $ETR_{x,t}$ the exposure to risk at age *x* last birthday during year *t*. The exposure to risk refers to the total number of persons-years in a given population over a calendar year and is estimated by the number of the population aged *x* in the middle of the calendar year (namely on 1 July of each year), meaning those who reached age x between 1 July of the previous year and 30 June of the observing year.

Figure 3-1 Slovenia mortality data in selected years between 1975 and 2008



Let us denote with $D_{x,t}$ the number of deaths recorded at age *x* last birthday during calendar year *t*. Then, the maximum likelihood estimator for $\hat{\mu}_x(x)$ (force of mortality) equals:

$$\hat{\mu}_x(t) = \frac{D_{x,t}}{ETR_{x,t}} \tag{3.1}$$

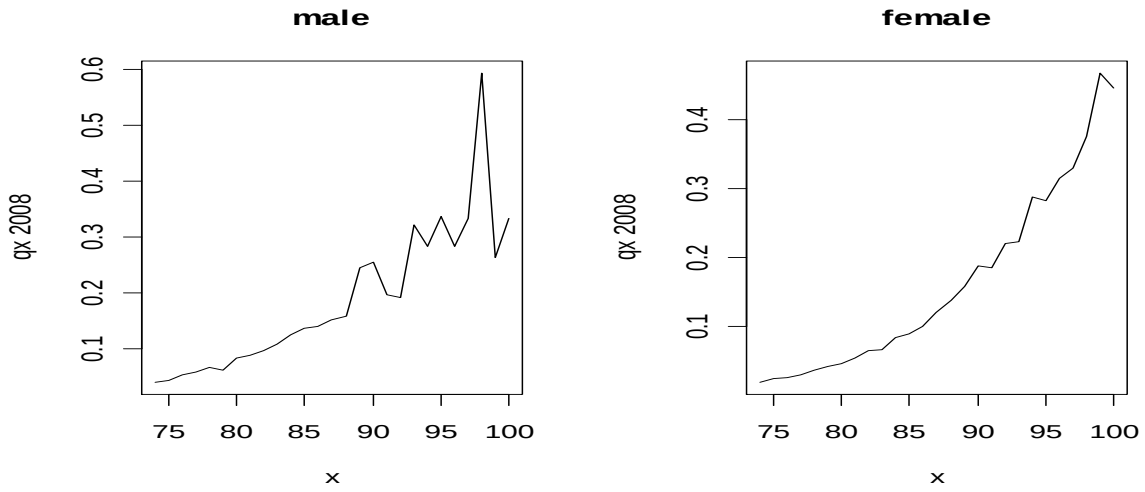
If we assume a constant force of mortality for non-integer years we have $\hat{\mu}_x(t) = \hat{m}_x(t)$. With this assumption we construct Slovenian mortality data for further analysis.

As Figure 3-1 shows, the improvements in mortality are highest for younger ages and lowest for older ages. Compared to other countries, the shape of the mortality curve exhibits a similar characteristic with a small hump present for males aged 18 to 20 years. This hump is less present in the case of females. This conclusion can be drawn by looking at the figures for raw data, although the conclusions are less robust at old ages due to the small exposure.

3.3 Mortality at very old ages

Slovenian population mortality data at very old ages have very low risk exposures, leading to large sampling errors and highly volatile crude death rates (see Figure 3-2). For example, risk exposures for males vary between 567 in 1971 and 1300 in 2007. For 1971–1980 the data for age groups above 85 are not available at all. Therefore, we need a method that can extrapolate a survival function at very old ages, without requiring accurate mortality data for that part of the population.

Figure 3-2 High volatility at very old ages



Several mathematical models have been developed to express mortality as a function of age. Most models (including the well-known Gompertz-Makeham model and Weibull model) are concentrated on describing adult mortality only. The mortality curve at very old ages suggests that the death rate increases exponentially with age. This law was first proposed by the British actuary Benjamin Gompertz. In the Gompertz model, the force of mortality is expressed in the form $\mu_x = Bc^x$.

Recent mortality studies suggested that the force of mortality is slowly increasing at very old ages, approaching a relatively flat shape (see Pitacco et al., 2010). In other words, the exponential rate of mortality increase at very old ages is not constant (as, for example, in Gompertz's model), but declines.

We apply the method proposed by Denuit and Goderniaux (2005) to extrapolate death rates at very old ages. Following this approach, the death rates for very old ages were estimated according to the logistic formula proposed below. Parameters were chosen in a way to maximise the fit.

The log-quadratic regression model is defined as

$$\ln \hat{q}_x(t) = a_t + b_t x + c_t x^2 + \varepsilon_{xt} \tag{3.2}$$

where one-year death probability at time t with ε_{xt} is independent and normally distributed with mean 0 and variance σ^2. If ω is limit age, then we have an additional constraint:

$$q_\omega(t) = 1 \tag{3.3}$$

To ensure the concave behaviour of $\ln \hat{q}_x(t)$ we implement a second constraint

$$\frac{\partial}{\partial x} q_x(t)\Big|_{x=\omega} = 0 \tag{3.4}$$

This two constraints yield the following regression model

$$\ln \hat{q}_x(t) = c_t(\omega - x)^2 + \varepsilon_{xt} \tag{3.5}$$

with $t = 1971,\ldots,2007$ and $x = x_t^{SA}, x_t^{SA}+1,\cdots$. We estimated c_t with a log-quadratic regression on the basis of set $\{\hat{q}_x(t)\}$ $x = x_t^{SA}, x_t^{SA}+1,\cdots$. As Figure 3-3 demonstrates, we obtain the optimal fit (highest R^2) with $\omega = 130$ and starting smoothing age $x_t^{SA} = 75$. We use 85 as an age for extrapolating mortality (as an example).

Figure 3-3 Testing R2

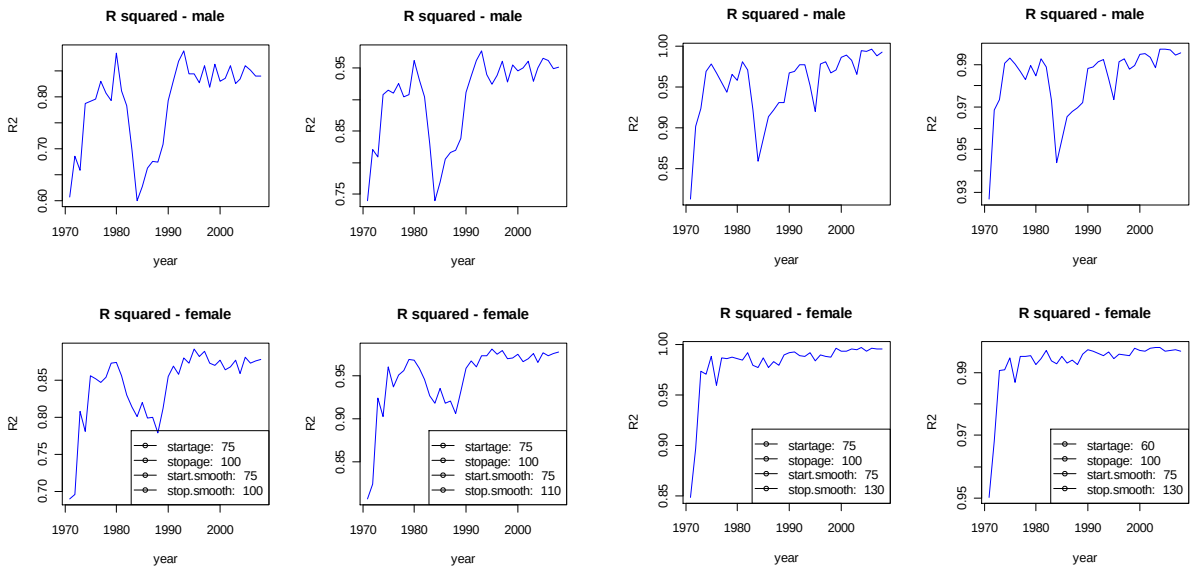
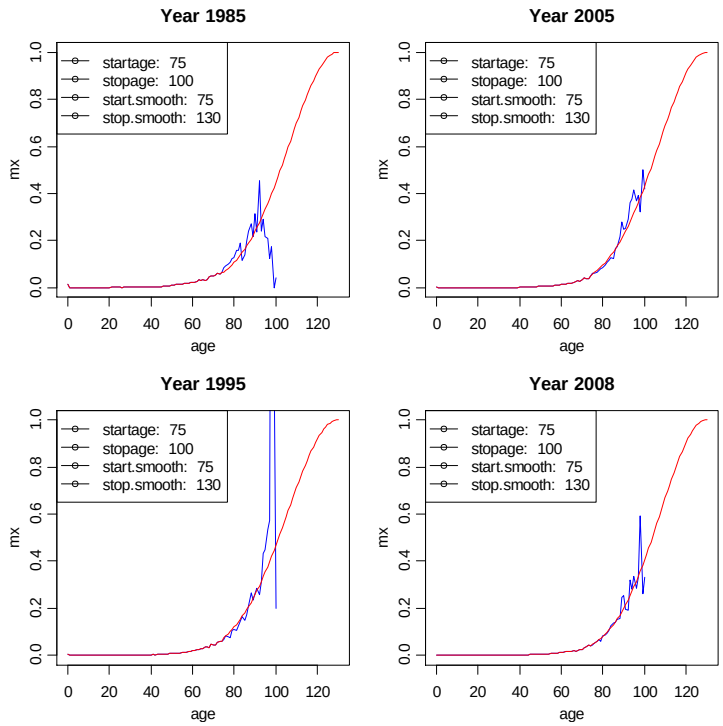


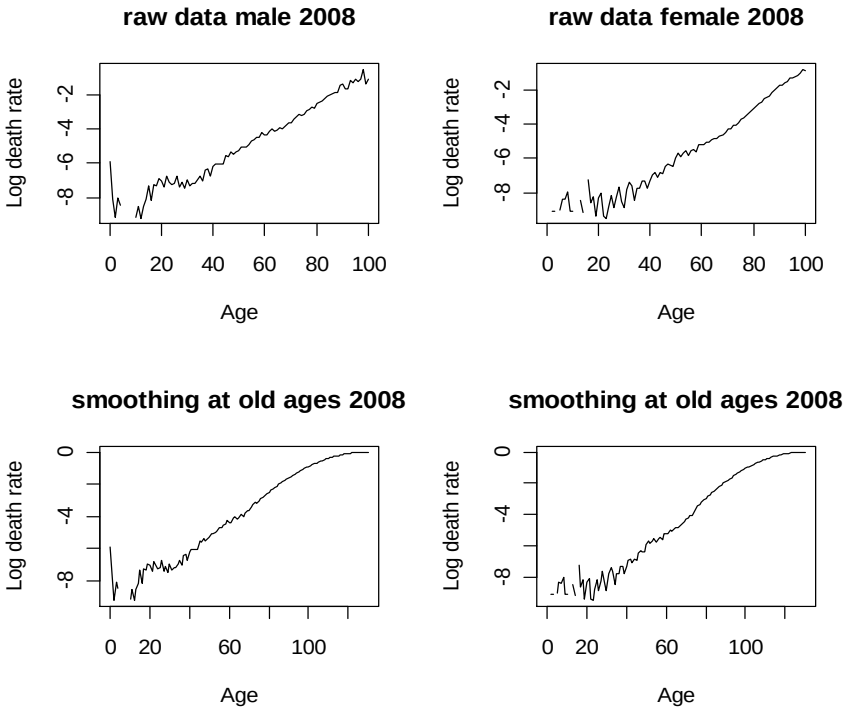
Figure 3-4 presents the extrapolation for 2008 in which the logistic nature of the extrapolated function at high ages can be seen.

Figure 3-4 Smoothing at very old ages



In Figure 3-5 we present smoothed Slovenian mortality data, which extend the data set up to age 130. Since at lower ages some central death rates are equal to zero, we also implement a smoothing procedure for lower ages, this time with m-splines. The results will be presented in the following section. The program for implementing the logistic formula was written in R.

Figure 3-5 Smoothed data at very old ages: males and females



3.4 Smoothing mortality data with splines

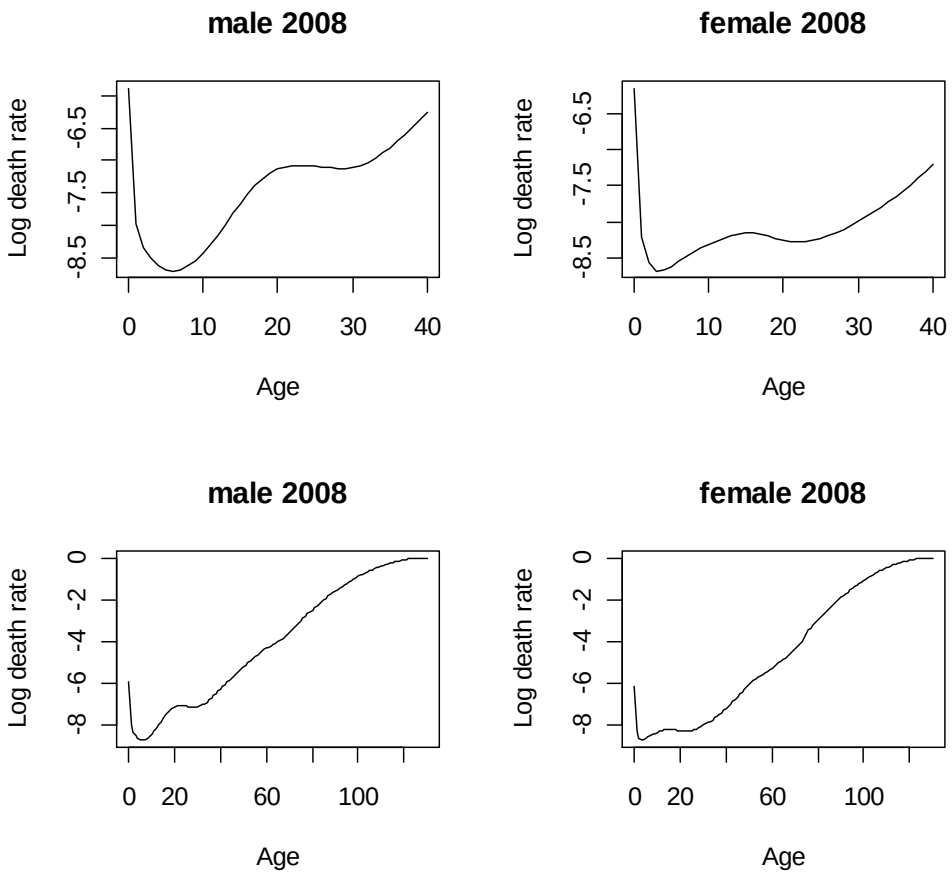
Slovenian population death rates exhibit considerable variations as seen in Figure 3-1. We therefore use smoothing techniques to obtain a better picture of the underlying mortality. We use weighted penalised regression splines with a monotonicity constraint proposed by Hyndman and Ullah (2007). We use code already available in the R library.

Let $y_t(x)$ denote the log of the observed mortality for age *x* in year *t*. We assume there is an underlying smooth function $f_t(x)$, such that at discrete points $\{x_i, y_t(x_i)\}$, $i = 1,\ldots, p$ we have

$$y_t(x_i) = f_t(x_i) + \sigma_t(x_i)\varepsilon_{t,i} \tag{3.6}$$

$\varepsilon_{t,i}$ is a standard normal random variable and $\sigma_t(x_i)$ allows the amount of noise to vary with x. The task is to smooth the data for each *t* using a nonparametric smoothing method to estimate $f_t(x_i)$ for *x* from $\{x_i, y_t(x_i)\}$. The smoothing is done with constrained and weighted penalised regression splines. Weighting eliminates heterogeneity due to $\sigma_t(x_i)$. We assume that $f_t(x_i)$ is monotonically increasing for *x* > 60. This monotonicity constraint reduces the noise in the estimated curves in high ages.

Figure 3-6 Smoothing Slovenian mortality data

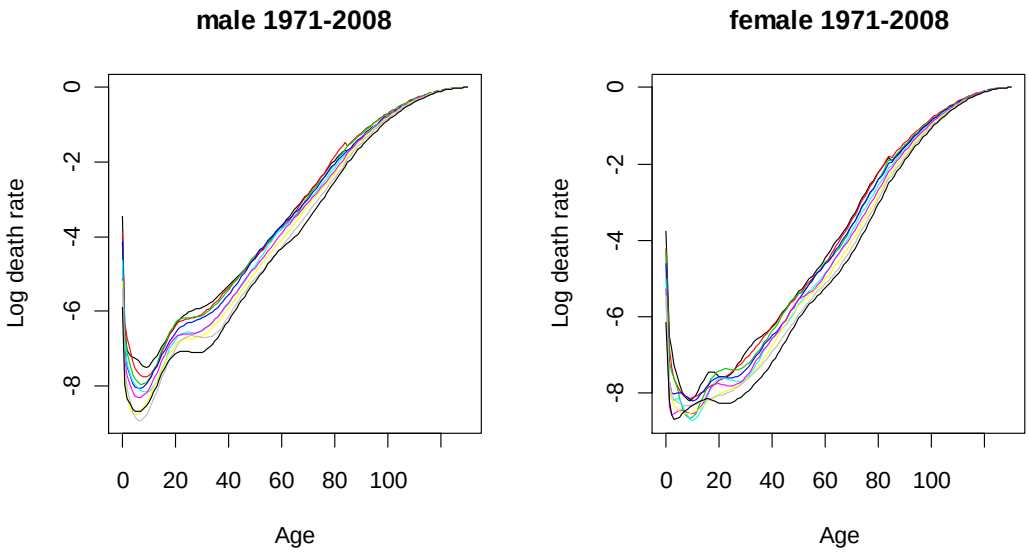


The results are shown in Figure 3-6. As one can see, a hump in mortality profiles for younger males is evident in contrast to the female mortality profile. This is mainly due to accidents, which more often occur to males at younger ages. At higher ages a concave shape of mortality is evident, as we would expect due to the smoothing at higher ages.

The mortality data obtained by the smoothing procedure explained in this section will be used in the forecasting.

In Figure 3-7, smoothed Slovenian death rates are presented for the period from 1971 up to 2008. The curves for this period follow from left to the right, indicating an improvement (decline) of mortality during that period. Not all the results (curves) are presented for this period – only for every 5 years (1971, 1976 ... 2008) to increase the clarity of the picture.

Figure 3-7 Mortality profile – smoothed data



3.5 Preparing mortality data for forecasting

3.5.1 Exposure to risk

The Slovenian mortality data had some irregularities that needed to be adjusted before we could use the data for forecasting. For this purpose, we employed the techniques presented in the previous sections.

Some deaths rates were equal to 0 (meaning there were no deaths in the observed period), which happens quite often at younger ages due to the small population. Since for forecasting we use logarithms of death rates, adjustment techniques were implemented to obtain positive values. In particular, we used interpolation techniques with neighbour central death rates to obtain the best estimate for such cases. At very old ages we observed two types of irregularities: first, the population at some very old ages was equal to 0 and, second, at some ages the number of deaths exceeded the total number of the population of the same age in the middle of the year. We extrapolated mortality rates at very old ages with the logistic formula explained in Section 3.3.

The following procedure was implemented to derive $m_x(t)$

1. $ETR_{x,t}$ = size of the population at 1 July of each year, $D_{x,t}$ = the number of observed deaths in year *t* at age *x*
2. $m_x(t) = \frac{D_{x,t}}{ETR_{x,t}}$
3. We used the following procedure to prepare basic raw data:
    a. replacing $m_x(t)$ which are NA with zero
    b. smoothing $m_x(t)$ at a very old age (above 85) – a regression with a logistic function from 75 with $q_x(t) = m_x(t) / (1 + 0.5 m_x(t))$ at limit age 130 – >$R^2 \cong 0.97$
    c. reverse back to $m_x(t) = q_x(t) / (1 - 0.5 q_x(t))$
    d. cut to upper age 100
    e. where $m_x(t) = 0$: interpolate $m_x(t)$ with neighbouring values; i.e. $m_x(t-s)$ and $m_x(t+k)$, if $m_x(t-s)$ and $m_x(t+k)$ >0 for the first *k* and *s,* and predict if 0 at the beginning or end of the time series
    f. leave the population data as original
    g. fix $D_{x,t}$ number as $D_{x,t} = ETR_{x,t} \cdot m_x(t)$ for ages over 85, otherwise $D_{x,t}$ as observed
    h. this data set is then considered raw data for further research.
4. Data used for the Lee-Carter method: $m_x(t)$ – smoothed with m-splines, weights are $D_{x,t}$ and $ETR_{x,t}$
5. Other methods: $D_{x,t}$ and $ETR_{x,t}$

The code for preparing the data was implemented in R.

3.5.2 Interpolation between 1945 and 1970

In order to build a life table in one cohort (say 1965), we need an assumption of $m_x(t)$ prior to 1971 since we only have data for each period from 1971 to 2008. The choice of methods should depend on the likely behaviour of mortality in Slovenia in the 1945–1970 period. In any case, since in many European countries the most important changes in the age pattern of mortality took place in the last decades of the 20th century, which also holds for Slovenia, the assumption of constant $m_x(t)$ could be accepted as a first estimation.

In the human mortality database we can find average central mortality rates for the years 1930–1933, 1948–1952, 1952–1954, 1960–1962. We will use this information to interpolate $m_x(t)$ for years from 1970 to 1945.

We used a log linear interpolation to interpolate the missing $m_x(t)$ in the 1945 to 1970 period. We calculated a log regression line between 1932 and 1985 and made an interpolation between 1945 and 1970 using a 95% confidence interval.

The estimated $m_x(t)$ from 1945 to 1970 will not be used for the projections, but are only used to construct the base cohort life table and to calculate cohort life expectancy. The results are presented in Figure 3-8 and Figure 3-9.

Figure 3-8 Central death rates from 1945 to 1970 – males

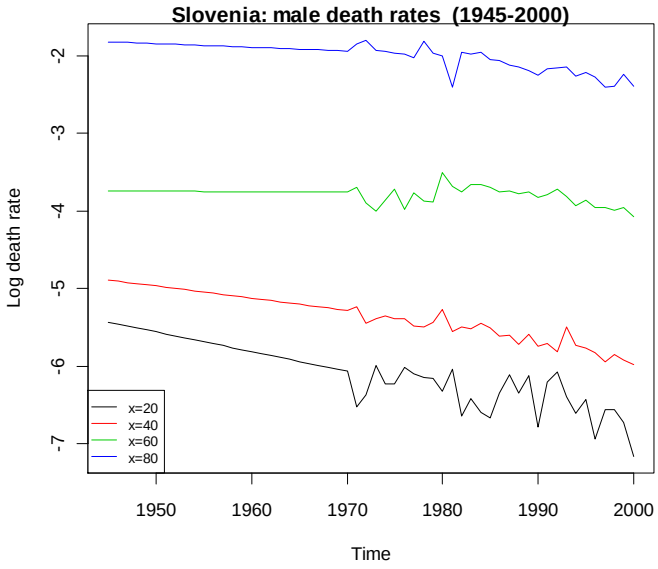
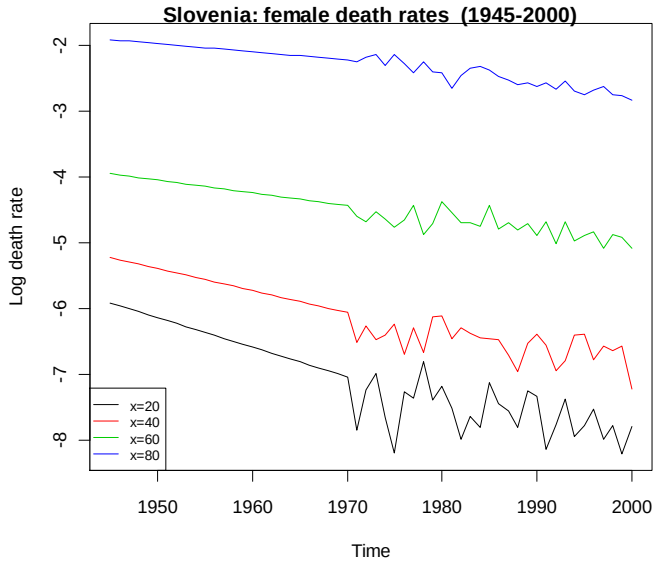


Figure 3-9 Central death rates from 1945 to 1970 – females



4 LEE-CARTER MODELS AND EXTENSIONS – THEORETICAL FRAMEWORK

The Lee-Carter (LC) method and its extensions are a powerful approach to mortality projections, as it combines a demographic model with a time-series model. In a stochastic framework, the results of LC projections consist of point and interval estimates. In this respect, the LC method allows for uncertainty in forecasts. This chapter explains the mathematical backgrounds of the basic Lee-Carter, Poisson log-bilinear and APC models. It also explains how models are fitted with observed data and how one can project mortality based on derived parameters.

4.1 The Lee-Carter model

In 1992 Lee and Carter established the standard for modelling longevity. Namely, in that year they proposed a model with three parameters that was very successful in explaining most of the variability of the central death rate. Lee and Carter proposed to model the central death rate as a bilinear model with an error term

$$\ln m_x(t) = \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t} \tag{4.1}$$

with α_x describing the age pattern of mortality averaged over time, and β_x describing the deviation from the average pattern when κ_t varies. Finally, κ_t gives the evolution of the level of mortality over time. $\varepsilon_{x,t}$ is the error term, which reflects the age-specific influences not captured by the model. It is assumed that the error term has a mean 0 and standard deviation σ_ε.

The usual approach to estimating the parameters is to use the least squares method. Further, one must impose additional constraints to obtain a unique solution. The usual approach is to assume

$$\sum_x \beta_x = 1 \tag{4.2}$$

$$\sum_t \kappa_t = 0 \tag{4.3}$$

which in turn forces α_x to be an average of the log central death rates over calendar years. Once the parameters α_x, β_x and κ_t are estimated with $\widehat{\alpha}_x$, $\widehat{\beta}_x$ and $\widehat{\kappa}_t$, we can forecast mortality by modelling the values of κ_t in future as a time series, for example, as a random walk with a drift or an ARIMA model.

4.2 The Poisson log-bilinear model

As noted by several authors, the Lee-Carter method assumes that random errors are homoscedastic. Namely, the error terms are assumed to have finite variance and, with the assumption of normality, share the same underlying probability density function. In the majority of cases this assumption is violated since the logarithm of the observed mortality rate has much greater variability at older ages than at younger ages. Thus, it makes sense to assume that the number of deaths follows a Poisson law with parameter

$$D_x(t) \propto Poisson(ERT_x(t)\mu_x(t)) \tag{4.4}$$

where $ERT_x(t)$ is the central number of exposed to risk and $\mu_x(t)$ is the force of mortality. The log of the force of mortality equals

$$\ln \mu_x(t) = \alpha_x + \beta_x \kappa_t \tag{4.5}$$

as in the LC model. The parameters have a similar meaning as in the LC model. The values of the parameters are determined by maximising the log-likelihood.

4.3 The APC model

One of the shortcomings of the one-factor Lee-Carter model is that we ignore the cohort effect, which may be significant in some cases (see Pittaco et al., 2009, p. 255).

Thus, Renshaw and Haberman (2006) considered a model for the force of mortality that adds an additional term to the Lee-Carter framework

$$\ln \mu_x(t) = \alpha_x + \beta_x^0 i_{t-x} + \beta_x^1 \kappa_t \tag{4.6}$$

With the related mortality-reduction factor

$$RF(x,t) = \exp(\beta_x^0 i_{t-x} + \beta_x^1 \kappa_t) \tag{4.7}$$

where α_x, β_x^0, β_x^1, i_{t-x} and κ_t are parameters of the model. In contrast to Lee-Carter, there are two additional parameters β_x^0 and i_{t-x} that capture the cohort effect. The first term β_x^0 captures the contribution of different ages within the cohort effect, whereas i_{t-x} gives the overall effect on reduced/increased mortality predicted by Lee-Carter from cohort *t-x* (those born in year *t-x*).

4.4 Methodology – econometrics, fitting the model

In this section we show how to estimate the parameters of the models presented in the previous section.

4.4.1 Fitting the original LC model

The simplest way to estimate the parameters of the LC model with white noise is to use the approach proposed by Lee and Carter. They estimate α_x by averaging log-rates over time and β_x and κ_t via a singular value decomposition of the residuals, essentially a method for approximating a matrix as the product of two vectors.

More precisely, alpha is given by

$$\overline{\alpha_x} = \frac{1}{t_n - t_1 + 1} \sum_{t=t_1}^{t_n} \ln \overline{m_x(t)} \tag{4.8}$$

Where $\overline{\alpha_x}$ is the best estimate of the average of the log central death rates over calendar years. The estimates of β_x and κ_t are obtained from the eigenvectors of *Z* with *Z* defined as

$$Z = \ln \overline{M} - \overline{\alpha} \tag{4.9}$$

and *M* equal to

$$M = \begin{pmatrix} m_{x_1}(t_1).....m_{x_1}(t_n) \\ . \\ . \\ m_{x_m}(t_1)....., m_{x_m}(t_n) \end{pmatrix} \tag{4.10}$$

Now $\overline{\beta}$ is equal to

$$\vec{\beta} = \frac{\vec{v}_1}{\sum_{j=1}^{x_m - x_1 + 1} v_{1j}} \tag{4.11}$$

and $\overline{\kappa}$ is equal to

$$\vec{\kappa} = \sqrt{\lambda}\vec{u}_1 \sum_{j=1}^{x_m - x_1 + 1} v_{1j} \tag{4.12}$$

with

$$Z = \sqrt{\lambda}\vec{u}_1\vec{v}_1 \tag{4.13}$$

4.4.2 Fitting the Poisson log-bilinear model

In the case of the Poisson response model, we adopt the following iterative procedure given in Brouhns et al. (2002). The values of parameters α_x, β_x and κ_t are estimated with $\overline{\alpha}_x$, $\overline{\beta}_x$ and $\overline{\kappa}_t$, obtained by maximising the log-likelihood.

$$L(\vec{\alpha}, \vec{\beta}, \vec{\kappa}) = \sum_{x,t} D_x(t)(\alpha_x + \beta_x\kappa_t) - ERT_x(t)\exp(\alpha_x + \beta_x\kappa_t)) \tag{4.14}$$

Because of the presence of the bilinear term, it is impossible to estimate the proposed model with commercial statistical packages that implement the Poisson regression. One of the paths to obtaining the estimates is to use the method proposed by Goodman (1979). He proposed the iterative method for estimating log-linear models with bilinear terms as follows.

Within this approach, we define the starting values for parameters as $\overline{\alpha_x^0} = 0, \overline{\beta_x^0} = 0, \overline{\kappa_t^0} = 0$. The parameters are then estimated using the following iteration:

$$\begin{aligned}
\overline{\alpha_x^{n+1}} &= \overline{\alpha_x^n} + \frac{\sum_t (D_{xt} - \overline{D_{xt}^n})}{\sum_t \overline{D_{xt}^n}}, \quad \overline{\beta_x^{n+1}} = \overline{\beta_x^n}, \overline{\kappa_t^{n+1}} = \overline{\kappa_t^n} \\
\overline{\kappa_t^{n+2}} &= \overline{\kappa_t^{n+1}} + \frac{\sum_t (D_{xt} - \overline{D_{xt}^{n+1}})\overline{\beta_x^{n+1}}}{\sum_t (\overline{\beta_x^{n+1}})^2 \overline{D_{xt}^n}}, \quad \overline{\beta_x^{n+2}} = \overline{\beta_x^{n+1}}, \overline{\alpha_x^{n+2}} = \overline{\alpha_x^{n+1}} \\
\overline{\beta_x^{n+3}} &= \overline{\beta_x^{n+2}}, + \frac{\sum_t (D_{xt} - \overline{D_{xt}^{n+2}})\overline{\kappa_t^{n+2}}}{\sum_t (\overline{\kappa_t^{n+2}})^2 \overline{D_{xt}^{n+2}}}, \quad \overline{\kappa_t^{n+3}} = \overline{\kappa_t^{n+2}}, \overline{\alpha_x^{n+3}} = \overline{\alpha_x^{n+2}}
\end{aligned} \tag{4.15}$$

with

$$\overline{D_{xt}^n} = ERT_x(t)\exp(\alpha_x^n + \overline{\beta_x^n} * \overline{\kappa_t^n})) \tag{4.16}$$

4.4.3 Fitting the APC model

In the case of the APC model with a Poisson error structure, the procedure for finding the values of the parameters is a little more complicated. The three factors of the model are constrained by the relationship

Cohort = period - age

The starting point in this case is to estimate alpha by

$$\overline{\alpha_x} = \frac{1}{t_n - t_1 + 1}\sum_{t=t_1}^{t_n} \ln \overline{m_x(t)} \tag{4.17}$$

Now, using the extended definition of $\overline{D_{xt}^n}$

$$\overline{D_{xt}^n} = ERT_x(t)\exp(\alpha_x^n + \overline{\beta_x^1}\overline{\kappa_x} + \overline{\beta_x^0}\overline{i_{t-x}}) \tag{4.18}$$

we can use a similar iterative procedure as in the case of the Poisson response model (Brouhns et al. 2002)

$$\overline{i_x^{n+1}} = \overline{i_x^n} + \frac{\sum_t (D_{xt} - \overline{D_{xt}^n})\overline{\beta_x^{0,n+1}}}{\sum_t \overline{(\beta_x^{0,n+1})^2} D_{xt}^n}, \overline{\beta_x^{1,n+1}} = \overline{\beta_x^{1,n}}, \overline{\kappa_t^{n+1}} = \overline{\kappa_t^n}, \overline{\beta_x^{0,n+1}} = \overline{\beta_x^{0,n}}$$

$$\overline{\beta_x^{0,n+2}} = \overline{\beta_x^{0,n+1}}, + \frac{\sum_t (D_{xt} - \overline{D_{xt}^{n+1}})\overline{i_x^{n+2}}}{\sum_t \overline{(i_x^{n+2})^2} \overline{D_{xt}^{n+1}}}, \overline{\kappa_t^{n+2}} = \overline{\kappa_t^{n+1}}, \overline{i_x^{n+2}} = \overline{i_x^{n+1}}, \overline{\beta_x^{1,n+2}} = \overline{\beta_x^{1,n+1}},$$

$$\overline{\beta_x^{n+3}} = \overline{\beta_x^{n+2}}, + \frac{\sum_t (D_{xt} - \overline{D_{xt}^{n+2}})\overline{\kappa_t^{n+2}}}{\sum_t \overline{(\kappa_t^{n+2})^2} \overline{D_{xt}^{n+2}}}, \overline{\kappa_t^{n+3}} = \overline{\kappa_t^{n+2}}, \overline{i_x^{n+3}} = \overline{i_x^{n+2}}, \overline{\beta_x^{0,n+3}} = \overline{\beta_x^{0,n+2}} \quad (4.19)$$

$$\overline{\kappa_t^{n+4}} = \overline{\kappa_t^{n+3}} + \frac{\sum_t (D_{xt} - \overline{D_{xt}^{n+3}})\overline{\beta_x^{1,n+1}}}{\sum_t \overline{(\beta_x^{1,n+3})^2} \overline{D_{xt}^{n+1}}}, \overline{\beta_x^{1,n+4}} = \overline{\beta_x^{1,n+3}}, \overline{i_x^{n+4}} = \overline{i_x^{n+3}}, \overline{\beta_x^{0,n+4}} = \overline{\beta_x^{0,n+3}}$$

The starting values are

$$\overline{\beta_x^{0,0}} = 1, \overline{\beta_x^{1,0}} = 1, \overline{\kappa_t^0} = 0 \quad (4.20)$$

4.5 Projecting future mortality

In order to obtain estimates of future mortality, one needs to estimate the dynamics of kappa for both men and women (Lee-Carter, 1992, Maria Rusolilloo, 2005). As noted by several authors (Haberman, 2005; Lee, 2000), κ_t can be regarded as a stochastic process that can be modelled by fitting an ARIMA(p,d,q) model.

The dynamics of κ_t can thus be described as

$$\nabla^d \kappa_t = \varphi_1 \nabla^d \kappa_t + \ldots + \varphi_p \nabla^d \kappa_t + \xi_t + \psi_1 \xi_{t-1} + \psi_q \xi_{t-q} \quad (4.21)$$

In most instances, the appropriate time series model takes a simpler form such as

$$\kappa_t = \kappa_{t-1} + c + \varepsilon_t + \rho \varepsilon_{t-1} \quad (4.22)$$

Based on the results of the time series model we can obtain forecasts of future mortality and its moments

$$\mu_x(t+n) = \exp(\alpha_x) RF(t+n) \quad (4.23)$$

In the case of Lee-Carter, this translates to

$$RF(x,t) = \exp(\beta_x \kappa_{t+n}) \quad (4.24)$$

and in the case of the APC model to

$$RF(x,t) = \exp(\beta_x^0 i_{t+n-x} + \beta_x^1 \kappa_{t+n}) \quad (4.25)$$

Thus when projecting the values of kappa under different scenarios for obtaining the estimates of future mortality we can use the following relationship

$$m_{x,2008+i} = m_{x,2008} \exp(\beta_x(\kappa_{2008+i} - \kappa_{2008})) \quad (4.26)$$

Where $m_{x,2008+i}$ is the mortality factor for year (2008+*i*) and age *x*. The formula (4.26) is essentially an extrapolation of the classical Lee-Carter using the projections of kappa obtained from ARIMA models. In determining future mortality we have to take the uncertainty of our estimates into account. Thus, we construct three scenarios that differ with respect to the values of kappa used when making projections. Under the best estimate scenario, we determine future values of kappa by taking kappa to be equal to the expected value. In this case, using equation (4.26) future values of kappa are obtained by using the following relationship

$$\kappa_{2008+t} = \kappa_{2008} + ct \quad (4.27)$$

In the case of a high mortality scenario, future values of kappa are obtained by using the following relationship

$$\kappa_{2008+t} = \kappa_{2008} + ct + 2\sigma_\varepsilon \sqrt{t} \quad (4.28)$$

In the case of a low mortality scenario, future values of kappa are obtained by assuming lower than expected values of kappa. In this case, the future values of kappa are obtained by taking

$$\kappa_{2008+t} = \kappa_{2008} + ct - 2\sigma_\varepsilon \sqrt{t} \quad (4.29)$$

5 FORECASTING MORTALITY USING EXTRAPOLATION TECHNIQUES

General approaches to projecting age-specific mortality rates can be categorised in various ways; for example, as process-based, explanatory, forecasting methods or a combination of these techniques.

Process-based methods concentrate on the factors that determine deaths and attempt to model mortality rates from a bio-medical perspective (see Section 6.2). Nevertheless, process-based methods are not generally used to make projections, but to confirm extrapolative methods.

Explanatory-based methods use econometric techniques based on variables such as economic or environmental factors. We would like to employ these techniques to forecast mortality trends by cause. For example, Tabeau et al. (2001) describe attempts to model Dutch mortality using various explanatory variables. However, not much data are available for the Slovenian case to allow deaths to be categorised by risk factors. Thus, we present trends only for two important lifestyle variables for which data are publicly available.

Forecasting is the process of projecting mortality based on historical trends. Forecasting methods include some element of subjective judgment, for example, the type of underlying function, the time series we take into account etc. Simple forecasting methods (for example, exponential formula) are only usable in the sense that the pattern of changing mortality in the past will continue in the future.

Parametric methods involve fitting a parameterised curve to data for previous years and then projecting trends in these parameters forward. However, the shape of the curve may not continue to satisfactorily describe mortality in the future. These methodologies can be used to provide deterministic projections of mortality in the future.

With a model fitted to historical data, most methods can be adapted in some way to provide stochastic projections.

5.1 Period tables and cohort tables

A projected mortality table is a rectangular matrix

$$\begin{pmatrix} q_{x_{\min}}(t_0)\cdots & q_{x_{\min}}(t_n) & \cdots & q_{x_{\min}}(t_{\max}) \\ \vdots & & & \\ \vdots & & & \\ q_{x_{\max}}(t_0) & q_{x_{\max}}(t_n) & & q_{x_{\max}}(t_{\max}) \end{pmatrix} \tag{5.1}$$

where

$\{q_x(t)\}, t \in (t_0,\ldots,t_n)$ presents observed smoothed mortality data and $\{q_x(t)\}, t \in (t_n+1,\ldots,t_{\max})$ represents projected mortality data. With t_n we denote the base year from which projections are made. For Slovenian mortality projection data, the parameters are as follows

$$\begin{pmatrix} x_{\min} = 0 \\ x_{\max} = 100 \\ t_0 = 1971 \\ t_n = 2008 \\ t_{\max} = 2118 \end{pmatrix} \tag{5.2}$$

The sequence $q_x(t), q_{x+1}(t+1),\ldots$ is called a cohort table. The sequence $q_x(t), q_{x+1}(t), q_{x+2}(t)\ldots$ is called a period table.

In this respect, the probabilities concerning the lifetime of a person aged *x* for each year *t* is derived from the diagonal of matrix (5.1):

$$q_x(t), q_{x+1}(t+1),\ldots \tag{5.3}$$

5.2 Mortality improvement over time

It is interesting to see how mortality is improving over time. In Figure 5-1 we plot typical time series for Slovenian mortality data. As one can see, the mortality improvement for the 60+ population is much stronger than for the middle-aged population (as we could expect). This is not very obvious for females in this age group, probably because they already enjoy better mortality statistics than males. As we discussed earlier, improvement in mortality at very old ages is not very strong. From these charts one can conclude that the majority of the improvement seen in the last 30 years in Slovenia is due to an improvement in mortality in ages 60+. This is an important fact that has to be incorporated into the projections.

Figure 5-1 Mortality improvement over time - males

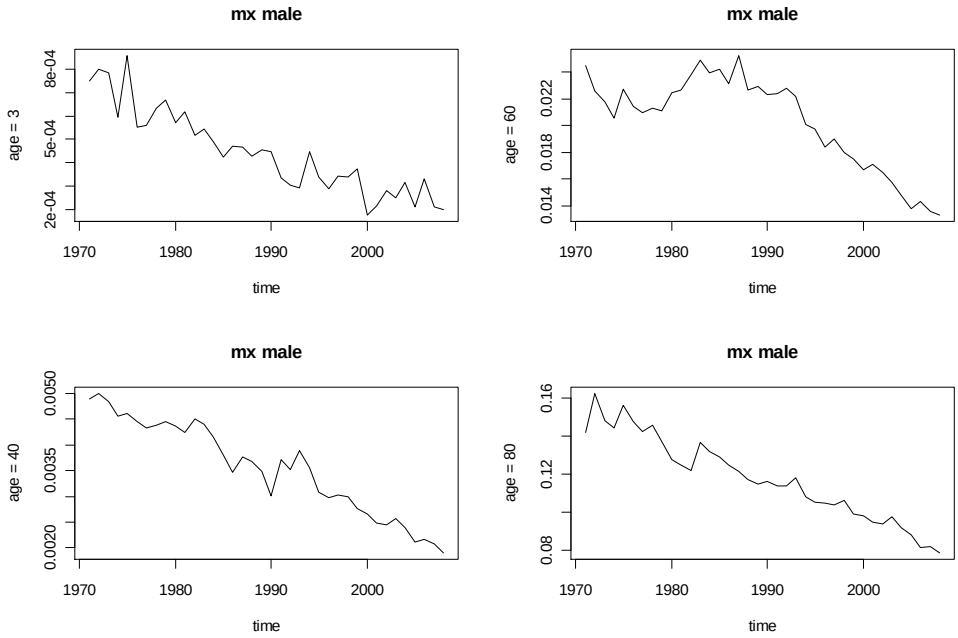
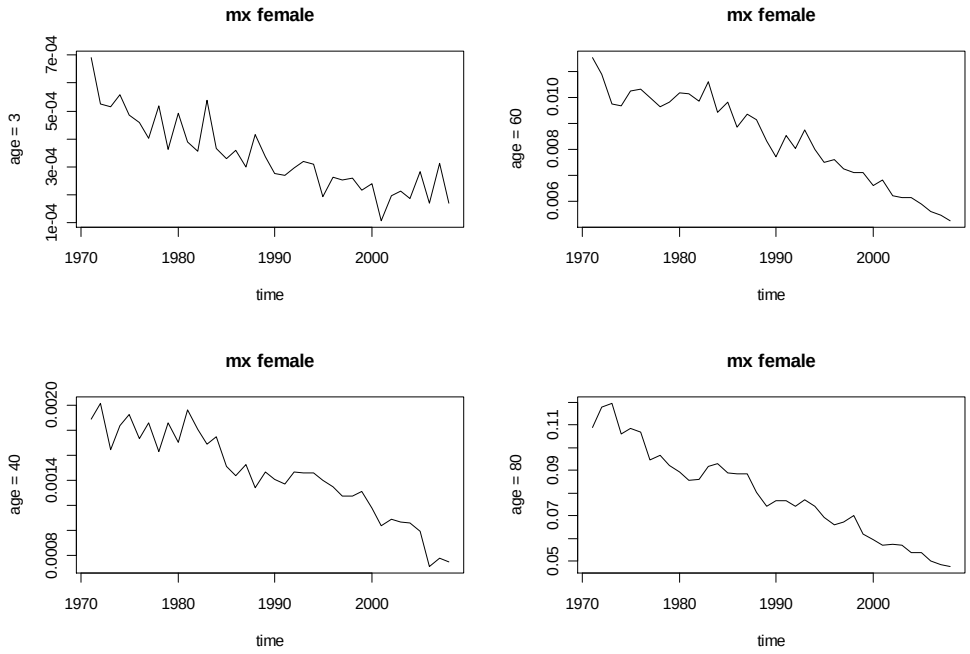


Figure 5-2 Mortality improvement over time - females



5.3 Extrapolation

Pure extrapolation of a time series assumes that all we need to know is contained in the historical values of the time series being forecasted. The main shortcoming of a time-series extrapolation is the assumption that nothing else besides the prior values of a series is relevant.

We will follow Pitacco's (2009) modelling of future mortality based on a reduction factor. Assuming that the mortality trend over time is decreasing, we define future mortality $q_x(t)$ at time *t* in respect of given starting year t_n as

$$q_x(t) = q_x(t_n)R_x(t - t_n). \tag{5.4}$$

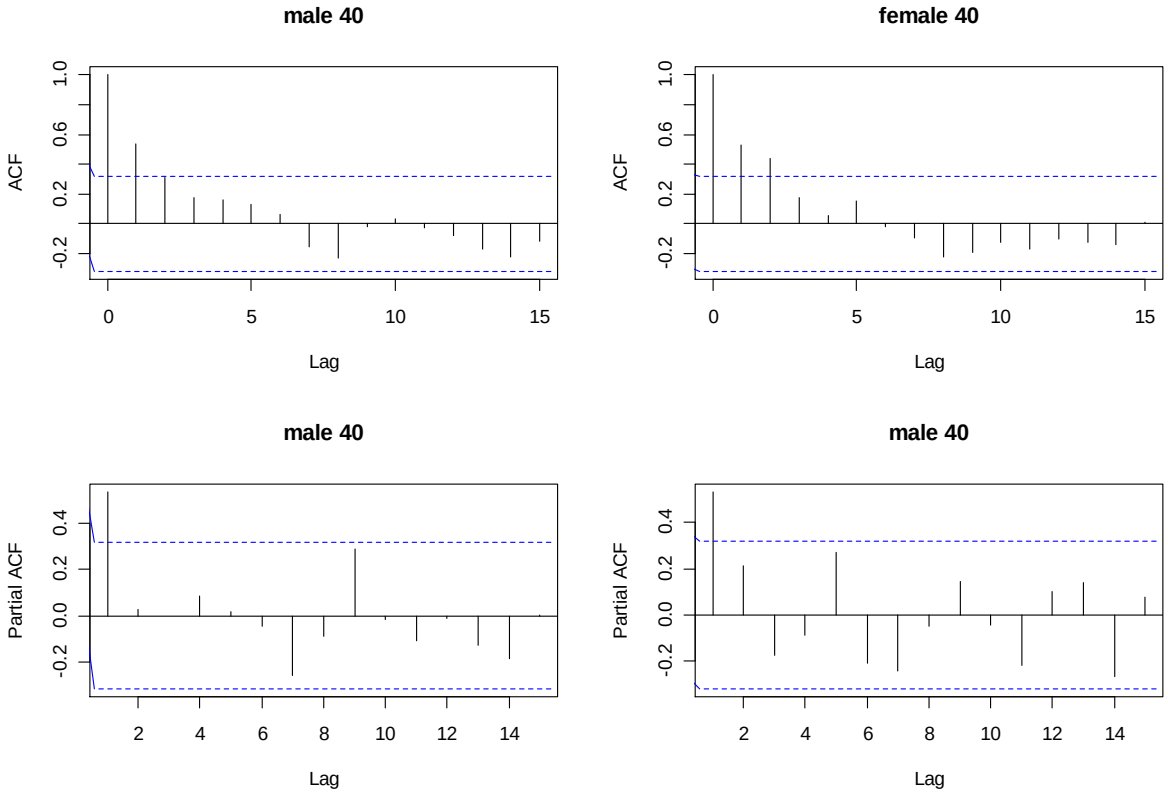
The quantity $R_x(t - t_n)$ is called reduction factor, as is expected to be less than 1. To project mortality in a deterministic context we use the exponential formula:

$$R_x(t - t_n) = e^{-\delta_x(t-t_n)} \tag{5.5}$$

where δ_x is derived from a least squares estimation for each *x*. In our case, we take $t_n = 2008$. The code for the extrapolation was implemented in the statistical package R.

Below we present correlograms of residuals for a typical age.

Figure 5-3 Correlogram of residuals



It is more realistic to limit mortality at arbitrary age *x* to positive values. To derive this, $R_x(t-t_0)$ is defined as

$$RF(x,t) = \alpha_x + (1-\alpha_x)(1-f_x)^{\frac{t}{20}} \tag{5.6}$$

where

$$f_x = \frac{q_x(t_n) - q_x(t_n+20)}{q_x(t_n) - q_x(\infty)} \tag{5.7}$$

The parameters of the model may be interpreted as follows:

(a) $\alpha_x \cdot q_x(t_n)$ is the ultimate rate of mortality at age *x* at infinity

(b) f_x is the proportion of the total mortality decline assumed to occur in the first 20 years

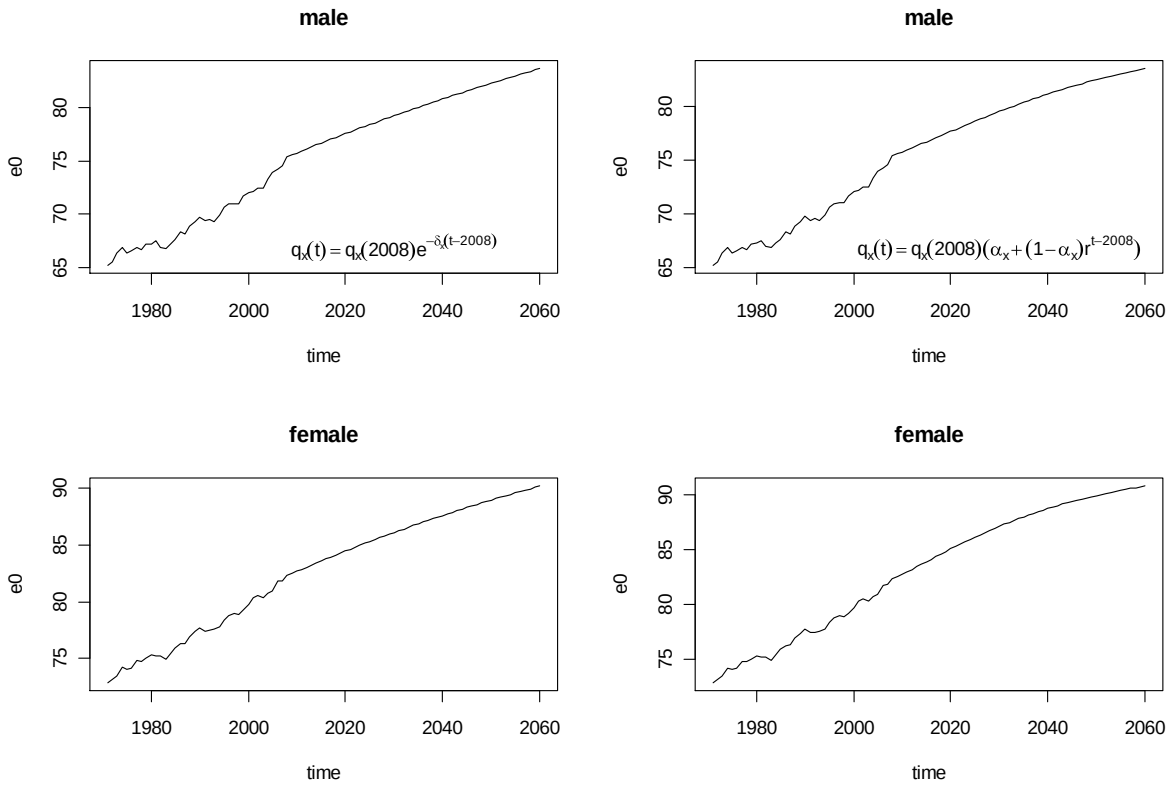
For the Slovenian population model we follow the Mortality Investigation Bureau (UK) approach and introduce

$$\alpha_x = \begin{cases} 0.4, & x < 60 \\ \dfrac{(110 - x)\cdot 0.4 + (x - 60)\cdot 0.2}{50}, & 60 \le x \le 110 \\ 0.2, & x > 110 \end{cases} \tag{5.8}$$

$$f_x = \begin{cases} 0.2, & x < 60 \\ 1 + 0.8 \cdot \dfrac{x - 110}{50}, & 60 \le x \le 110 \\ 1, & x > 110 \end{cases} \tag{5.9}$$

We changed the parameters proposed by the Mortality Investigation Bureau (1999) because life expectancy at birth in Slovenia is still lower than in the UK (so α_x should be lower and f_x higher than in the UK). In Figure 5-4 we can see the development of life expectancy using the exponential formula.

Figure 5-4 Deterministic approach for forecasting e0



It is apparent that life expectancy at birth (calculated on a "period" basis) does not differ much between the two methods.

Table 1: Forecast of life expectancy at birth

year	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
male - exp1	75.57	75.76	75.94	76.13	76.32	76.50	76.68	76.86	77.04	77.22
male - exp 2	75.56	75.74	75.92	76.10	76.29	76.48	76.66	76.85	77.05	77.24
female - exp1	82.49	82.68	82.86	83.05	83.23	83.41	83.59	83.77	83.94	84.12
female - exp 2	82.56	82.78	83.01	83.23	83.46	83.69	83.93	84.16	84.39	84.62
year	**2019**	**2020**	**2021**	**2022**	**2023**	**2024**	**2025**	**2026**	**2027**	**2028**
male - exp1	77.40	77.57	77.74	77.92	78.09	78.26	78.43	78.60	78.76	78.93
male - exp 2	77.43	77.62	77.82	78.01	78.20	78.39	78.58	78.77	78.95	79.14
female - exp1	84.29	84.46	84.63	84.80	84.97	85.13	85.30	85.46	85.62	85.78
female - exp 2	84.85	85.08	85.31	85.53	85.75	85.96	86.17	86.38	86.58	86.78
year	**2029**	**2030**	**2031**	**2032**	**2033**	**2034**	**2035**	**2036**	**2037**	**2038**
male - exp1	79.09	79.26	79.42	79.58	79.74	79.90	80.06	80.22	80.37	80.53
male - exp 2	79.32	79.50	79.67	79.85	80.02	80.18	80.35	80.51	80.67	80.82
female - exp1	85.94	86.10	86.25	86.40	86.56	86.71	86.86	87.01	87.15	87.30
female - exp 2	86.97	87.16	87.35	87.52	87.70	87.87	88.03	88.19	88.35	88.50
year	**2039**	**2040**	**2041**	**2042**	**2043**	**2044**	**2045**	**2046**	**2047**	**2048**
male - exp1	80.68	80.84	80.99	81.14	81.29	81.44	81.59	81.73	81.88	82.02
male - exp 2	80.98	81.13	81.27	81.41	81.55	81.69	81.82	81.96	82.08	82.21
female - exp1	87.44	87.59	87.73	87.87	88.01	88.14	88.28	88.42	88.55	88.68
female - exp 2	88.64	88.78	88.92	89.05	89.18	89.31	89.43	89.54	89.66	89.77
year	**2049**	**2050**	**2051**	**2052**	**2053**	**2054**	**2055**	**2056**	**2057**	**2058**
male - exp1	82.17	82.31	82.45	82.59	82.73	82.87	83.01	83.15	83.29	83.42
male - exp 2	82.33	82.45	82.56	82.68	82.79	82.89	83.00	83.10	83.20	83.30
female - exp1	88.81	88.94	89.07	89.20	89.33	89.45	89.57	89.70	89.82	89.94
female - exp 2	89.87	89.97	90.07	90.17	90.26	90.35	90.44	90.52	90.60	90.68
year	**2059**	**2060**								
male - exp1	83.56	83.69								
male - exp 2	83.39	83.49								
female - exp1	90.06	90.17								
female - exp 2	90.76	90.83								

6 PROJECTING CAUSE-SPECIFIC MORTALITY

The aim of this chapter is to present the mortality development by cause of death in Slovenia in the past, and to use these results to project future development in longevity. Using a deterministic approach, we produce life-tables for three different scenarios.

6.1 Cause-specific approach

An advantage of the cause-specific approach is that full information on behavioural and environmental changes, as well as expert medical knowledge, can be taken into account when projecting mortality from specific causes.

While the cause-specific approach is needed to investigate the effects of improvements in mortality from specific causes, there are a number of difficulties when using such approaches to project aggregate mortality (taken from CMI Report 39):

a) Deaths from specific causes are not always independent and the complex interrelationships are not always well understood. The same risk factor can affect several causes; for example, smoking affects both lung cancer and heart disease. States of health are very complex, particularly at older ages. If these complex inter-relationships are incorrectly modelled, the projected aggregate mortality can be seriously misestimated.

b) There is limited understanding of how various risk factors affect causes of death, making them difficult to model even at the population level. Even smoking prevalence is not always a good predictor of mortality.

c) The proportion of deaths due to a particular cause (the cause structure) shifts over time as a cause appears, peaks and then disappears. If we do not die for one particular reason, we will die

for some other reason. Therefore, the aggregate projected mortality improvements arising from cause-specific approaches will have a tendency to undershoot historical aggregate improvement rates.

d) The death of a very old person may have multiple causes, but only one cause may be recorded on the death certificate, or an incorrect or general cause may be recorded, leading to significant misclassification.

e) There may be causes of mortality at extreme old ages that have not yet been identified, as other, known, causes have resulted in deaths at earlier ages. If only the known causes of death are projected, future aggregate mortality would be underestimated.

6.2 Mortality trends in a bio-medical perspective

This section was prepared on the basis of an interview with a medical doctor in which we asked questions regarding which risks of mortality will be most influential in modern society.

Infectious Diseases

Risks of mortality are represented by:

- influenza virus H1N1 and others – epidemics, pandemics;
- MRSA – nosocomial infections; and
- HIV, Hepatitis B and C infections – injecting drug use and sexual habits.

Respiratory Diseases

Risks are represented by:

- chronic obstructive pulmonary disease, asthma, pneumonia, especially in the elderly and in patients with other chronic diseases – smoking, occupational exposure, smog, appropriate care in nursing or rest homes, adequacy of home health care services, timeliness of hospitalisation in terms of the acute aggravation of a disease.

When people are confined to bed due to illness, they usually die from pneumonia or urinary tract infections. It is advisable to look at the correlation between the number of people in nursing homes and the quantity of home health care services.

External Causes

Risks are represented by:

- traffic accidents – alcohol, drugs, adequacy of roads, technology of motor vehicles, risky driving behaviour, preventive measures for safer driving, road visibility;
- suicide – unrecognised depression, loss of work, unemployment, divorce, partner conflict, financial issues, single life, low social network support, alcohol, drugs; and
- industrial accidents – change of technology in the industry, number of employees, safety measures and regulations.

The reduction of traffic accident numbers has an important impact on the total reduction of death by external causes.

Neoplasms

Risks are represented by:

- various types of cancer: early identification of disease and adequate diagnosis is crucial, smoking, inadequate nutrition (too high energy food, too much fat, especially saturated fat, insufficient fruit, a small number of daily meals, poor nutrition rhythm), obesity, physical inactivity, introduction of preventive medical examinations.

The different types of cancer have to be examined in detail. For men:

- lungs: smoking, pneumoconiosis, especially in the past (working in mines, dust);
- colon: bad habits (too much meat, no vegetables; alcohol); and
- prostate: reduction due to diagnosis and prevention.

Endocrine Diseases

Risks are represented by:

- diabetes: a stressful lifestyle, inadequate nutrition, obesity, physical inactivity.

With endocrine diseases there has to be a great emphasis on disease surveillance. Death due to endocrine diseases is sex-dependent – women have a higher risk of thyroid disease.

Gastrointestinal Diseases

Risks are represented by:

- pancreatitis: inadequate nutrition, obesity, presence of gallstones; and
- chronic colitis complications, Crohn's disease, bleeding gastric ulcer: alcohol, inadequate nutrition, and obesity.

As with cancer these diseases occur due to poor nutrition and stress; fatal complications can arise from disorders of the gallstones not surgically removed, a bleeding ulcer etc.

Cardiovascular Diseases

Risks are represented by:

- inflammation of the heart muscle, cardiomyopathy with heart failure, heart rhythm disturbances, acute myocardial infarction, complication of hypertension, stroke – appropriate targeted preventive examinations, appropriate, timely and intensive treatment with aggressive surgical methods and statins, adequate diagnostics; and
- the impact of the following risk factors: inadequate nutrition, obesity, smoking, poorly treated diabetes, physical inactivity, high cholesterol and blood fat, alcohol, a stressful lifestyle.

Why is there a reduction of death for these reasons?

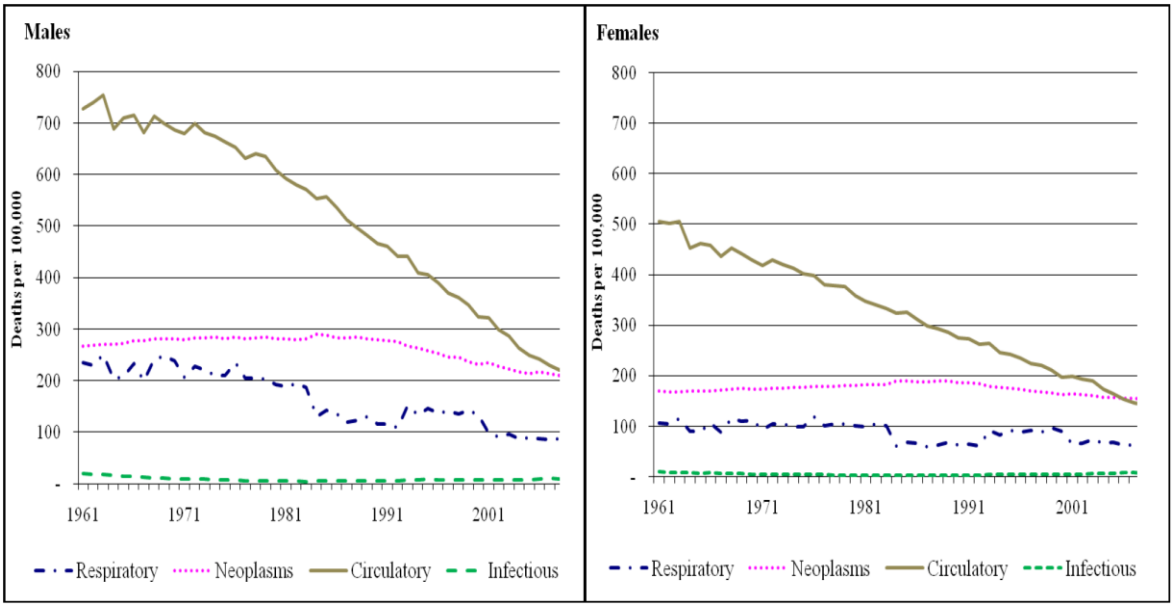
- Great attention to the treatment or monitoring of high blood pressure and strokes (at one time you could die from just one of these diseases, today you can survive many).
- Preventive measures – statins – although recent surveys show they do not have any real impact.
- Surgery: dilatation – cardiovascular surgery is so advanced that the risk of death is significantly reduced.
- A greater emphasis on nutrition in the case of a significant exposure to risk factors – this was not practiced in the past.
- Promotion of a healthy and active lifestyle.

6.3 Mortality trends for major cause groups of death

There are many factors that affect people's mortality. For example, there is a strong link between mortality and the socio-economic characteristics of individuals. Poorer, less-educated people are likely to die sooner. In addition, where people live can be an indicator of how long they may expect to live. However, in our analysis we do not go into these dimensions. Instead, we will try to shed some light on the development of mortality by cause of death.

We would like to analyse trends for major cause groups of death. The trends would rest on past trends of factors influencing mortality in major cause groups. A small number of major "cause of death groups" have contributed the bulk of improvements. While in the first part of the 20th century reductions in mortality were predominantly due to declining mortality from infectious diseases, later reductions were due to a decline in coronary heart disease (Ridsdale & Gallop, 2010a). Figure 6-1 shows the development in three major cause groups in England and Wales for 1961–2008. In addition, the category of infectious diseases is presented – having once been (and in less developed countries they still are) one of the leading causes of death, but in the last several decades they have only had a negligible share of total deaths in developed countries.

Figure 6-1: Mortality by major cause group in England and Wales, 1961 – 2008, age-standardised

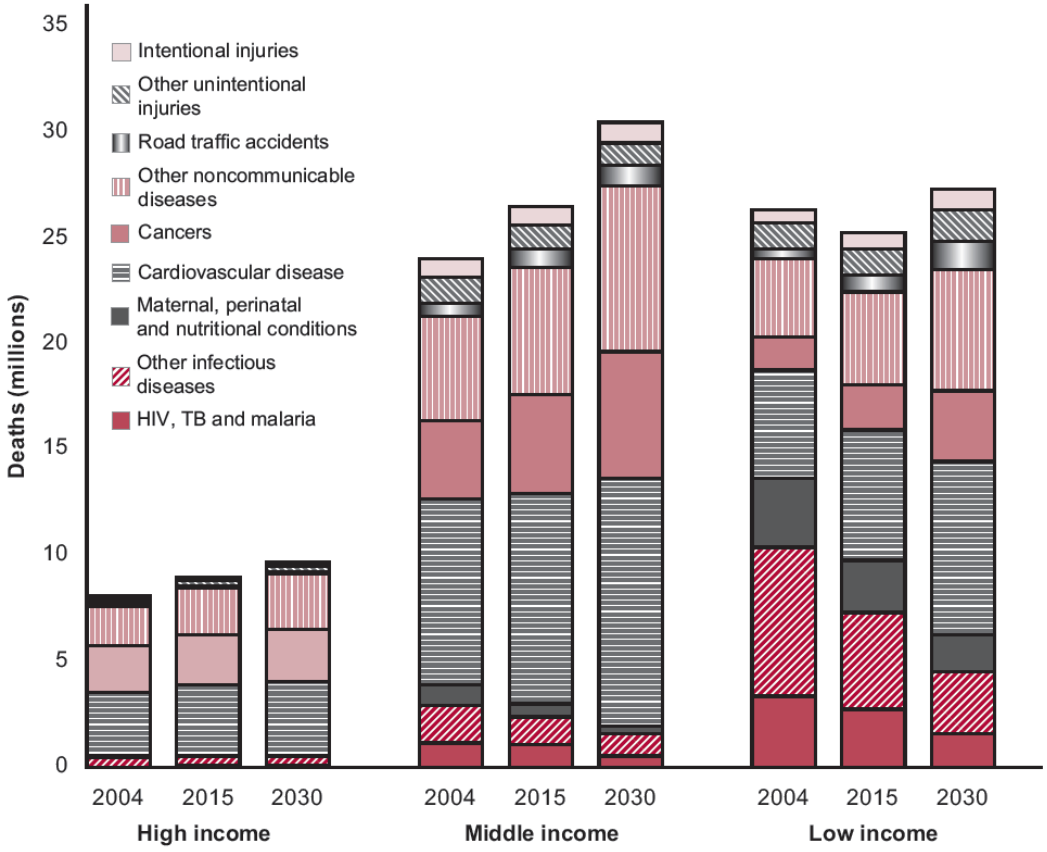


Source: Ridsdale & Gallop, 2010a, p. 15

According to past research, the important factors contributing to an increase in longevity have been improvements in medical treatment (based on medical research), social change and health education – and they are expected to contribute substantially to further longevity improvements in the future. Based on a review of the literature, Ridsdale and Gallop (2010b) conclude that many papers stress that social change and health education contribute more to future improvements in longevity than medical treatments.

In 2008 the World Health Organisation prepared projections of deaths by causes of death. As presented in Figure 6-2, the level of deaths between high- and low-income countries differs (and is expected to continue to differ in the future) although the composition is also different. While in low-income countries there is still high mortality (the current one and projections for 2030) because of infectious diseases, in high-income countries mortality is concentrated in three major cause groups – cardiovascular disease, cancers and other non-communicable diseases.

Figure 6-2: Projected deaths by cause for high-, middle- and low-income countries

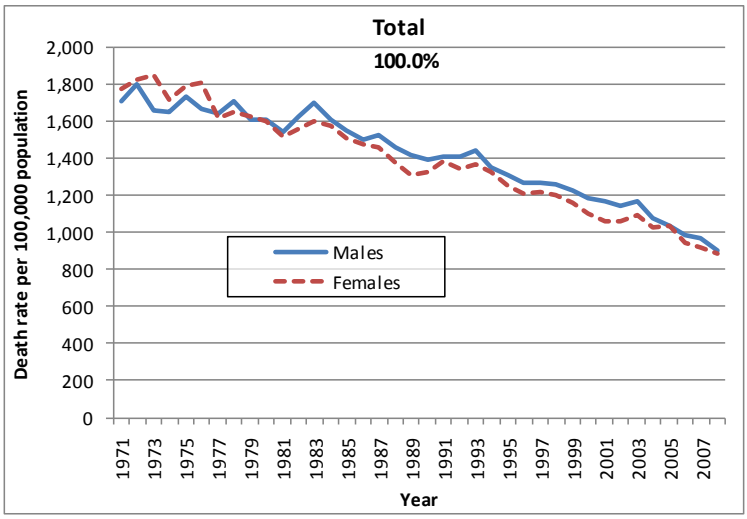


Source: World Health Organisation, 2008, p. 29

Focusing on the Slovenian case, Figure 6-3 presents the development of mortality in the 1971–2008 period. To eliminate the effect of a changing age structure in time, we applied age-specific mortality rates by years to the same (standard) population. The age structure of the Slovenian population in 2008 was taken as a standard population. Thus, in Figure 6-3 the hypothetical number of deaths per 100,000 inhabitants is presented assuming the population age (males and females together) structure from 2008. In the 1971–2008 period mortality has almost halved – from about 1,800 to

about 900 deaths per 100,000 population (standardised). Males and females are treated separately – deaths for males are expressed per 100,000 males and deaths for females are expressed per 100,000 females. The death rate for both genders combined would lie somewhere in-between and be calculated as a weighted average of these two values.

Figure 6-3: Age-standardised total death rates per 100,000 population, 1971–2008



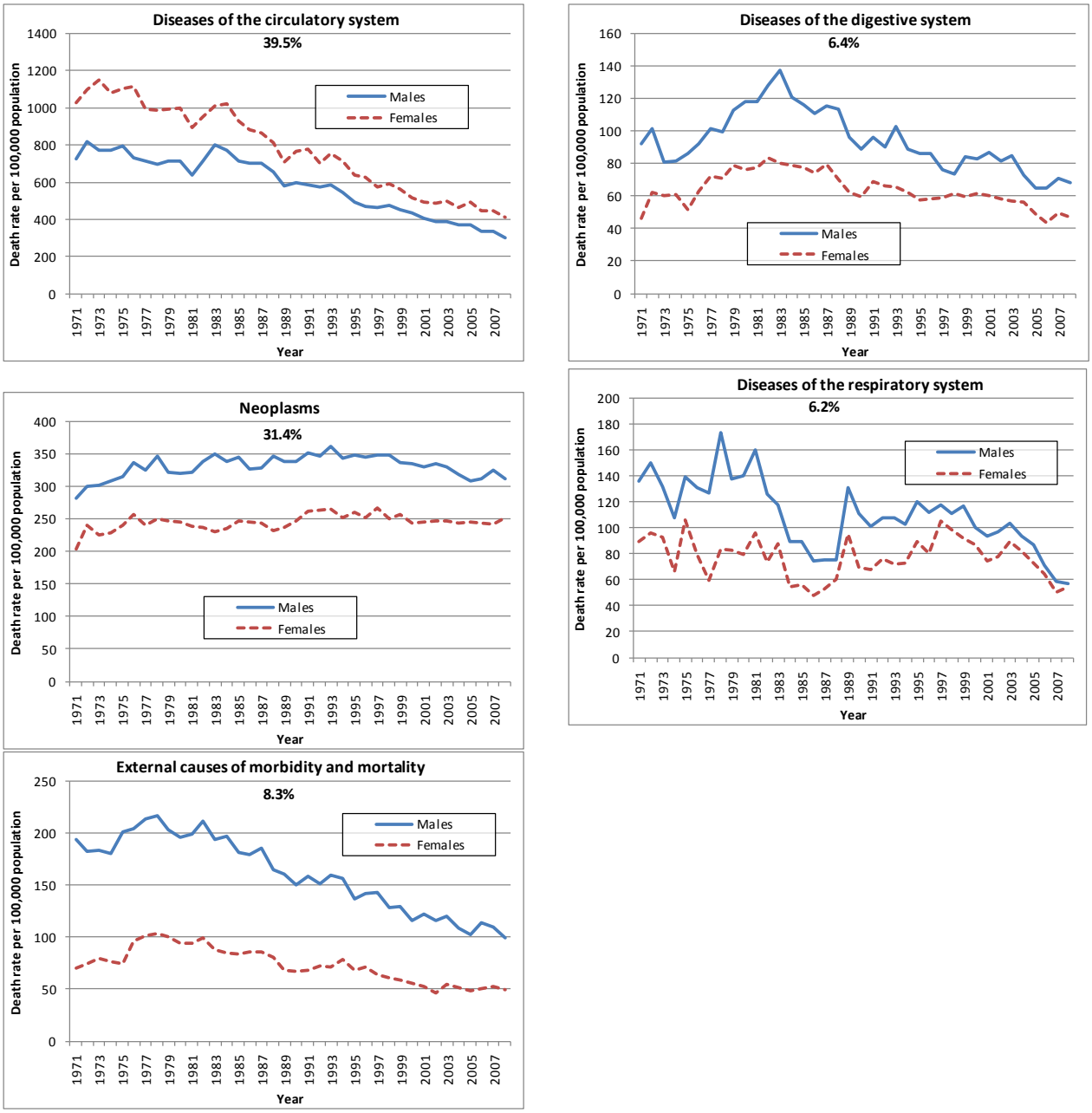
Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia; authors' calculations

In Figure 6-4 the total number of deaths per 100,000 population is decomposed according to the causes of death (major cause groups). The measurement units on the vertical scale are the same for all groups of diseases (standardised number of deaths per 100,000 inhabitants). Therefore, the results are comparable across different groups of diseases, although the scales are differently stretched in each of them according to their maximum values.

In Section 7 we conduct a Lee-Carter analysis on the total death count. However, the results of the Lee-Carter model depend on the model's assumptions. Therefore, we also present trends in mortality on original data and further decompose them by causes of death. In Figure 6-4 we present the results for major causes of death with an above 5% share among all deaths in 2008. The percentage values under the titles in Figure 6-4 (cause of death groups) denote deaths caused by a particular disease as a share of total deaths in 2008. To identify the main cause groups of deaths, the pictures in Figure 6-4 are sorted according to those shares. Mortality developments for the remaining cause groups of death are presented in Appendix 6.

By far the most important causes of death in 2008 were "diseases of the circulatory system" (39.5% of all deaths) and "neoplasms", i.e. cancer (31.4% of all deaths), together representing 71% of all deaths. By also including the third largest group – deaths caused by external causes (8.3% of all deaths) – about 80% of all deaths are included in the analysis. However, the trends of these three cause groups of death differ. Diseases of the circulatory system exhibit a strong decline in the analysed period – from over 900 to less than 400 deaths per 100,000 population (using a standard population from 2008) in the 1971–2008 period. Because it represents such a high share among all causes of death and because of its strongly negative trend, this group is a main driving force in the

Figure 6-4: Age-standardised death rates by cause of death (five main cause groups with the biggest share among total deaths)



Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia; authors' calculations

decline of total mortality in the 1971–2008 period in Slovenia. Although the share of external causes of morbidity and mortality is much smaller and the negative trend is less pronounced, they contribute substantially to the total mortality decline as well. In contrast, neoplasms do not show any clear trend. If the trends for these two groups of diseases were to continue, neoplasms would become the no. 1 cause of death in the near future. The results for Slovenia are in line with the results for England and Wales presented in Figure 6-1, where the numbers of deaths for both causes of death are already about the same.

In Slovenia in 2008, diseases of the circulatory system, neoplasms and external causes of morbidity and mortality were followed by the next two cause groups, representing somewhat more than 6% of all deaths in 2008: “diseases of the digestive system” and “diseases of the respiratory system”. These two groups also exhibit a generally declining trend during the analysed period. However, the number of deaths in the group “diseases of the digestive system” (a 6.4% share among all deaths in 2008) increased considerably during the 1970s. The group “external causes of morbidity and mortality”, mentioned earlier, also faced a noticeable increase during the 1970s.

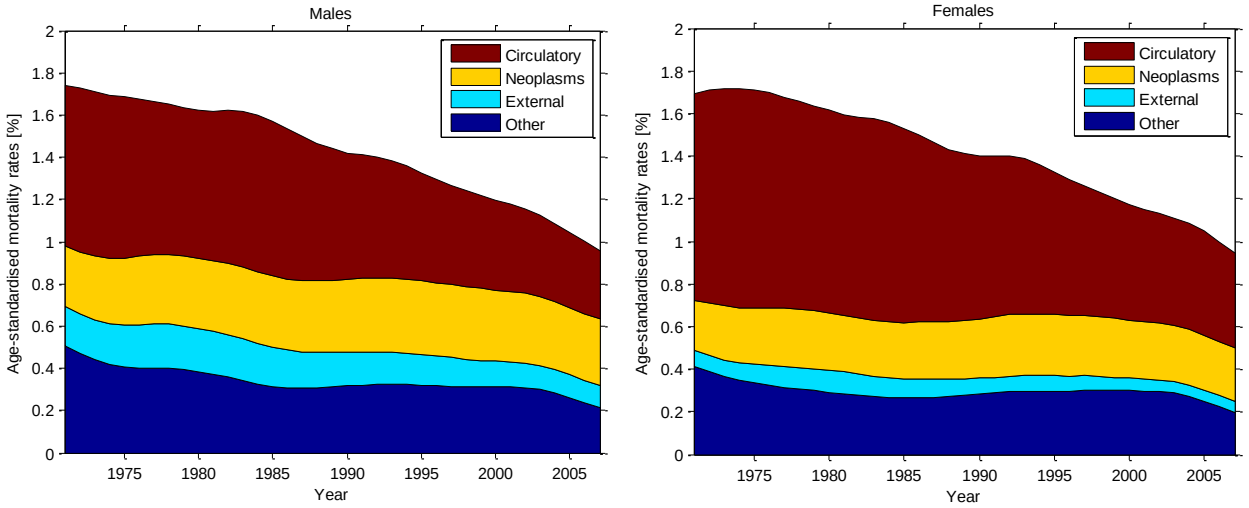
During the 1970s a high share of deaths was still classified in the “symptoms, signs and abnormal clinical and laboratory findings, not elsewhere classified” group. The share of this group declined sharply during this period. Thus, a smaller decline or even increase in death rates for some groups of diseases in the 1970s could be due to the higher share of deaths being categorised into this group of diseases as a result of better identification of causes of death.

All other major groups of diseases accounted for less than 2% of total deaths, or even just a few cases of deaths, among all deaths in 2008 (they are presented in Appendix 9).

Figure 6-5 presents the improvement in mortality in the 1971–2008 period in the form of declining age-standardised mortality rates. Again, the age structure of males and females from 2008 is used. Note that the standard population is different for males and females. Thus, the results for males and females are not directly comparable in their levels. The total age-standardised mortality rate for males declined from 1.71% in 1971 to 0.91% in 2008, whereas Figure 6-5 presents its decomposition by cause groups. We see therefore how much individual cause groups have contributed to the total decline in mortality. We analyse separately only the three largest cause groups: “diseases of the circulatory system”, “neoplasms” and “external causes of morbidity and mortality”. We present smoothed results (using the Spline toolbox in the Matlab software) to reduce variability due to the random factor contained in Figure 6-4. Original and smoothed age-standardised death rates for these three groups of death causes are presented in Figure 11-4 in Appendix 7. All other cause groups are summed up in the “other” category, which is calculated as a residual between the smoothed total age-standardised death rates and the smoothed age-standardised death rates for the three main cause groups. Data for one period (2008) were lost due to the smoothing procedure.

Figure 6-5 summarises the results from Figure 6-4 about the strong decline in diseases of the circulatory system group. However, this cause group of death is more relevant for females than males. Although the decline during the analysed period was about the same for both genders, the share of this cause group among total deaths is much higher among females. It still represents almost half of the total age-standardised mortality rate, while during the 1970s and 1980s it was even considerably more than one-half. For males the share of this cause group of death is just about one-third, the next third represents neoplasms while the last third represents remaining causes – external causes of morbidity and mortality, together with all other causes.

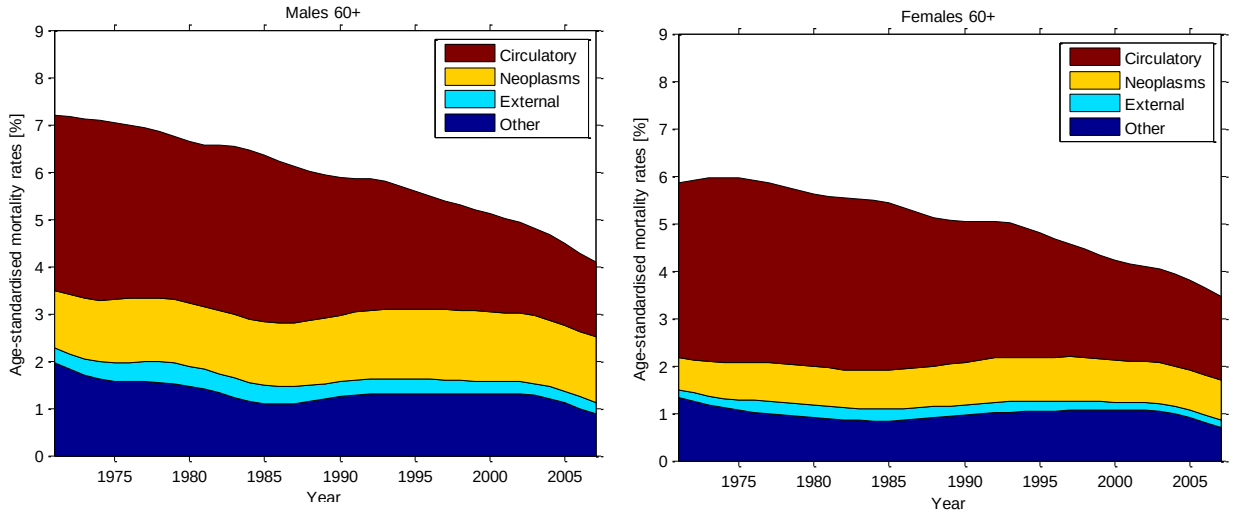
Figure 6-5: Age-standardised death rates by cause of death (three main cause groups with the biggest share among total deaths); for all age groups together



Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia; authors' calculations

Figure 6-6 is the same as Figure 6-5, but it is focused on the age group 60+, which is the group most relevant in the calculation of annuity values or the valuation of pension liabilities (Continuous Mortality Investigation, 2009a). The results are similar to the results in Figure 6-5, but at much higher levels (see the vertical axis). The share of external causes of morbidity and mortality cause group for the 60+ age group (Figure 6-6) is distinctively smaller than all age groups together (Figure 6-5). In addition, neoplasms among women have a much lower share when focusing only on the 60+ age group instead of all ages.

Figure 6-6: Age-standardised death rates by cause of death (three main cause groups with the biggest share among total deaths); for the age group 60+



Source: Institute of Public Health of the Republic of Slovenia; Statistical Office of the Republic of Slovenia; authors' calculations

Table 2 complements the results from Figure 6-5 and Figure 6-6 by presenting the components of mortality improvement in 2008 relative to 1971. As already discussed, most of the mortality

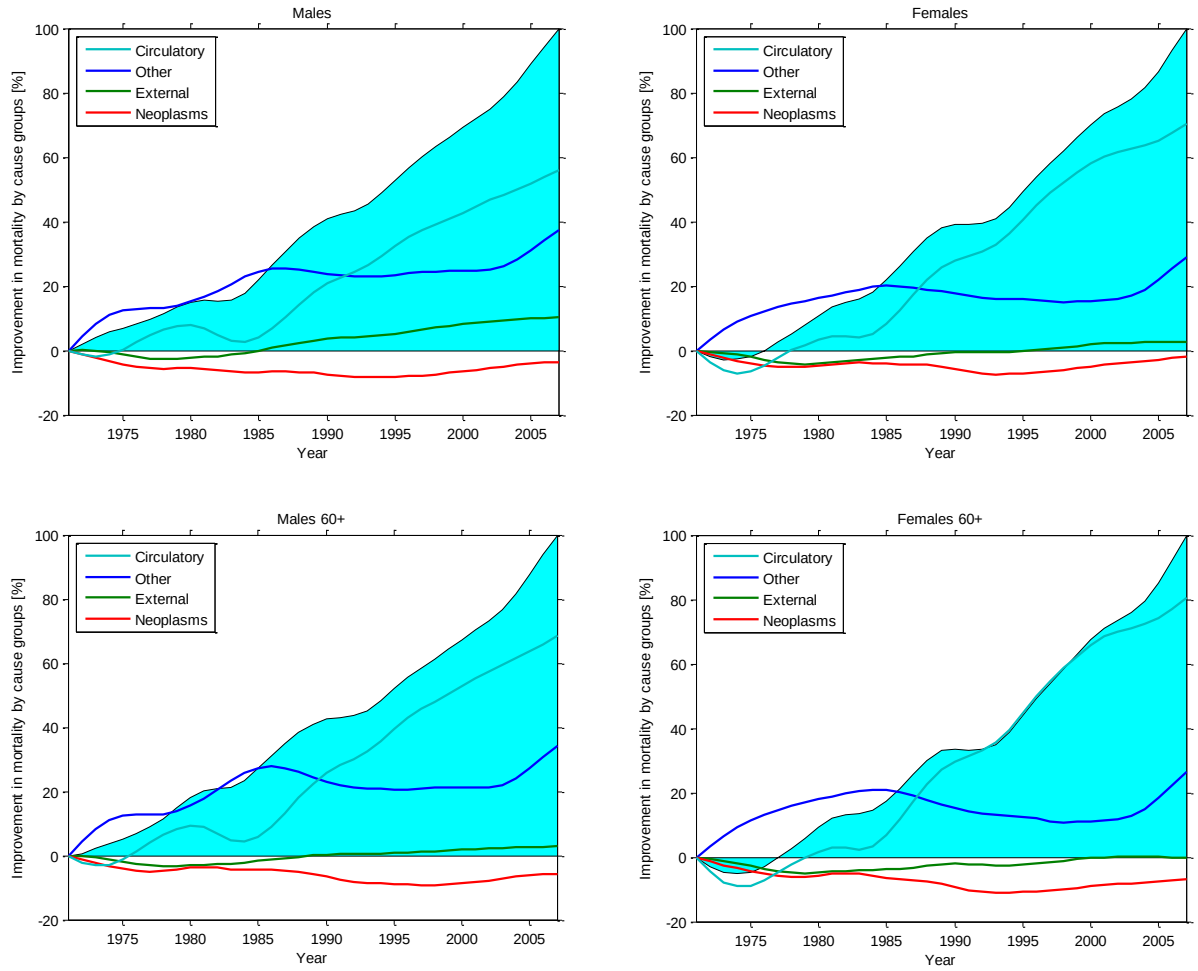
improvement was due to the reduced mortality in diseases of the circulatory system cause group of death. The contribution of the external causes of morbidity and mortality cause group was fairly limited, while mortality because of neoplasms even increased during the analysed period.

Table 2: Components of improved mortality in 2008 relative to 1971 [% of total improvement]

	Males	Females	Males 60+	Females 60+
Diseases of the circulatory system	53	70	69	80
Neoplasms	-4	-5	-6	-7
External causes of morbidity and mortality	12	2	3	0
Other	39	33	34	26
Total	**100**	**100**	**100**	**100**

Source: Institute of Public Health of the Republic of Slovenia; Statistical Office of the Republic of Slovenia; authors' calculations

Figure 6-7: Components of improved mortality in the 1971–2007 period [% of total improvement presented as the blue area]



Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia

Figure 6-7 presents the development of individual components also for the intermediate period between 1971 and 2007. The shaded area represents the percentage gain in mortality improvement in all cause groups together, while the four lines represent the contribution of individual cause groups and sum up to the blue area. The improvement in mortality was generally faster in the second half of the analysed period, especially for females aged 60+, encountering a strong mortality improvement in the circulatory system cause group.

In Table 3 the average annual rates of increase (+) or decline (-) by cause groups are presented for the entire 1971–2008 period along with its sub periods. The last sub period is especially relevant for our analysis because the average annual decline of mortality in the last period (by age groups) will be used for projecting mortality development in the future.

Table 3: Average annual increase [in %] in age-standardised mortality rates for males and females in Slovenia in the 1971–2008 period and its sub periods

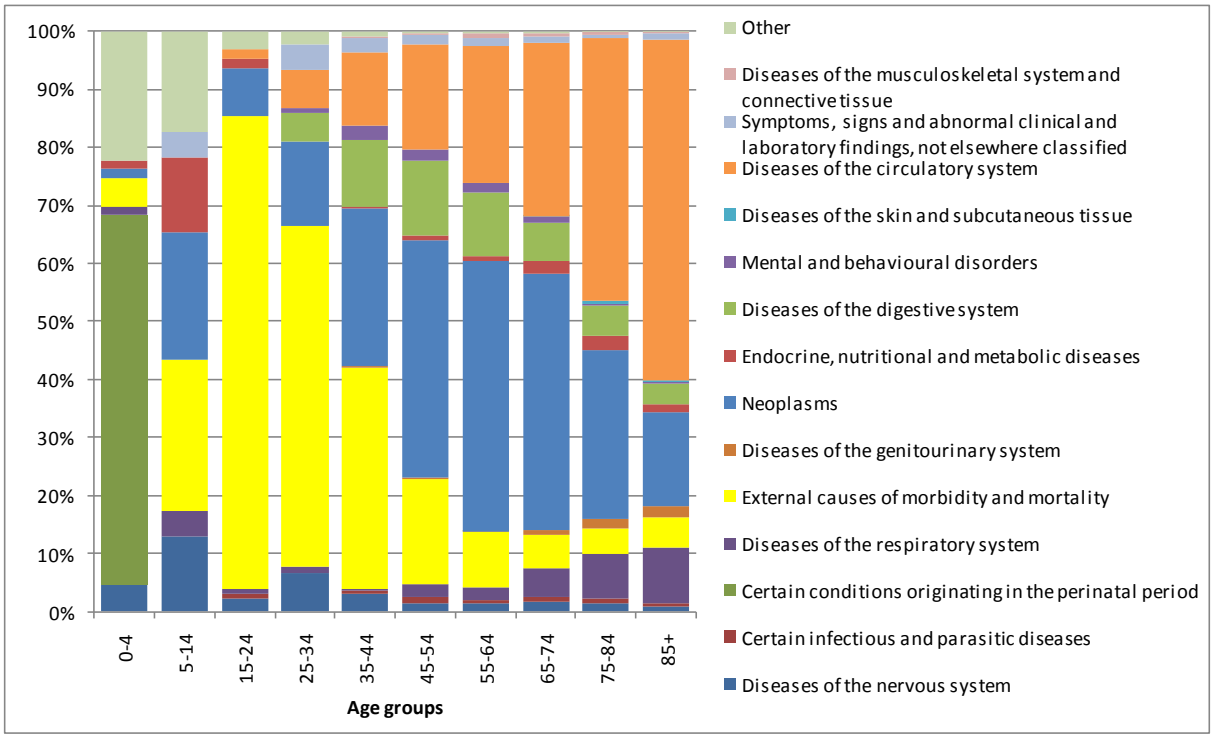
			Males		
	1971-2008	1971-1981	1981-1991	1991-2001	2001-2008
Diseases of the circulatory system	**-2.37**	-1.26	-0.94	-3.56	-2.98
Neoplasms	**0.27**	1.30	0.88	-0.61	-0.58
External causes of morbidity and mortality	**-1.79**	0.24	-2.23	-2.58	-2.03
Other	**-2.55**	-2.71	-2.04	-0.17	-4.49
All causes	**-1.70**	**-1.01**	**-0.95**	**-1.85**	**-1.70**
			Females		
	1971-2008	1971-1981	1981-1991	1991-2001	2001-2008
Diseases of the circulatory system	**-2.46**	-1.36	-1.36	-4.53	-1.82
Neoplasms	**0.57**	1.59	0.93	-0.70	0.30
External causes of morbidity and mortality	**-0.97**	2.90	-3.15	-2.54	-0.67
Other	**-2.58**	-4.75	-0.68	0.13	-4.16
All causes	**-1.85**	**-1.54**	**-0.93**	**-2.59**	**-1.77**

Source: Institute of Public Health of the Republic of Slovenia; Statistical Office of the Republic of Slovenia; authors' calculations

6.4 Mortality by age groups

As illustrated in Figure 6-8, the leading cause of death varies by age groups. In the 0–4 age group conditions originating in the perinatal period dominate. Thereafter, until the 35–44 age group, external causes of death account for the biggest share among all deaths. Although the total number of deaths in these age groups is relatively low (see Table 31 in Appendix 8), the contribution to life expectancy is substantial. Namely, if people in this age group do not die from external causes of mortality, they can expect to live for many more years. Therefore, they will contribute a high number of person years to the *Total number of person years* (T_x) in life tables from which *Life expectancy* (e_x) is calculated. In the age group 45–54 years, neoplasms have the highest share among major cause groups of death. Finally, in the age group 75–84 diseases of the circulatory system are becoming the leading cause of death and this is even more so in the highest age group (85+) where deaths in this group represent almost 60% of all deaths.

Figure 6-8: Age-standardised death rates by cause of death (five main cause groups with the biggest share among total deaths), 2008



Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia

In Appendix 9 the same analysis is presented separately by gender. There are some distinctive differences between both genders. External causes of death have lower shares among total deaths for females than males during the entire lifetime. Neoplasms represent considerably higher shares for females than males in the 25–64 age group, while in the same age diseases of the circulatory system have higher shares among males than among females. The gender differences by all causes of death and age groups are presented in Table 34 in Appendix 9.

6.5 Age-specific trends in the top three major cause groups of death

Next we analyse trends in mortality by age groups. We only analyse the three main cause groups of death with the highest shares in 2008, presented in Figure 6-4. The standardisation procedure is not needed this time.

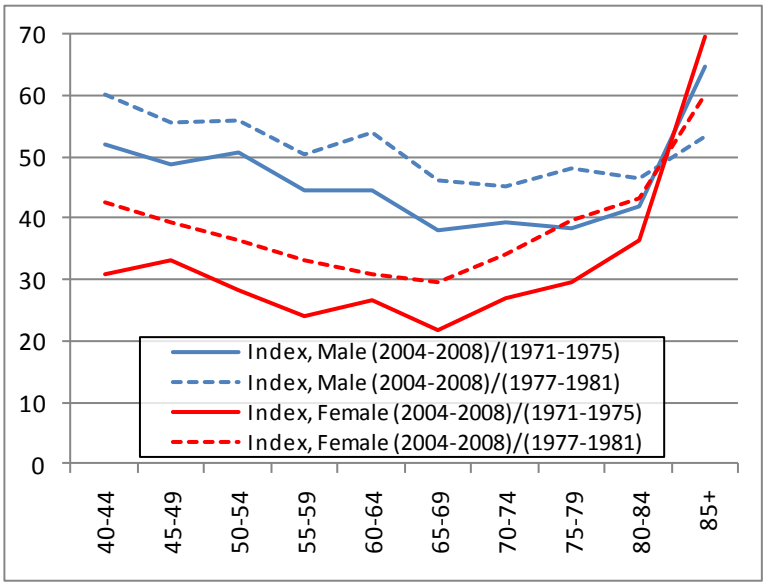
6.5.1 Diseases of the circulatory system

The decline was not equal for all age groups. In Figure 6-9 we present the decline during the analysed period by age groups. We compare the central death rate (${}_5m_x$) from the end of the analysed period with those from the beginning of the analysed period. Mortality rates decomposed by 5-year age groups in the Slovenian population of just 2 million are subject to a large random factor, especially in age groups where mortality is low. To alleviate the random factor, we use 5-year averages at the beginning and at the end of the analysed period. However, this results in the loss of some dynamics because the analysed period is somehow shortened by 4 years. The results are presented in the form

of indexes. Thus, we present averages of ${}_5m_x$ for 2004–2008 compared to ${}_5m_x$ averages for 1971–1975.

The biggest decline in mortality for males was in ages from 65 to 84 years – by about 60% – while in lower age groups it was 50%–55%. In the highest age group (85+), the decline was only about 35%. Observing the development in time reveals that the result is also affected by increasing mortality in the 85+ age group in the 1971–1976 period (see Figure 6-10). As discussed, this could be a result of better identification of causes of death – i.e. relocating cases from the "not elsewhere classified" category). However, dropping these first 6 years (1971–1976) from the analysis still results in a smaller mortality decline in this age group compared to the 65–84 age group.

Figure 6-9: Diseases of the circulatory system: level of age-specific mortality rates in 2004–2008 compared to 1971–1975 and 1977–1981 by age groups [indexes]



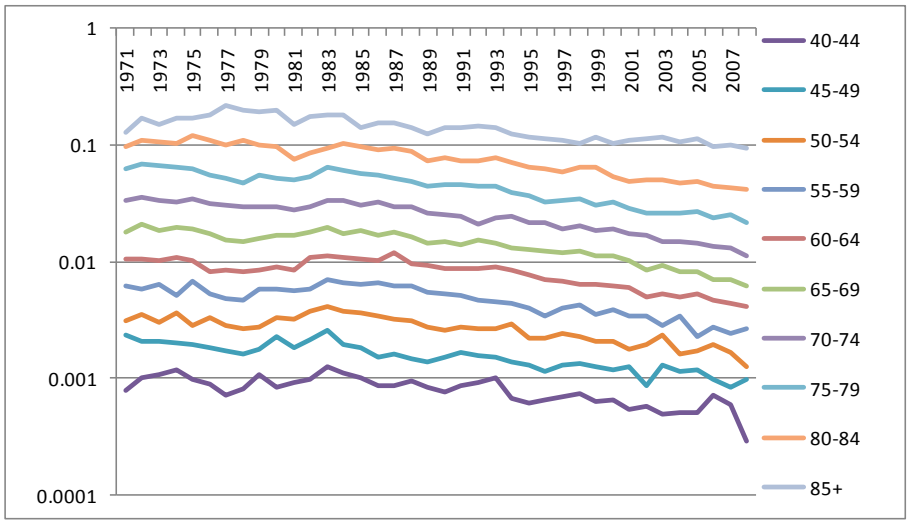
Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia; authors' calculations

For females the pattern is similar, but much more pronounced. The largest decline in mortality was in the age group 65–69 – in 2004–2008 it fell to just 20% of the level in 1971–1975. For females in other age groups the decline was also very strong – to about 30%. However, the exception is the decline in the 85+ age group, which was modest. Mortality declined by less than one-third – to about 70% of the 1971–1975 level.

Figure 6-10 and Figure 6-11 present the development of central death rates through time by age groups. To preserve clarity in these figures we do not present the results for the lowest age groups because of the small number of cases – note that the scale on the y-axis in these figures is logarithmic. Because of the small number of cases (about 0–2 cases per 100,000 population in the last several years), the random effect is large and mortality in these age groups does not have a substantial impact on total deaths. In any case, the patterns in these age groups are not that different than the patters in the lowest age groups for which the results are presented in figures.

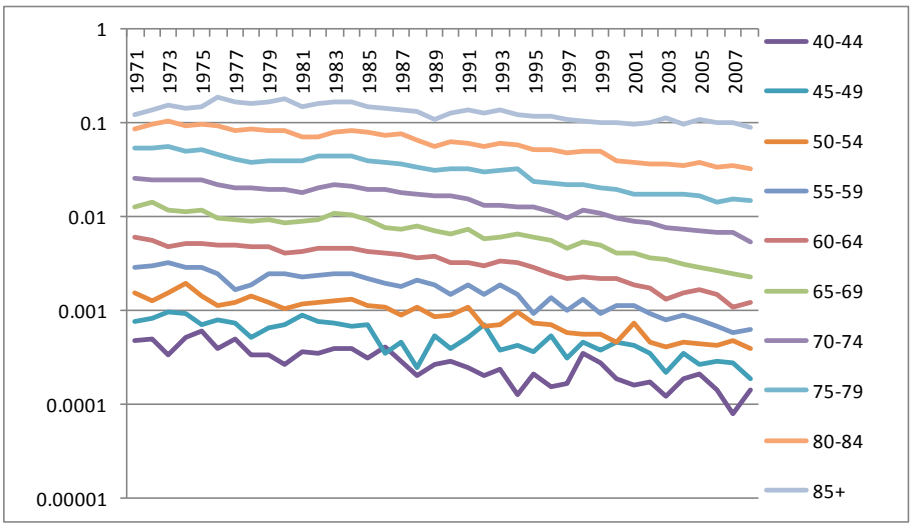
In diseases of the circulatory system cause group of death, the evidence stresses the importance of reducing the risk factor. In his work, Capewell cites analyses discovering that in Scotland, England and Wales, Ireland, Finland, New Zealand and the USA about 45%–75% of the decline in mortality related to this cause can be attributed to changes in risk factors, while only the remaining 25%–55% can be attributed to treatments (Ridsdale & Gallop, 2010b, p. 24).

Figure 6-10: Diseases of the circulatory system: age-specific mortality rates in 1971–2008 by age groups; males



Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia

Figure 6-11: Diseases of the circulatory system: age-specific mortality rates in 1971–2008 by age groups; females



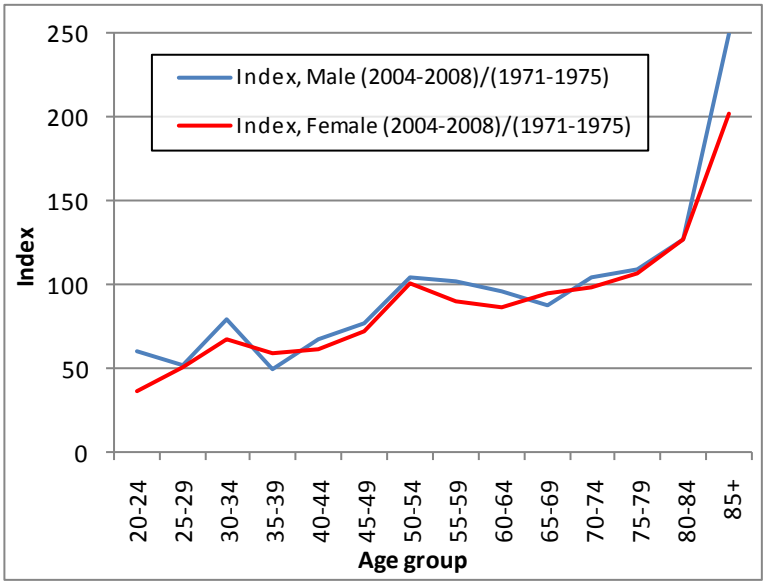
Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia

6.5.2 Neoplasms

During the analysed period there was practically no change in central death rates in ages between 50 and 79 in the neoplasms cause group of death (the index is around 100, see Figure 6-12). In lower

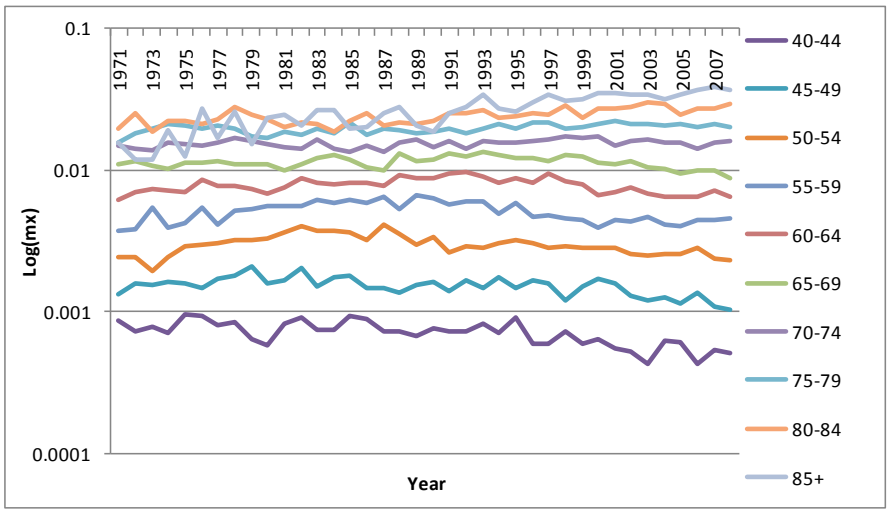
age groups, there was a substantial decline in mortality, while in the highest age group (85+) mortality more than doubled. This could be due to an increase in risk factors, but it can also be suspected that with the strong decline of mortality in *diseases of the circulatory system* cause group of death people "hit" this cause group of death instead.

Figure 6-12: Neoplasms: level of age-specific mortality rates in 2004–2008 compared to 1971–1975 by age groups [indexes]



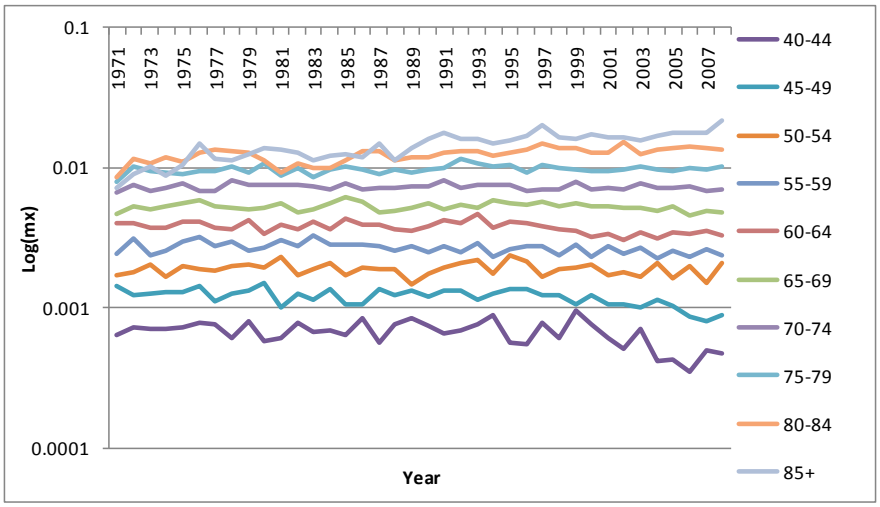
Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia; authors' calculations

Figure 6-13: Neoplasms: age-specific mortality rates in 1971–2008 by age groups; males



Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia

Figure 6-14: Neoplasms: age-specific mortality rates in 1971–2008 by age groups [indexes]; females

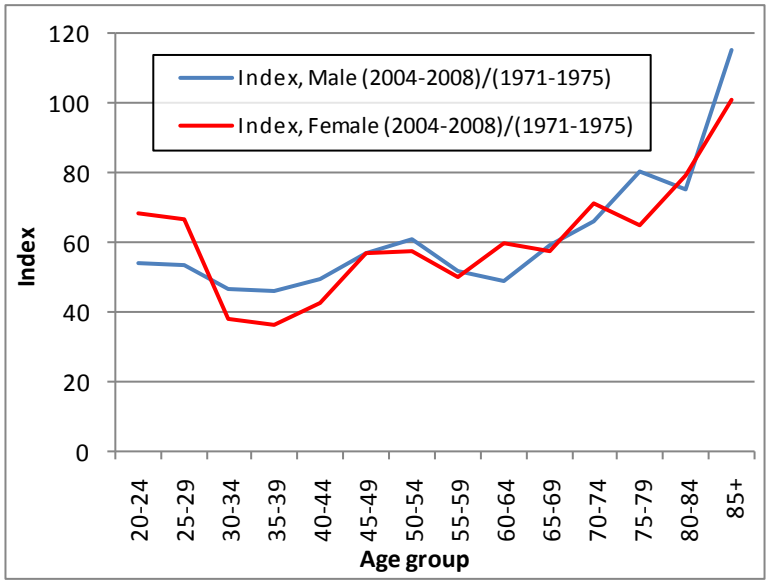


Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia

6.5.3 External causes of morbidity and mortality

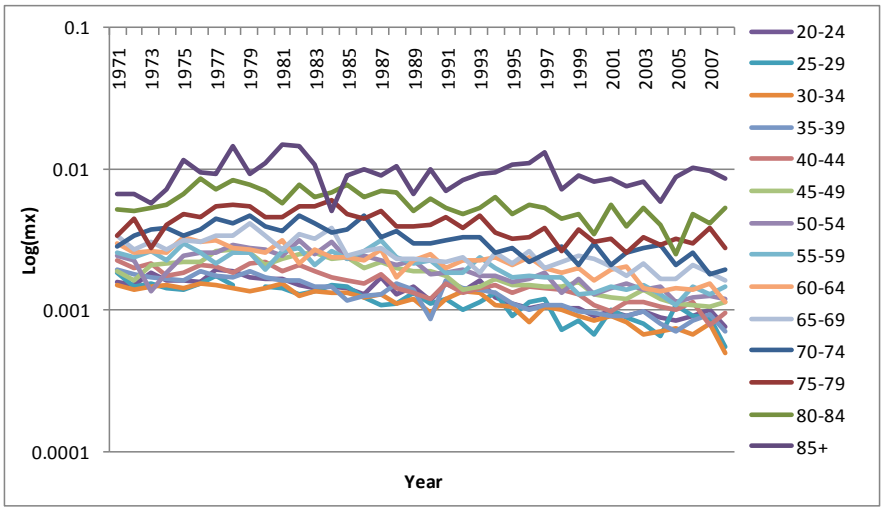
In this cause group of death the mortality decline for people below age 70 was about 40% – in the ages of 30s and early 40s, the decline was even about 50% (see Figure 6-15). Above age 70 the mortality improvement was more modest, while in the highest age group (85+) mortality has not improved at all.

Figure 6-15: External causes of morbidity and mortality: level of age-specific mortality rates in 2004–2008 compared to 1971–1975 by age groups [indexes]



Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia; authors' calculations

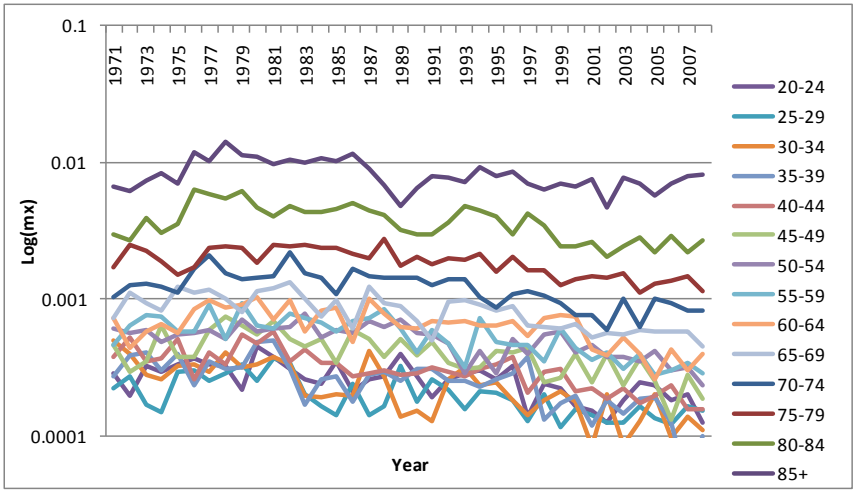
Figure 6-16: External causes of morbidity and mortality: age-specific mortality rates in 1971–2008 by age groups; males



Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia

Again, in Figure 6-16 and Figure 6-17 the mortality development by age groups is presented. Female mortality differs from male mortality in lower age groups – while female mortality during the analysed period does not exceed the 0.1% threshold (0.001 threshold in Figure 6-17) up until age 60, males' mortality exceeds this threshold already practically in all age groups above the age of 20.

Figure 6-17: External causes of morbidity and mortality: age-specific mortality rates in 1971–2008 by age groups; females



Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia

6.6 Age-specific trends in the top three major cause groups of death

Based on the past development in mortality by age groups, we present mortality and longevity projections for the next 50 years. The model assumes a gradual transition from the recently observed annual rates of mortality improvement into "long-term" rates of mortality improvement that are arbitrarily determined by the user. A linear transition is assumed. The model results in deterministic mortality projections. However, we also present a sensitivity analysis for several sets of assumptions.

This approach was also used in studies by the research group Continuous Mortality Investigation (2009a, 2009b).

As mentioned, the model builds on the assumption of an annual improvement (decline) in mortality. It is gradually converging from initial rates of mortality improvement (annual growth rates) to the long-term mortality rates – by age groups.

To capture recent development in mortality by age groups we would like to use annual growth rates of mortality (by age groups) from the last year. However, because of the small number of deaths – especially in lower age groups – the variability is huge. Therefore, we use the average annual growth rate by age groups in the 2000–2008 period instead. The variability of mortality growth rates by age using this approach is still high.

If the average annual growth of mortality rates for the 2000–2008 period is generally below the average annual growth rate of mortality for the 1971–2008 period and is only above it in one age group, while in the "neighbour" age groups it is not, we prefer to take a lower value (from the 1971–2008 time period). Namely, we assume that it is a consequence of the random effect, especially if the average annual growth of the mortality rate for the 1971–2008 period is similar to the neighbour age groups. Further, for the 0–24 age groups the average annual mortality rate growth in the entire 1971–2008 period is taken. Due to the small number of death cases in these age groups, the variability is too high otherwise. The described solutions introduce personal judgement. However, it does not affect the main results much because the solutions predominately refer to age groups with low mortality. In any case, the model is based on expert opinions and, therefore, the model is subjective as such. Actual mortality rates from 2008 are taken as starting values of mortality rates by age. Results of the projections are presented for various scenarios.

6.6.1 Scenario 1

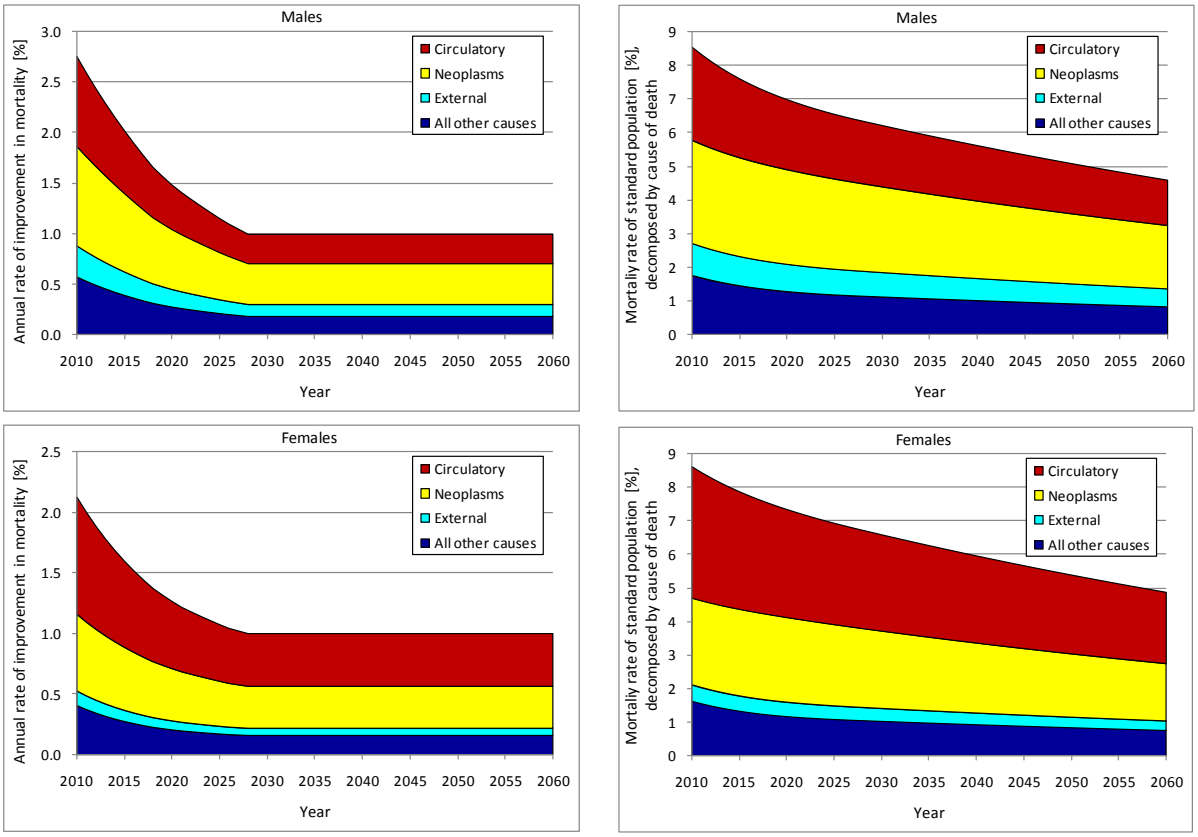
In Scenario 1 we follow the work of the Continuous Mortality Investigation research group (2009a, 2009b). They assume a 10-year transition period for ages up to 50 and, from age 50 to 60, a gradual increase in transition period by one year for each consecutive age (from a transition period of 10 years for 50 years of age to a transition period of 20 years for 60 years of age). However, since our model uses 5-year age groups we use a 12-year transition period for the 50–54 age group and 17 years for the 55–59 age group. From 60 to 80 years a transition period of 20 years is taken. From 80 to 95 a gradual decrease from 20 to a 5-year transition period is assumed, keeping it at a 5-year transition period thereafter (Continuous Mortality Investigation, 2009a, p. 14). In our model with 5-year age groups we use a transition period of 18 years for the age group 80–84, while for age 85+ we assume a transition period of 13 years.

A 1% long-term rate of mortality improvement is assumed, without a constant addition to rates of mortality improvement. In Scenario 1 we assume a long-term annual mortality decline of 1% for all cause groups of death.

Following the described approach we obtain the results presented in Figure 6-18 and Table 4 and Table 5. A strong decline in mortality rates is projected for the starting years of the projections. Consequently, life expectancy for all ages is also growing fast in this period, which is in line with the rapid increase in life expectancy that we have been facing in Slovenia over the last one to two

decades. A further improvement in mortality depends on the assumed long-term mortality decline – which in Scenario 1 is 1%.

Figure 6-18: Annual rate of improvement in mortality [%] and mortality rate of standard population [%] in 2010–2060 projection period, decomposed by cause of death; Scenario 1



Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia; authors' calculations

We obtained the presented results from central death rates obtained as the sum of central death rates by four cause groups of death: “diseases of the circulatory system”, “neoplasms”, “external causes of morbidity and mortality” and “other causes of death”. In Appendix 10 we also present results for life expectancy by age and gender whereby the rate of mortality improvement is calculated using the described procedure using the total mortality rate as an aggregate category – not as the sum of the individual cause groups of death.

Table 4: Life expectancy for males, by age; Scenario 1, mortality rates are calculated as the sum of assumed mortality rates by cause groups of death

	2010	2015	2020	2025	2030	2035	2040	2045	2050	2055	2060
0	76.31	77.68	78.63	79.38	80.01	80.64	81.27	81.92	82.58	83.25	83.94
1–4	75.51	76.84	77.79	78.53	79.15	79.77	80.40	81.04	81.70	82.37	83.05
5–9	71.57	72.90	73.84	74.58	75.20	75.82	76.45	77.09	77.74	78.41	79.09
10–14	66.58	67.90	68.85	69.59	70.21	70.83	71.45	72.09	72.74	73.41	74.09
15–19	61.63	62.95	63.90	64.63	65.25	65.87	66.49	67.13	67.78	68.45	69.13
20–24	56.82	58.13	59.06	59.79	60.41	61.02	61.63	62.27	62.91	63.57	64.25
25–29	52.04	53.33	54.26	54.98	55.59	56.19	56.80	57.43	58.07	58.72	59.39
30–34	47.22	48.50	49.42	50.14	50.74	51.33	51.94	52.56	53.19	53.84	54.51
35–39	42.39	43.66	44.57	45.28	45.88	46.47	47.07	47.69	48.32	48.96	49.62
40–44	37.64	38.88	39.79	40.49	41.08	41.66	42.26	42.87	43.49	44.13	44.78
45–49	33.02	34.24	35.14	35.83	36.41	36.98	37.56	38.16	38.77	39.40	40.05
50–54	28.64	29.81	30.69	31.37	31.93	32.48	33.05	33.63	34.23	34.84	35.48
55–59	24.44	25.57	26.43	27.09	27.62	28.15	28.70	29.26	29.84	30.43	31.05
60–64	20.62	21.69	22.50	23.13	23.63	24.14	24.65	25.19	25.74	26.31	26.89
65–69	16.92	17.90	18.65	19.23	19.70	20.18	20.67	21.17	21.70	22.24	22.80
70–74	13.34	14.12	14.74	15.25	15.69	16.14	16.61	17.09	17.59	18.12	18.66
75–79	10.32	10.94	11.45	11.89	12.29	12.71	13.15	13.60	14.07	14.57	15.09
80–84	7.69	8.15	8.55	8.92	9.29	9.68	10.09	10.52	10.97	11.44	11.94
85+	5.85	6.23	6.58	6.92	7.28	7.65	8.05	8.46	8.90	9.36	9.84

Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia; authors' calculations

Table 5: Life expectancy for females, by age; Scenario 1, mortality rates are calculated as the sum of assumed mortality rates by cause groups of death

	2010	2015	2020	2025	2030	2035	2040	2045	2050	2055	2060
0	83.06	84.06	84.83	85.47	86.07	86.67	87.29	87.93	88.58	89.26	89.97
1–4	82.24	83.24	84.00	84.64	85.22	85.82	86.43	87.06	87.72	88.39	89.09
5–9	78.26	79.26	80.01	80.65	81.24	81.83	82.45	83.08	83.73	84.40	85.10
10–14	73.33	74.33	75.08	75.72	76.30	76.89	77.50	78.13	78.78	79.45	80.15
15–19	68.36	69.36	70.11	70.74	71.33	71.92	72.53	73.16	73.80	74.47	75.17
20–24	63.43	64.43	65.17	65.81	66.39	66.97	67.58	68.21	68.85	69.52	70.21
25–29	58.48	59.47	60.22	60.85	61.42	62.01	62.62	63.24	63.89	64.55	65.24
30–34	53.56	54.54	55.28	55.91	56.49	57.07	57.67	58.30	58.94	59.60	60.29
35–39	48.65	49.62	50.36	50.99	51.56	52.14	52.74	53.36	54.00	54.66	55.35
40–44	43.75	44.72	45.45	46.07	46.64	47.22	47.82	48.44	49.07	49.73	50.42
45–49	38.93	39.87	40.60	41.22	41.78	42.36	42.95	43.56	44.19	44.85	45.53
50–54	34.18	35.09	35.81	36.42	36.98	37.55	38.13	38.74	39.37	40.01	40.69
55–59	29.68	30.57	31.27	31.86	32.41	32.96	33.53	34.12	34.74	35.38	36.04
60–64	25.20	26.06	26.74	27.32	27.84	28.38	28.94	29.52	30.12	30.75	31.40
65–69	20.87	21.67	22.32	22.87	23.38	23.90	24.44	25.01	25.60	26.21	26.85
70–74	16.69	17.41	18.00	18.52	19.01	19.52	20.05	20.60	21.17	21.77	22.39
75–79	12.84	13.46	13.99	14.47	14.94	15.43	15.94	16.47	17.02	17.61	18.22
80–84	9.55	10.08	10.55	10.98	11.43	11.89	12.38	12.89	13.43	13.99	14.58
85+	7.03	7.51	7.92	8.33	8.76	9.21	9.69	10.19	10.71	11.26	11.84

Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia; authors' calculations

6.6.2 Scenario 2 and Scenario 3

Following the work of the Continuous Mortality Investigation research group (2009a, 2009b), in Scenario 1 we have a 1% long-term rate of mortality improvement for all cause groups of death. This (uniform) 1% level of long-term mortality improvement could be the subject of discussion.

For example, in a survey conducted by the Society of Actuaries in 1998, experts from different fields were asked to provide their best guess for the ultimate annual rate of improvement in mortality rates beyond the year 2020 for the age group 65+. From a total of 59 experts coming from Canada, Mexico and the United States, there were 37 actuaries, 9 demographers, 8 economists and 5 from other professions (Rosenberg & Luckner, 1998, p. 65). For males their mean estimates were 0.58% (Canadian males), 0.76% (Mexican males) and 0.67% (US males), while for females their estimates were 0.64% (Canadian females), 0.83% (Mexican females) and 0.70% (US females) (Rosenberg & Luckner, 1998, p. 75). Thirteen per cent of respondents believed that a method using cause-specific mortality rates will produce a better forecast than a method projecting aggregate mortality, while 41 of the respondents thought that a cause-specific projection is a good first step for the aggregate study and 42 (8 of the 9 demographers) agreed it was a useful indicator of the reasonableness of the projection (Rosenberg & Luckner, 1998, p. 66). In some other projections somewhat lower values were used, while for some European countries (Germany, Switzerland and Austria) much higher values were assumed – for most age groups between 1% and 3% per annum (see Continuous Mortality Investigation, 2009b, p. 55).

In Scenario 2 we will therefore present results assuming a lower long-term rate of mortality improvement than in Scenario 1 – of 0.5% per annum – while in Scenario 3 we will assume a higher long-term rate of mortality improvement – of 1.5% per annum. In addition, in Scenario 2 we assume a higher long-term rate for the neoplasms cause group of death. In particular, we assume a 1.0% long-term rate of mortality improvement (while for other cause groups of death we assume 0.5% annual growth). We have already mentioned the rationale for this assumption – low improvements in the past may be due to decreasing mortality in the "diseases of the circulatory system" cause group. It might be the case that for people not dying from a circulatory cause of death, people "hit" the neoplasms cause of death instead, although there might be a substantial improvement in identifying and curing this disease. Scenarios 2 and 3 are presented in Appendix 11: Figure 11-7 and Figure 11-8 and Tables 37–40.

The presented results should be regarded with caution. To all the previously mentioned problems (see Section 6.1) we can also add broadly defined cause of death groups, which can therefore be very heterogenic. We have to bear in mind that there are complex problems of comorbidity, subsidiary causes and distinctions between the proximal and underlying causes of death. In older ages people often have more than one disease interacting with each other, making the identification of each of them more difficult, and only one of them is identified as a (main) cause of death. Further, there might be a big time lag between the change in risk factors and their effect on mortality (Ridsdale & Gallop, 2010b).

6.7 Two lifestyle factors affecting mortality

In a report from 2009 the World Health Organisation (WHO) summarises that "the leading global risks for mortality in the world are high blood pressure (responsible for 13% of deaths globally), tobacco use (9%), high blood glucose (6%), physical inactivity (6%), and overweight and obesity (5%)"

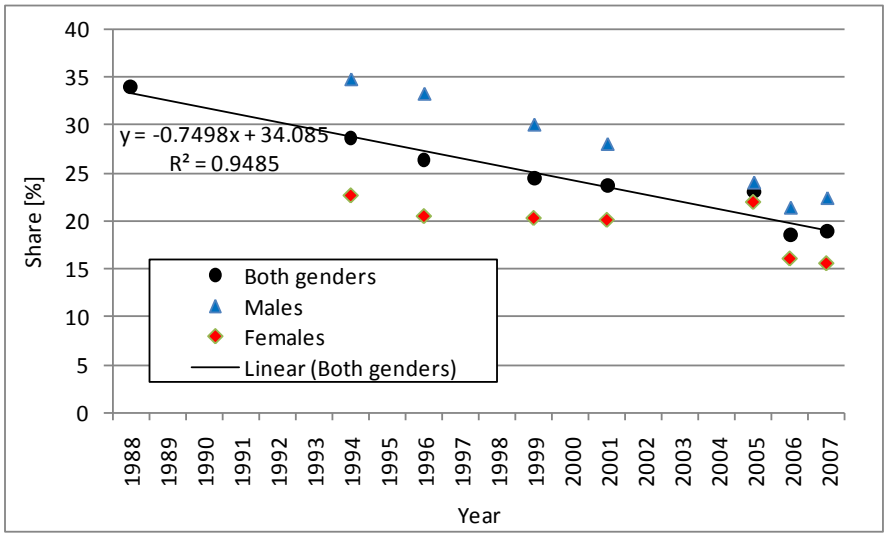
(World Health Organisation, 2009). We will present data available for some of the factors that are believed to affect mortality. Analysing the trends of these factors can be helpful for preparing assumptions about the future development of central death rates. The available data for the Slovenian case are not detailed enough to allow a more advanced analysis of the connections between these factors and deaths by major cause groups.

6.7.1 Smoking

The decline in tobacco consumption is supposed to be one of the key factors underpinning the mortality decline seen in *diseases of the circulatory system* cause group of death during the last few decades. It is believed that tobacco also contributes to many other causes of death. Tobacco accounted for 18% of deaths in high-income countries, while for specific cancers its impact is especially high. It is reported that tobacco smoking alone causes 71% of lung cancer deaths worldwide (World Health Organisation, 2009, p. V). Peto et al. conducted a study based on surveys in 45 developed countries for the 1950–2000 period. They conclude that smoking is currently responsible for about two million deaths per year, about half of which are concentrated in ages between 35 and 69 (Ridsdale & Gallop, 2010b, p. 25).

For the Slovenian case, rich data sets on smokers are not available. However, in the WHO databases data on "regular daily smokers in the population, age 15+" for some years are available. The data are presented in Figure 6-19. They show a clear negative trend in the 1988–2007 period. While in 1988 more than one-third of people aged 15+ were regularly smoking, in 2007 this share had dropped to less than one-fifth. As indicated by the linear trend line, the share of regularly daily smokers was declining in the 1988–2007 period on average by 0.75 of a percentage point per annum. The trend is negative for both males and females, but in the 1988–2007 period the trend was more negative for males than for females and therefore the difference between the two genders decreased.

Figure 6-19: Percentage of regular daily smokers in the population, age 15+



Source: World Health Organisation

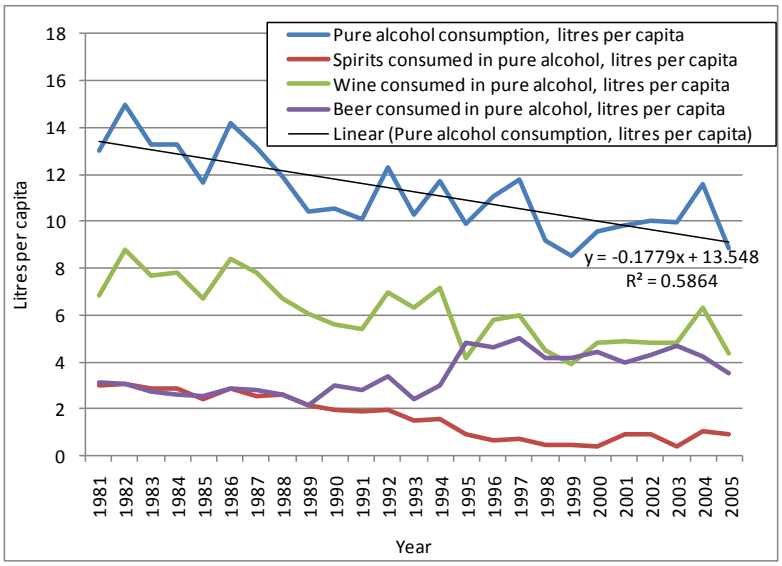
Results for other countries suggest that smoking is closely related to various cause groups of death. Based on these results, which reveal a strong decline in smoking habits among the Slovenian

population, we conclude that smoking is one of the very important factors that also explain the mortality decline seen in the last few decades in the Slovenian case.

6.7.2 Alcohol

Alcohol consumption is another lifestyle variable for which we have available data. From Figure 6-20, a substantial decline during the 1981–2005 period can be observed. Alcohol has a negative effect on people's health, especially if people drink it in large quantities for a longer period.

Figure 6-20: Alcohol consumption in the 1981–2005 period – total consumption and the consumption by sorts of alcoholic drinks



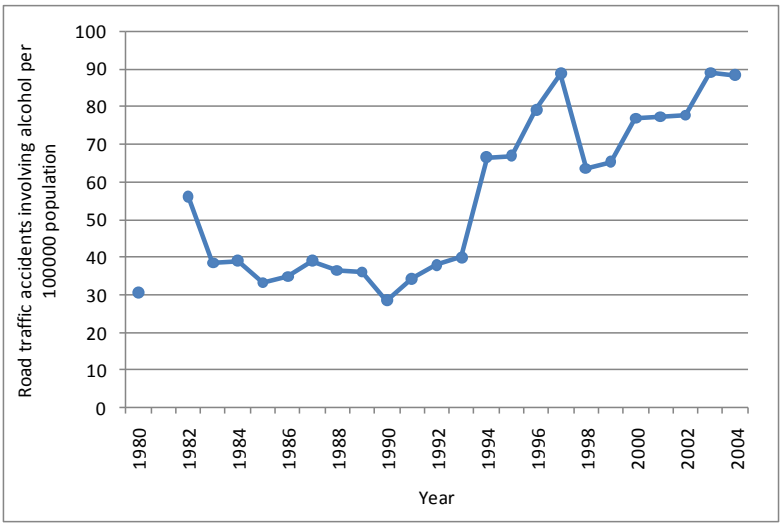
Source: World Health Organisation

In addition, if one drives with alcohol in the blood the probability of causing a traffic accident rises strongly. In Many other risk factors have an important impact on mortality. For example, we would like to have data on body mass index, data on the consumption of medications that substantially reduce mortality for big groups of diseases etc.

Figure 6-21 the number of traffic accidents per 100,000 population is presented. However, the increasing trend could be misleading because the number of register vehicles and the number of people driving vehicles has also substantially increased, so data for these aspects should be obtained for the same period. In addition, time spent in traffic could change. The decomposition of traffic accidents by cause could be another candidate for appropriately measuring the effect of alcohol on traffic accidents.

Many other risk factors have an important impact on mortality. For example, we would like to have data on body mass index, data on the consumption of medications that substantially reduce mortality for big groups of diseases etc.

Figure 6-21: Road traffic accidents involving alcohol per 100,000 population



Source: World Health Organisation

6.8 Conclusions

By far the most important causes of death in Slovenia in 2008 were "diseases of the circulatory system" (39.5% of all deaths) and "neoplasms", i.e. cancer (31.4% of all deaths). By also including the third largest group – deaths caused by external causes (8.3% of all deaths) – about 80% of all deaths are included in the analysis. The trends of these three cause groups of death differ. Diseases of the circulatory system exhibit a strong decline in the 2071–2008 period: from over 900 to less than 400 deaths per 100,000 population (using a standard population from 2008) in the 1971–2008 period. Representing a high share among all causes of death, this group is a main driving force in the decline of total mortality in the 1971–2008 period in Slovenia. Although the share of external causes of morbidity and mortality is much smaller and the negative trend is less pronounced, they contribute substantially to the total mortality decline as well. In contrast, neoplasms do not show any clear trend. If the trends for these two groups of diseases were to continue, neoplasms would become the no. 1 cause of death in the near future.

By taking into account the trends in mortality by the three main cause of death groups in the past, we project the future development of longevity. In a baseline scenario we assume gradual convergence from initial rates of mortality improvement (average annual growth rates in 2001–2008 period) to a 1% long-term rate of mortality improvement. Under these assumptions, life expectancy at birth for males would increase from 76.3 years in 2010 to 83.9 years in 2060 whereas for females it would increase from 83.1 years in 2010 to 90.0 years in 2060.

7 FORECASTING MORTALITY USING THE LEE-CARTER METHODOLOGY

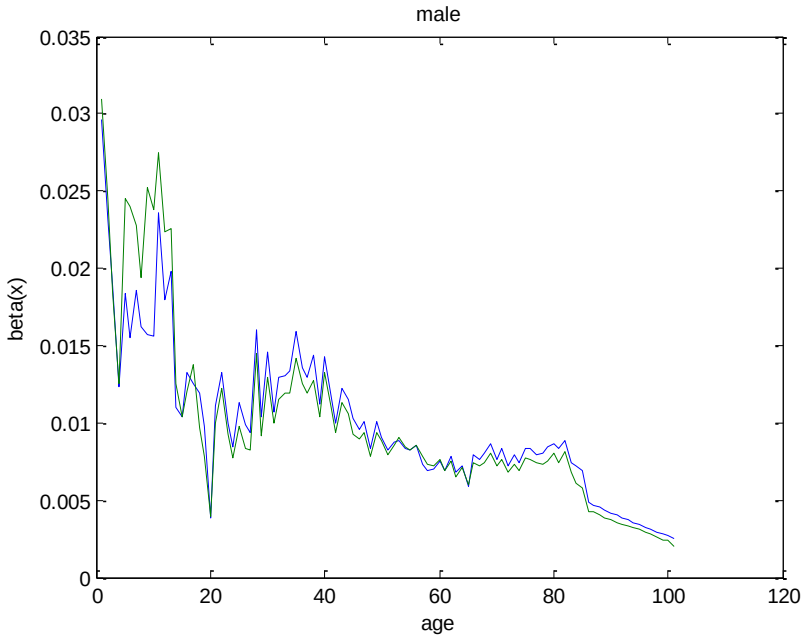
In this chapter we give the results for all of the three stochastic methods briefly introduced in Chapter 4. Based on back-testing results we will decide upon the method for projecting future mortality. In the next subsection we present the results for the Lee-Carter, Poisson log-bilinear and APC models. First, we outline the results for the Lee-Carter model vs. Poisson log-bilinear model based on non-smoothed data due to the fact that both models are three-parameter models and are thus easier to compare graphically/visually, whereas APC is a five-factor model and hence we analyse the result in a separate subsection. The code for this chapter was entirely programmed in Matlab.

7.1 Stochastic methods implemented in Slovenian mortality data

7.1.1 The Lee-Carter model vs. the Poisson log-bilinear model

In this chapter we present the results of the model introduced in Brouhns et al. 2002. The results of the model are compared to the results of the Lee-Carter model (as in Lee and Carter 1992). As mentioned in the previous sections, we use data from the Slovenian Statistical Office on mortality by age for the period between 1971 and 2008. For the purpose of our analysis we had to manipulate the data in order to make the computations feasible. Namely, one assumes when using the LC model that all $m_x(t)$ are larger than 0. In some instances for Slovenia, especially at older ages due to the small population involved, this is not the case. In addition, due to sampling issues, data for ages above 85 for the 1971 to 1980 period are relatively unreliable. Thus, we used some of the methods mentioned in Pitacco et al. (2010) to adjust the data as described in Section 3.5.

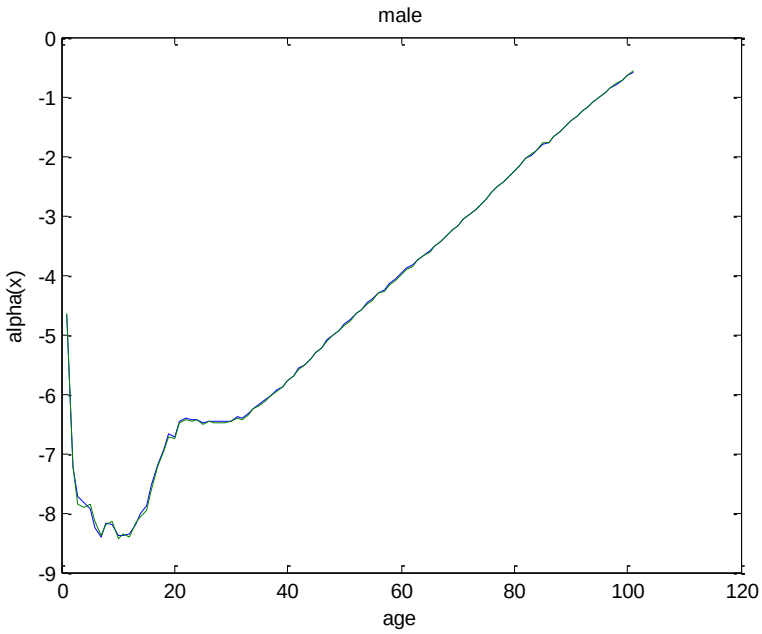
Figure 7-1: Beta(x) as a function of age (males): the Poisson vs. the LC model



As one can see from the figure, beta exhibits highly erratic behaviour when using both methods. This is mainly a consequence of the small population and relatively small number of both exposures and deaths, which contribute to the relatively higher volatility of estimates than is the case for larger

countries. If one compares these results with the results of the LC model obtained from smoothed data using m-splines (see Appendix 5), one notices that the trend behaviour is similar in both cases but with much less noise in the case of smoothed data. Looking at the figure, we observe that over the past 40 years the biggest improvements in mortality were in the age group of minors, especially new-borns and kids aged between 10 and 14 years. This is also the age group for which the discrepancy between the LC and Poisson log-bilinear model is largest. In the case of Poisson log-bilinear, beta is somewhat larger for this age group than the LC method and slightly lower for most of other age groups. The results in beta indicate similar trends as in other countries, with the largest improvements in mortality for lower age groups.

Figure 7-2: Alpha as a function of age (males): the Poisson vs. the LC model



The results for alpha for both models indicate there are hardly any differences in calculated values. The only difference is a small discrepancy in the values of alpha for ages between 2 and 10 years. Overall, the results of both models are similar to those for other countries. Mortality is relatively high for new-borns and drops considerably for minors. For teenagers and young adults, mortality is increasing with a noticeable hump ("the testosterone hump") around the age of 20. After that age, mortality is slightly decreasing or constant and starts to linearly increase around the age of 30.

Figure 7-3 shows that with both methods kappa decreases substantially over the observed period (1971–2008). During that time, we have witnessed a continuous improvement in mortality for all age groups (also see the corresponding figure for beta). Given an average value of beta of around 0.01, mortality has on average more than halved for the observed period. Of course, some age groups (such as minors) have observed decreases in mortality far higher than the average, whereas old age groups have improvements below or well below the average. If we look at the trend for kappa we can see that with the LC method kappa is almost linearly decreasing, whereas for the Brounhs et al. method it seems to be increasing at an even higher rate and exhibits a mild curvature.

Figure 7-3: Kappa as a function of year (males): the Poisson vs. the LC model

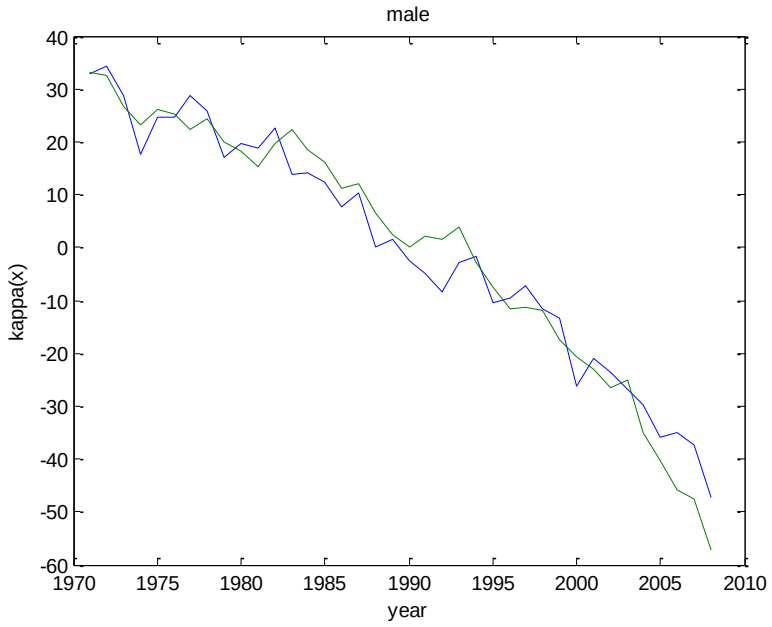
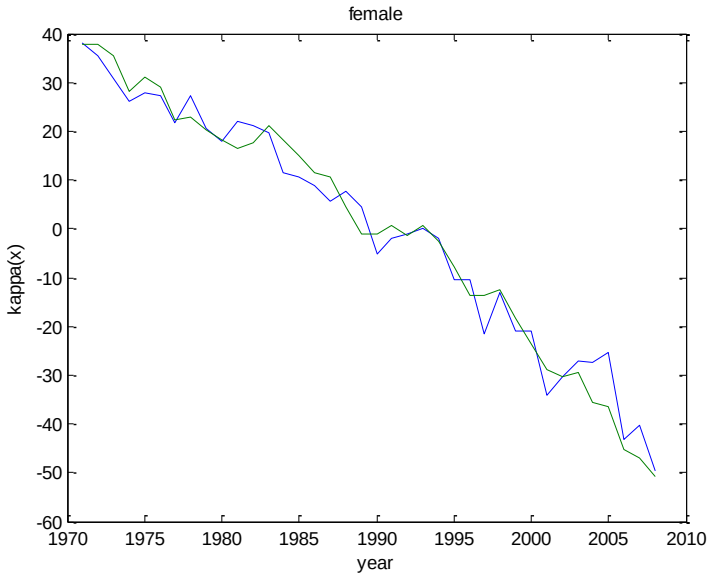


Figure 7-4: Kappa as a function of year (females): the Poisson vs. the LC model



Looking at the results for females, we can see that the trend in kappa is similar as for males. Kappa is decreasing almost linearly in time with both methods. Again, both methods yield very similar results with only slight differences in calculated values for some years. Overall, one can conclude that the results of the two methods are relatively robust in terms of the obtained values of kappa. From the values of beta we can again conclude that there are no significant differences between the two methods. Once again the estimates are not as smooth as the estimates based case on the LC method using smoothed data. This is a consequence of a small population and relatively small number of both exposures and deaths, which contribute to a higher volatility of estimates as compared to larger countries. The biggest difference between the two methods occurs for minors and teenagers, with the Poisson model giving estimates above those obtained with LC; on the other hand, for age groups

above 20 LC gives estimates somewhat higher although the estimates obtained with the two methods are comparable. Looking at Figure 7-6 presenting alpha, one can see that both methods yield similar results. Differences in values exist primarily for age groups between 15 and 25. In both cases, mortality drops significantly for minors and rises thereafter. After age 30 the trend is almost linear (as already observed by Cairns et al. 2006).

Figure 7-5: Beta(x) as a function of age (females): the Poisson vs. the LC model

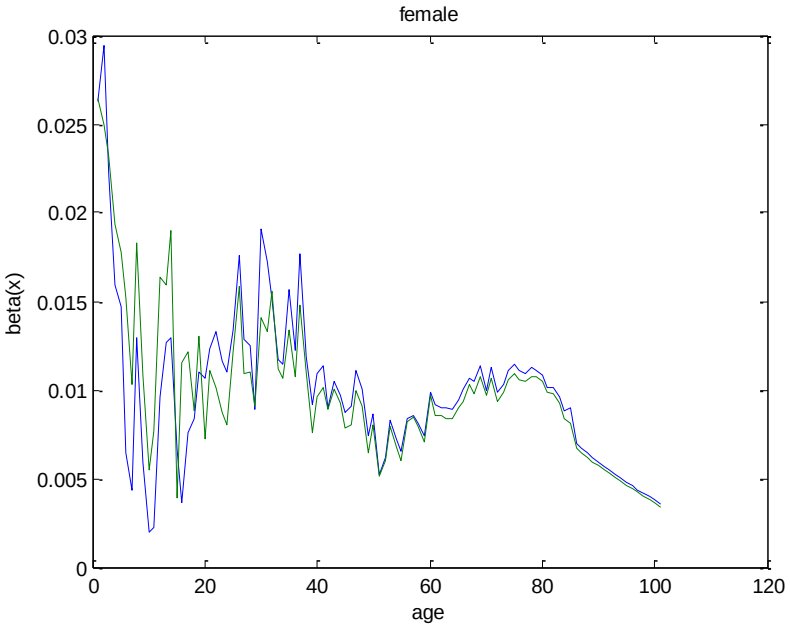
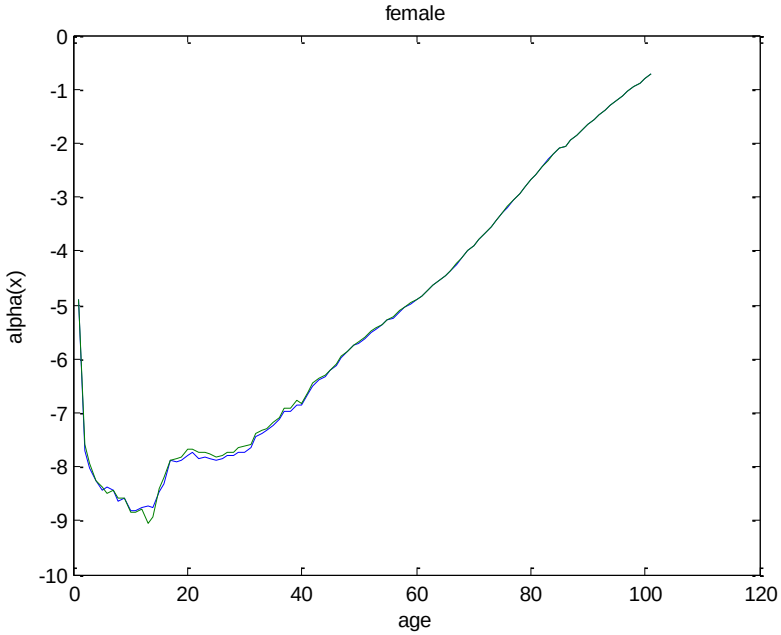


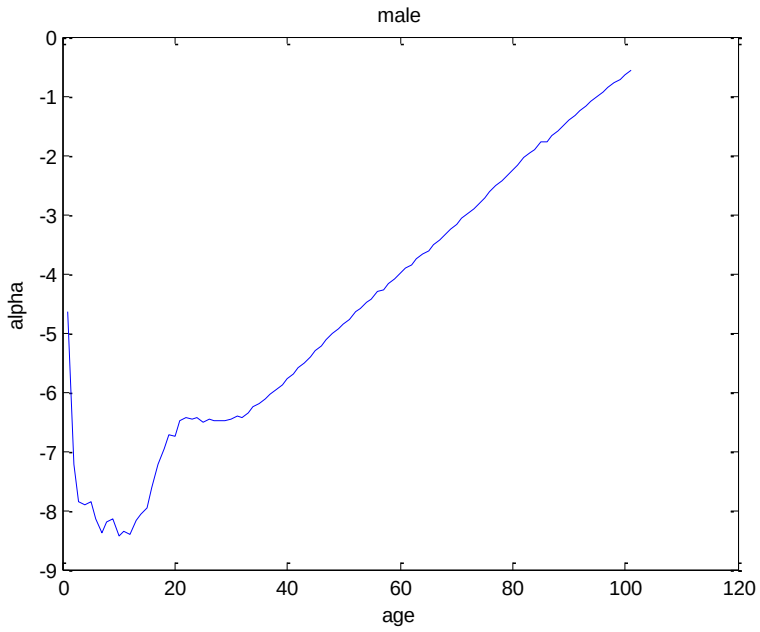
Figure 7-6: Alpha as a function of age (females): the Poisson vs. the LC model



7.1.2 The APC model

In the next section we give the results of the APC model for raw (non-smoothed) data for both males and females. We start with males first.

Figure 7-7: Alpha as a function of age (males): the APC model



As one can see from Figure 7-7, the values for alpha are similar to the values obtained with other methods such as the Poisson model (Brouhns et al. 2002) and the simple Lee-Carter model. The dynamics of beta (see Figure 7-8) are also similar to the Lee-Carter case, with the largest values occurring at young ages and the lowest values for old ages. There are occasional downward spikes in beta such as a spike at age 20 and another one around the age of 50, but on average the improvements in mortality over time have favoured younger generations more than older generations. Looking at kappa (see Figure 7-9), we observe a similar downward trend as noticed in the Poisson model and LC model. As shown by the graph, the trend is almost linear and significantly negative which indicates that the improvements in mortality were significant in the 1971 to 2008 period. Examining the cohort effect given by iota (see Figure 7-10) as a function of birth year, we can see that the overall effect for cohorts born after 1900 (males) is relatively small. If we compare the values of iota for Slovenia with the values of iota for the UK (there is also around 40 years of data with age groups divided between one-year classes from zero to 99 years), we can see that in the Slovenian case they amount to less than one-tenth of the values for the UK. This indicates that, compared to the UK, the cohort effect is not as significant. Given this fact when calculating future mortality, one is led to assume that the differences in projected mortality using the APC, LC or Poisson models are not significant.

Figure 7-8: Beta as a function of age (males): the APC model

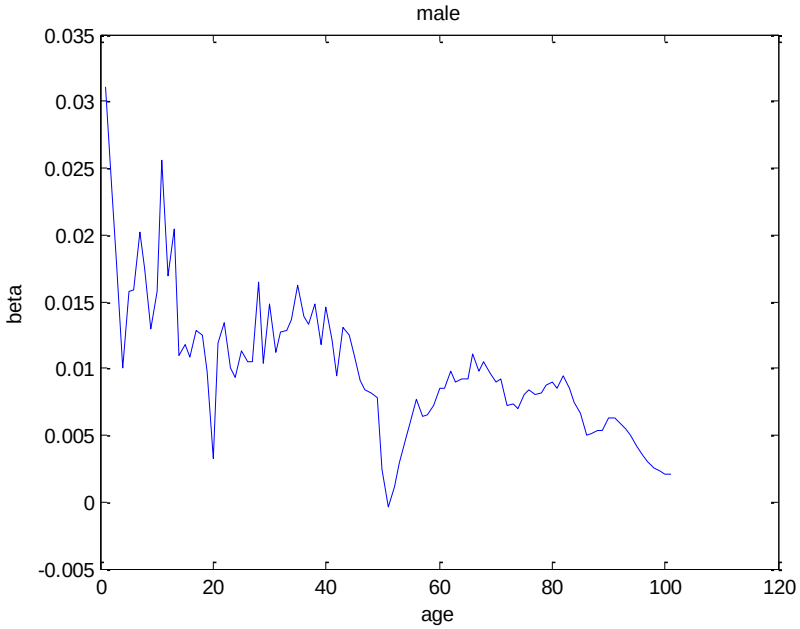
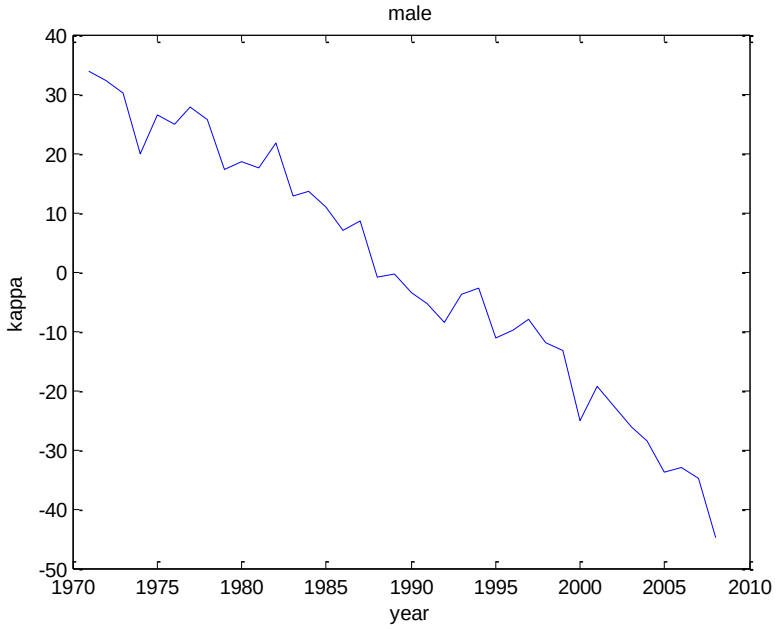


Figure 7-9: Kappa as a function of year (males): the APC model

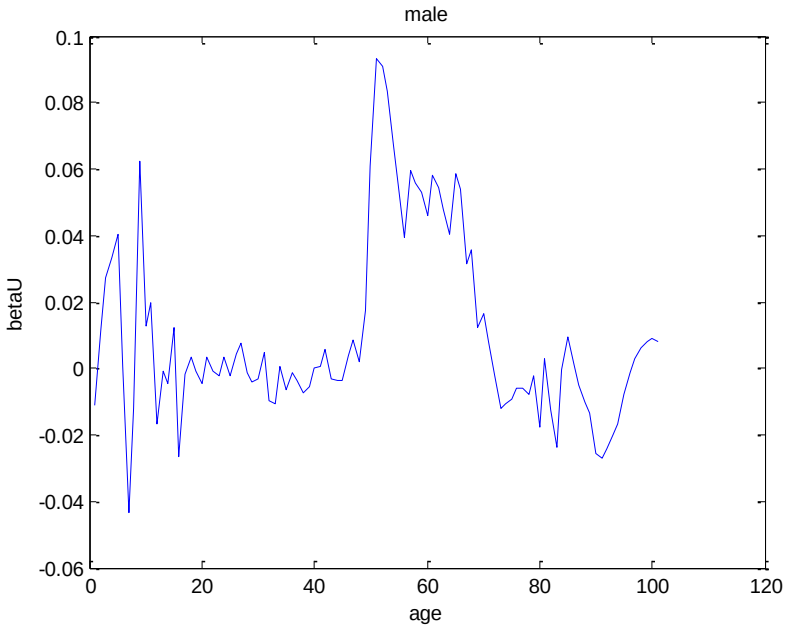


Looking at the values for alpha again, we can conclude that there are no significant differences among all three models. The dynamics of beta for females are quite different than for males, especially for teenagers where the effect of the improvement in mortality over time caused only a mild improvement in mortality. A similar result is observed for the LC model, although the spike (downward) is not as significant as with the APC model. Observing the values of kappa, we can see that the trend is again negative and significant with only a mild curvature. By contrast, the cohort effect is not as straightforward as is the case for males since the 1972 cohort has a significant and

negative value of iota. When examining this effect, we do not find any reasonable explanation except that there might be a problem with the data. We also observe high and highly volatile values of iota for cohorts born before 1905, which is simply a consequence of the small population size and probably also the scarcity of data (high age groups with a relatively small exposure and relatively few deaths).

Figure 7-10: The cohort effect (males): the APC model

Figure 7-11: Beta0(x) as a function of age (males): the APC model



Let us now consider the case of the APC model's results for females.

Figure 7-12: Alpha(x) as a function of age (females): the APC model

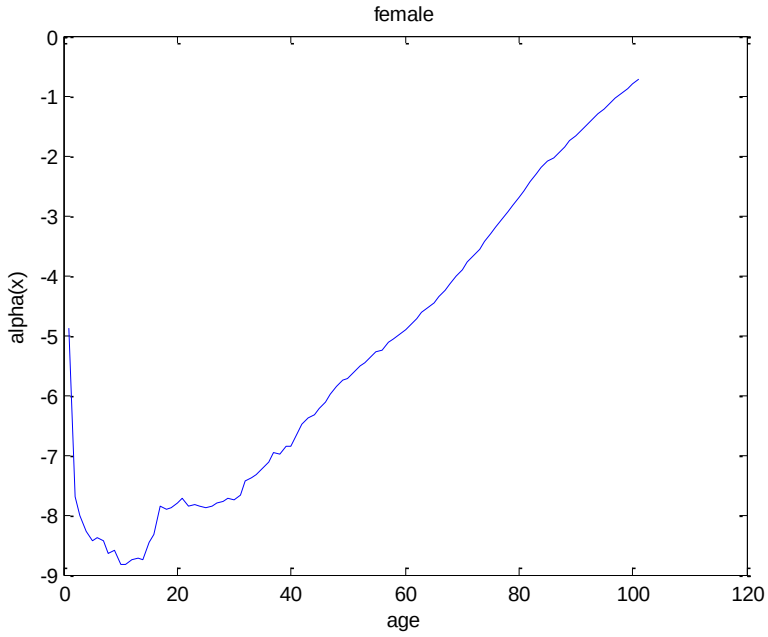


Figure 7-13: Beta(x) as a function of age (females): the APC model

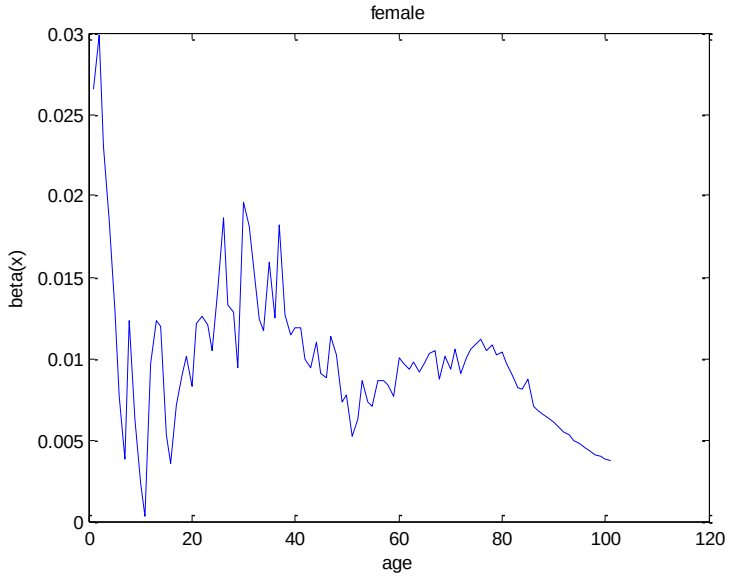


Figure 7-14: Beta0(x) as a function of age (females): the APC model

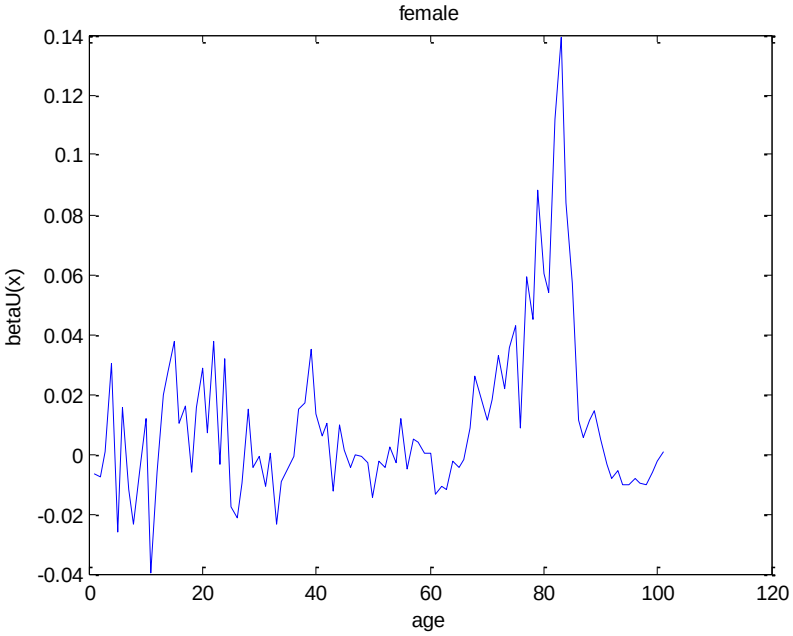


Figure 7-15: Kappa(t) as a function of year (females): the APC model

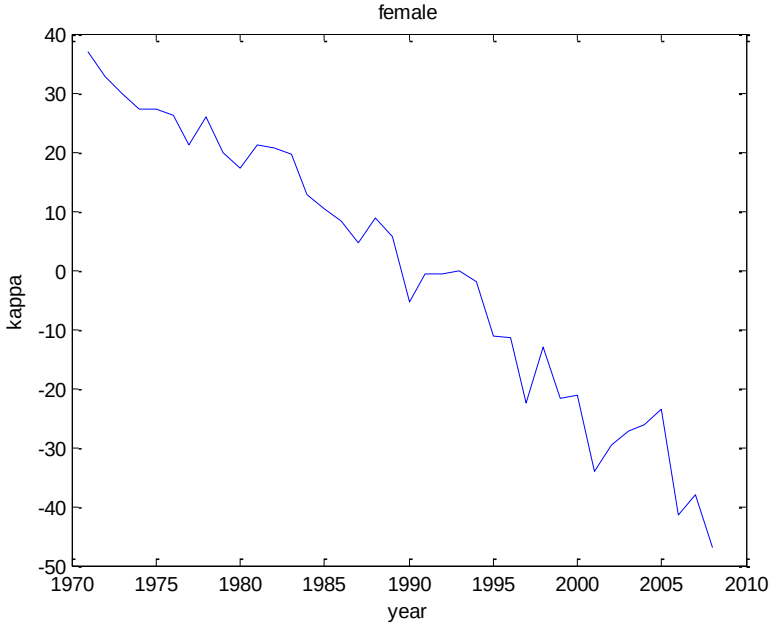
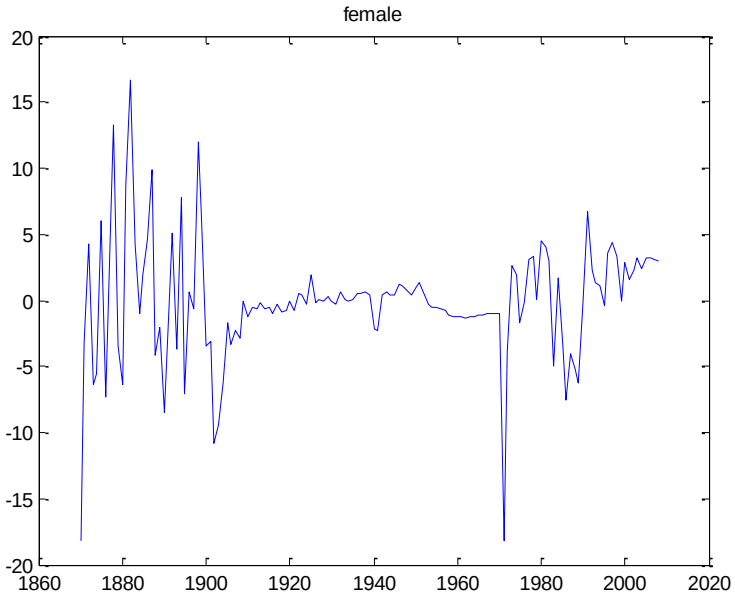


Figure 7-16: The cohort effect (females): the APC model



7.2 Modelling kappa

In order to obtain estimates of future mortality one needs to estimate the dynamics of kappa for both men and women. As noted by several authors, κ_t can be regarded as a stochastic process that can be modelled by fitting an ARIMA (p, d, q) model.

7.2.1 Modelling kappa using the Poisson log bilinear model

Now let us look first at the results of the Poisson model (Brouhns et al. 2002). In this case, the value of c is equal to -2.43 (equation (4.27)). In order to check the validity of the model we have to examine the statistical properties of the residuals.

Table 6: Summary statistics for residuals using Poisson model (males)

Mean	2.88E-16
Median	-0.307543
Maximum	6.586123
Minimum	-7.556903
Std. Dev.	3.349367
Skewness	-0.049737
Kurtosis	2.642194
Jarque-Bera	0.212627
Probability	0.899143
Sum	1.95E-14
Sum Sq. Dev.	403.8574
No. of observations	37

Source: SORS and own work

Table 6 shows we cannot reject the hypothesis of normally distributed residuals. Both the Jarqure Berra test and the value of the kurtosis and skewness indicate that the normal distribution is a good approximation for the distribution of the residuals. Further, when looking at Q-statistics (see

Appendix 4) for autocorrelation we observe that there is no statistically significant autocorrelation.

Figure 7-17: Kappa(t) as a function of time (males): the APC model

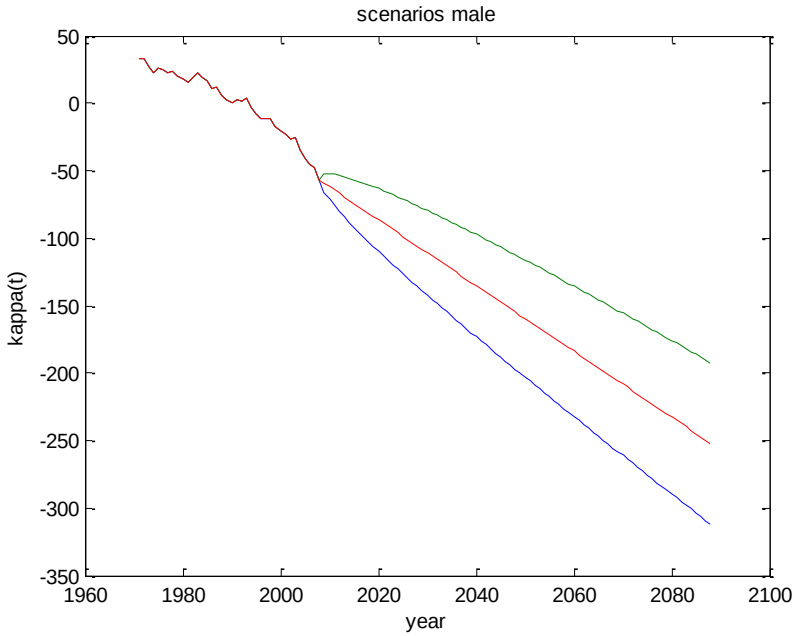


Figure 7-17 reveals quite a strong downward trend in mortality. For the low mortality scenario the trend is somewhat higher due to the higher than expected decrease in kappa. Likewise for the high mortality scenario, the decrease in kappa is somewhat lower. We can see that in both cases the trend is negative, meaning that the reduction in kappa is highly statistically significant.

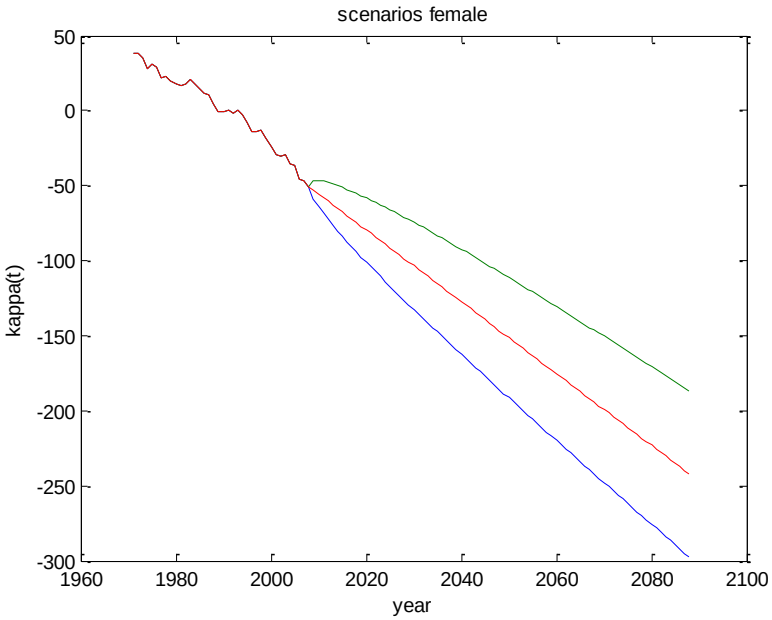
Now let us consider the result for females. Again the ARIMA (0,1,0) proves to be an adequate model for modelling the dynamics of kappa.

Table 7: Summary statistics for residuals using the Poisson log bilinear model

Mean	-3.24E-16
Median	0.199426
Maximum	6.081238
Minimum	-6.402445
Std. Dev.	3.093559
Skewness	-0.028232
Kurtosis	2.224110
Jarque-Bera	0.933006
Probability	0.627192
Sum	-7.99E-15
Sum Sq. Dev.	344.5238
No. of observations	37

Source: SORS and own work

Figure 7-18: Kappa(t) as a function of time (females): the APC model



As one may see from Table 7 and the corresponding table in, the residuals obtained using the ARIMA (0,1,0) model are uncorrelated and approximately normally distributed. Figure 7-18 shows that the difference between scenarios is increasing in time but the estimates still remain fairly close.

7.2.2 Modelling kappa using the LC model

Now we consider the case of modelling kappa when using the original Lee-Carter model for males. Again we can see that the simple ARIMA (0,1,0) does a good job of capturing the dynamics of kappa. The residuals are not autocorrelated, whereas the skewness, kurtosis and Jarqure Berra tests indicate that the hypothesis of normally distributed residuals cannot be rejected. Another important consequence of the residual test is the fact that with the LC model the standard deviation of residuals is higher than with the Poisson log bilinear model. Thus the difference between the forecasts under the three scenarios is much smaller in the case of the Poisson log bilinear model.

Figure 7-19 reveals there is again quite a strong downward trend in mortality. The trend is somewhat lower than for the case of the Poisson model but, on the other hand, the standard deviation of residuals is higher than with the Poisson model. The values for kappa under the high mortality scenario are thus higher in the original LC model than in the Poisson model. The same goes for the middle scenario. The values for the low mortality scenario in the LC model are comparable to the values in the Poisson model.

Now we consider the LC model for females and try to estimate the dynamics of kappa. In contrast to the case for males, in the case of females the dynamics cannot be modelled as an ARIMA (0,1,0) model, but can best be fitted by an ARMA(2,2) model. In this case, as one can see from Table 9, the residuals are not autocorrelated (at least up to the relevant number of lags) and the residuals can be assumed to be normally distributed.

Table 8. Summary statistics for residuals using the LC model (males)

Mean	-5.96E-16
Median	-3.33E-15
Maximum	10.12461
Minimum	-11.78582
Std. Dev.	4.889802
Skewness	-0.409455
Kurtosis	3.041657
Jarque-Bera	1.036536
Probability	0.595551
Sum	-2.13E-14
Sum Sq. Dev.	860.7658
No. of observations	37

Source: SORS and own work

Figure 7-19: Projecting kappa using the LC model for the 2008–2088 period (males)

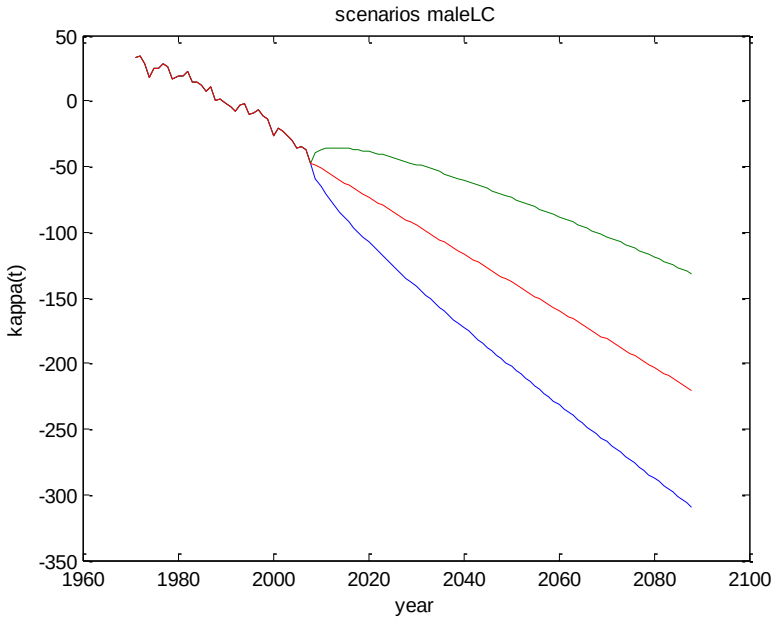


Table 9. Summary statistics for residuals using the LC model (females)

Mean	-0.052288
Median	0.183130
Maximum	7.569289
Minimum	-11.08728
Std. Dev.	4.813961
Skewness	-0.559959
Kurtosis	2.718712
Jarque-Bera	2.000007
Probability	0.367878
Sum	-1.882381
Sum Sq. Dev.	811.0976
No. of observations	36

Source: SORS and own work

Once again, we observe that the standard deviation of estimates of kappa is significantly higher than with the case of the Poisson log bilinear model. This is again another argument in favour of using Poisson log-bilinear for modelling future mortality. Namely, with the Poisson log bilinear model the confidence interval of estimates is much narrower than with the LC and APC models.

7.2.3 Modelling kappa using the APC model

In contrast to the LC and Poisson log bilinear models, the APC model does not allow us to use ARIMA (0,1,0) to model the evolution of kappa. As one can see, the appropriate model is achieved by using the AR(2) model, which gives us both uncorrelated residuals that are approximately normally distributed.

Table 10 demonstrates that the standard error of the kappa model is much higher than with the Poisson log bilinear model, hence there is a much larger deviance among the three scenarios.

Table 10: Summary statistics for residuals using the APC model (males)

Mean	-3.36E-10
Median	0.790038
Maximum	7.113411
Minimum	-14.58855
Std. Dev.	5.188256
Skewness	-0.785145
Kurtosis	3.209650
Jarque-Bera	3.764647
Probability	0.152236
Sum	-1.21E-08
Sum Sq. Dev.	942.1301
No. of observations	36

Source: SORS and own work

In the case of the APC model (females) we obtain a good fit using the ARMA(1,1,0) model. In this case, the residuals are again uncorrelated (see Appendix 4).

It is apparent when looking at Table 11 that we cannot reject the hypothesis of normally distributed residuals. Looking at the standard deviation of the estimates of the kappa model, we see that the values are again higher than in the case of the Poisson log bilinear model.

Table 11: Summary statistics for residuals using the APC model (females)

Mean	1.09E-08
Median	0.122306
Maximum	6.732146
Minimum	-9.420403
Std. Dev.	4.217108
Skewness	-0.440758
Kurtosis	2.614452
Jarque-Bera	1.388577
Probability	0.499430
Sum	3.92E-07
Sum Sq. Dev.	622.4401
No. of observations	36

Source: SORS and own work

7.3 Back-testing

When making future projections we need to make a quantitative assessment of all three models. This cannot, of course, be done graphically, so we use the back-testing technique to determine which model would be most appropriate for estimating future mortality. More precisely, the models considered are tested against real data (in our case the number of deaths) for the period 2001 to 2008. In the first step, we fit the model parameters to the data for the period 1971 to 2000. In the second step, the values of the parameters obtained in step one are used to predict the number of deaths D_{xt} for the 2001–2008 period. Several quantities of fit measures are then used to compare the methods.

Table 12. Comparison of methods using back-testing for the 2001–2008 period (males)

	LC	APC	Brouhns
MSE	21	25	20.8
MPE	11.7	13.6	11.6
R^2	0.955	0.935	0.96

Source: SORS and own work

Let us first examine the results for males for the period 2001 to 2008. As mentioned, we consider the number of deaths as a function of age and year as an explanatory variable. As Table 12 shows, the LC method is, according to all criteria, second best and comes very close to the Poisson log-bilinear method. On the other hand, according to all measures, APC is clearly last. The supremacy of the Poisson log-bilinear method is even more convincing in the case of females. Ninety-nine per cent of all variation in the number of deaths for the period 2001 to 2008 can be explained via this method (see Table 13).

Table 13. Comparison of methods using back-testing for the 2001–2008 period (females)

	LC	APC	Brouhns
MSE	18.5	25.1	11.5
MPE	11.2	24.9	7
R^2	0.975	0.93	0.99

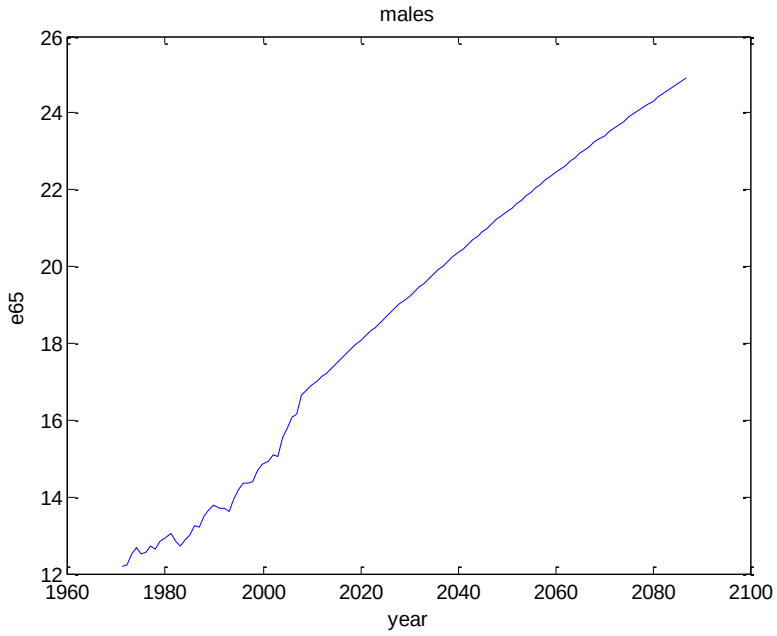
Source: SORS and own work

Based on the back-testing analysis, we may conclude that the Poisson log bilinear model best fits the actual central death rates and will, therefore, use this model for forecasting in the future.

7.4 Expected future lifetime at age 65 by period and population annuity factor

Next we examine the effects of increased longevity on annuities by looking initially at expected lifetime at age 65 e_{65} and later the immediate pension annuity factor at age 65 a_{65}.

Figure 7-20: The expected remaining lifetime as a function of time for males (using projections for kappa obtained by the Poisson log-bilinear method) by period



Looking first at the expected remaining lifetime at 65, we can see that there is a clear upward trend for males. Over a period of 80 years the expected remaining lifetime at age 65 is expected to further increase by 8.5 years. If we translate this prolongation of lifetime into monetary units, the increase in longevity translates into an increase of approximately 21%. The pension annuity factor is expected to rise from approximately 9.3 to a little less than 11.3.

Figure 7-21: The immediate annuity factor as a function of time for males (using projections for kappa obtained by the Poisson log-bilinear method) by period

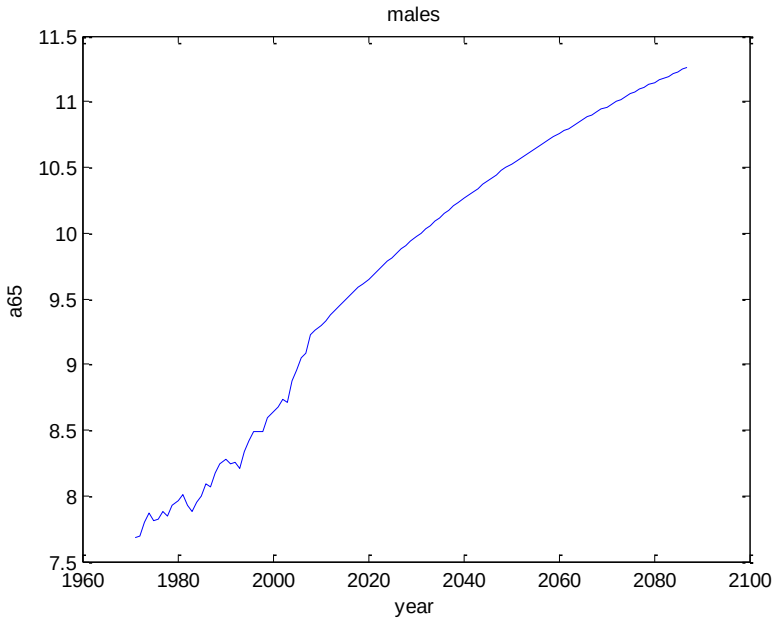
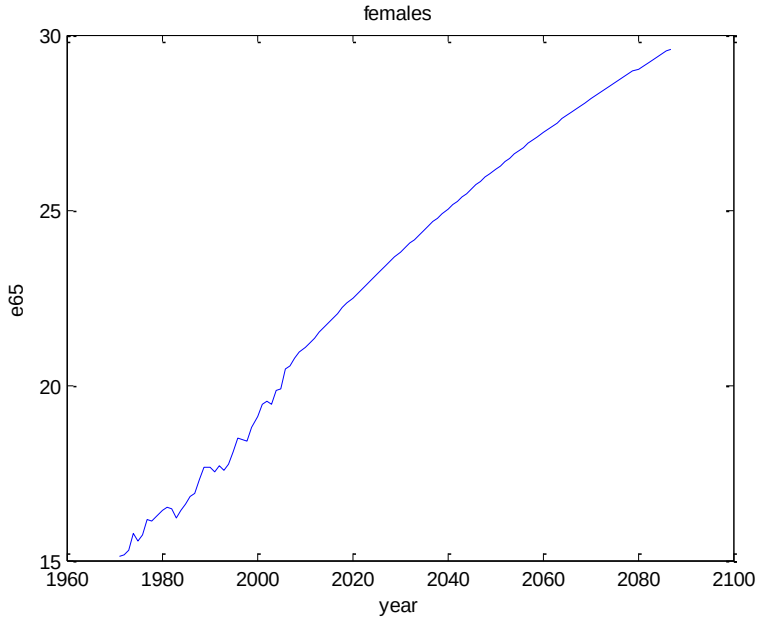
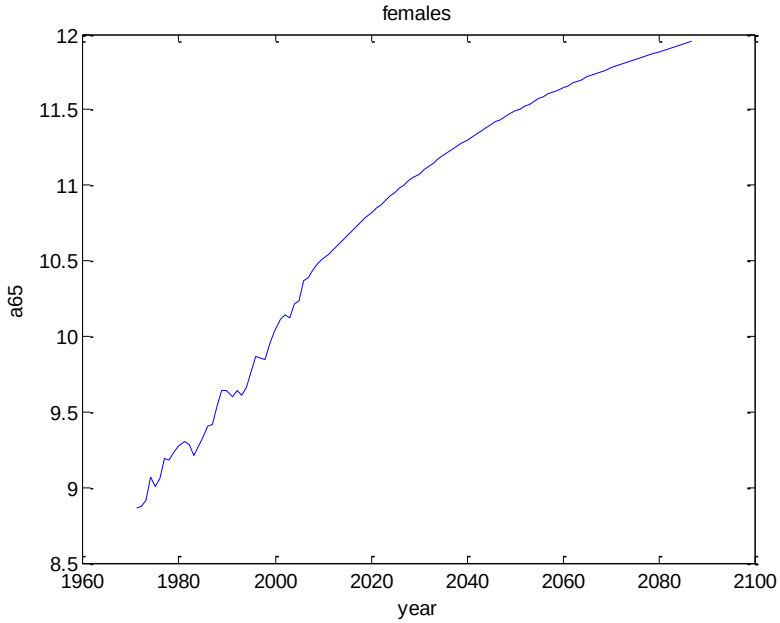


Figure 7-22: The expected remaining lifetime as a function of time for females (using projections for kappa obtained by the Poisson log-bilinear by period



If we consider the case of females, the expected remaining lifetime at age 65 is expected to rise by approximately an equal number of years as with the case for males (approximately seven years). Similarly, we can also observe that the immediate annuity factor is expected to increase by 16%.

Figure 7-23: The immediate annuity factor as a function of time for females (using projections for kappa obtained by the Poisson log-bilinear method) by period



8 CONSTRUCTION OF A SLOVENIAN ANNUITY LIFE TABLE

To calculate the present value of a future obligation arising from life annuity payments, actuaries must develop a life table. A life table shows for each age the probability that a person of that age will die before his or her next birthday. This chapter explains the method used to derive the first Slovenian annuity tables.

8.1 Life tables

In a **period life table**, life expectancy estimates correspond to age-specific mortality rates at age *x* and older ages observed in a given calendar year. In a **cohort life table**, life expectancy estimates correspond to age-specific mortality rates at age *x* and older ages observed for a birth cohort during a long calendar period.

To construct a life table we must first convert central death rates $m_x(t)$ into probabilities of death, $q_x(t)$. Let f_x be the average number of years lived within the age interval [*x*, *x* +1) for people dying at that age. We follow Renshaw and Haberman (2003a) and assume

$$f_x = \begin{cases} \frac{1}{2} & x > 0 \\ 0.15 & x = 0, \, male \\ 0.16 & x = 0, \, female \end{cases} \qquad (8.1)$$

We calculate from the following equation using the piecewise constant forces of mortality assumption

$$q_x(t) = \frac{m_x(t)}{1 + f_x \cdot m_x(t)} \tag{8.2}$$

To complete the period life table calculation we follow the methodology described on the Human Life Table Database webpage. For the period life table we calculate $l_x(t)$ number of survivals and $d_x(t)$ number of deaths by age at period t as

$$\begin{aligned} l_{x+1}(t) &= (1 - q_x(t)) \cdot l_x(t) \\ d_x(t) &= q_x(t) \cdot l_x(t) \end{aligned} \tag{8.3}$$

We denote the number of person-years lived within the elementary age interval [x,x+1) with $L_x(t)$, number of person years lived after the exact age x with $T_x(t)$ and life expectancy at exact age x with $e_x(t)$. The formula for period life expectancy is as follows:

$$e_x(t) = \frac{\sum_{k=1}^{\omega - x} l_{x+k}(t)}{l_x(t)} + \frac{1}{2} \tag{8.4}$$

The calculation of the other life table function is shown below:

$$\begin{aligned} L_x(t) &= \frac{1}{2}(l_x(t) + l_{x+1}(t)) \\ T_x(t) &= l_x(t) \cdot e_x(t) \end{aligned} \tag{8.5}$$

To calculate the **age cohort life table** we must first choose the base cohort birth year τ. We then calculate the life table to take diagonal probabilities from birth year τ as follows

$$\begin{aligned} l_{x+1}(\tau) &= (1 - q_x(\tau + x)) \cdot l_x(\tau) \\ d_x(\tau) &= q_x(\tau + x) \cdot l_x(\tau) \end{aligned} \tag{8.6}$$

Life expectancy is then calculated from

$$e_x(\tau) = \frac{\sum_{k=1}^{\omega - x} l_{x+k}(\tau)}{l_x(\tau)} + \frac{1}{2} \tag{8.7}$$

We can calculate an annuity of size 1 which is payable yearly at the beginning of each year while an insured is alive from

$$\ddot{a}_x(\tau) = \sum_{k=0}^{x-\omega} \left(\begin{cases} 1, & k = 0 \\ \prod_{j=0}^{k-1} (1 - q_{x+j}(\tau + x + j), & k > 0 \end{cases} \right) \cdot (1+i)^{-k} \tag{8.8}$$

8.2 Data used for the cohort projections

To build the cohort projected life table we first construct the following period tables:

from 1945 to 1970 – the log linear interpolation explained in Section 3.5

from 1971 to 2008 – observed central death rates (see Section 3.5.1)

from 2009 to 2118 – projected with the Poisson log-bilinear method – a point estimate is used

from 2118 – no change in mortality beyond 2118

In this way, we obtain a two-dimensional $m_x(t)$ table, where $0 \le x \le 100$ and $1945 \le t \le 2118$.

8.3 Age shifting

In this section the approximation method, which leads the calculation for Slovenian population cohort life tables back to a one-dimensional table, will be explained.

An annuity calculated with a cohort life table not only depends on age at entry, but also on the individual birth year of the insured person. This would lead to the construction of a cohort life table for every generation, in turn involving the use of a large series of tables. This is, of course, impractical and cannot be used in everyday actuarial calculations.

That is why a simplified solution used by several European countries, the so-called Rueff method, has been adopted. The first cohort period is chosen among a series of generation tables, which we call a fundamental (or select) cohort. By shifting the actual age, depending on the birth year, the exact actuarial values will be approximated by using the fundamental cohort with an age shift.

In this respect, the insured persons with birth years above than the birth year of the fundamental cohort will be made younger, while those with birth years below the birth year of the fundamental cohort will be made older to account for the trend of the mortality improvement.
Let us denote $\bar{\tau}$ the chosen fundamental cohort year. Then the adjustment involves an age shift $h(\tau)$ years (plus or minus). Assuming that mortality declines over time, the function $h(\tau)$ must satisfy the following relations:

$$h(\tau) = \begin{cases} \ge 0, & \tau \le \bar{\tau} \\ = 0, & \tau = \bar{\tau} \\ \le 0, & \tau \ge \bar{\tau} \end{cases} \tag{8.9}$$

To determine age shifts $h(\tau)$, a criterion is needed that is clearly based on the "differences" or "distances" between "values" correctly calculated with the intention to use appropriate cohort table

and values calculated with the fundamental cohort table.

As regards the type of criteria, usually the expected present value of a life annuity is used. When a criterion of this type is adopted, function $h(\tau)$ depends on the interest rate used in calculating the actuarial values. This is the approach employed by Germany for the construction of life tables. In the German case, $\ddot{a}_x(\tau)$ is the present value benefit of a life annuity with an annual payment and a discount rate of 2.75% according to the generation table DAV 2004. In practice, a 2.75% interest rate could be problematic if we want to calculate expected present values with a rate very far from 2.75%.

If we take 0%, the expected present values coincide with the expected remaining lifetimes (say, at age 65). Since we want to calculate age shifts appropriate to the Slovenian projected tables, we will use a combination of both 0% (focusing on the expected lifetime) and the rate used in Slovenia for pricing and/or reserving for life annuities (currently 2.75%).

We took the following steps to determine $h(\tau)$:
For each birth year $\tau = 1955,..,2020$ and for all ages $x_{\min} \le x < x_{\max}$, where $x_{\min} = 55$ and $x_{\max} = 65$, the integer shift $h^i(\tau, x)$ is determined, which satisfies the following condition:

$$\ddot{a}^i_{x+h^i(\tau,x)+1}(\bar{\tau}) < \ddot{a}^i_x(\tau) \le \ddot{a}^i_{x+h^i(\tau,x)}(\bar{\tau}) \tag{8.10}$$

We calculated an annuity with both a 2.75% interest rate and a 0% interest rate (i.e. $i \in (2.75\%, 0\%)$).

The birth year $\bar{\tau}$ =1965 was used as the fundamental cohort year because 1965, meaning those who turned 45 in 2010, is an age that can be considered intermediate between those who enter into a deferred annuity and insured persons who buy an immediate annuity. This choice will surely produce an error, which would be reasonably small by using the proposed parameters. The error will increase with the distance of the actual birth year from the fundamental cohort year.

For each period τ we obtain a set of $h^i(\tau, x)$ and we choose to select the average value of them to obtain a single value:

$$h^{2.75}(\tau) = \frac{1}{x_{\max} - x_{\min} + 1} \sum_{x=x_{\min}}^{x_{\max}} h^{2.75}(\tau, x) \tag{8.11}$$

$$h^{0}(\tau) = \frac{1}{x_{\max} - x_{\min} + 1} \sum_{x=x_{\min}}^{x_{\max}} h^{0}(\tau, x) \tag{8.12}$$

and

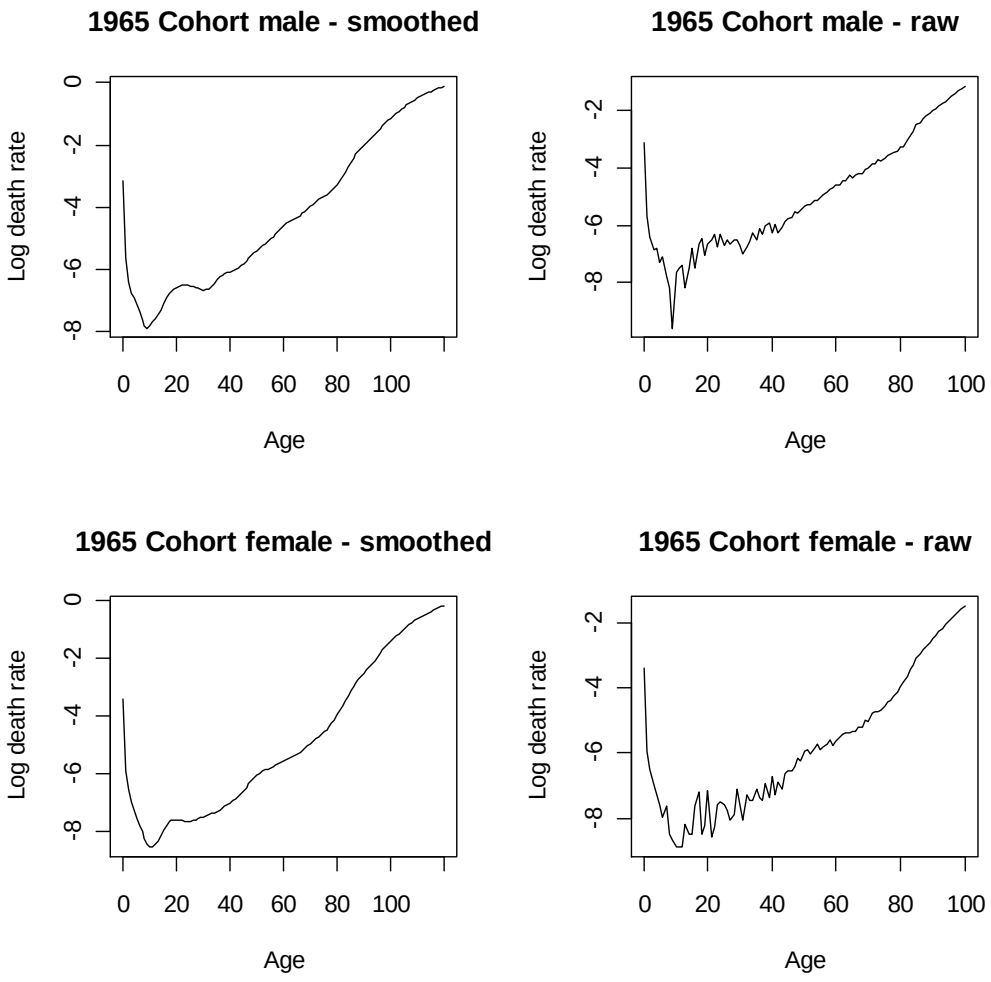
$$h(\tau) = \frac{h^{2.75}(\tau) + h^{0}(\tau)}{2} \tag{8.13}$$

This age shift $h(\tau)$ now depends only on the birth year. The resulting age shifts were rounded up to full years. The result of the procedure is shown in Appendix 2.

8.4 Slovenian reference population mortality table

The birth year $\bar{\tau}$ =1965 was used for the fundamental cohort year to generate a Slovenian base cohort population mortality table. We then used a smoothing procedure to obtain less variability in the data. At old ages we used the same procedure as in Section 3.3 for smoothing at very old ages, with the start smoothing age at 95. Then we smooth the overall data with m-splines with the limit age at 120. The age cohort life table based on the birth year 1965 is denoted **SCO65** (reference population mortality table) and shown in Appendix 1. Figure 8-1 below shows reference population mortality rates for males and females (smoothed rates) and non-smoothed rates.

Figure 8-1 Population fundamental mortality rates



8.5 Selection – theoretical background

The standardised mortality ratio (SMR) is used as an index for comparing mortality experiences: actual deaths in a particular population (for example, annuity owners) are compared with those that would be expected if "standard" age-specific rates were to be applied. The SMR is defined as (Pitacco et al. 2009)

$$SMR = \frac{\sum ETR_{xt}\hat{m}_x(t)}{\sum ETR_{xt}\hat{m}_x^{stand}(t)} \tag{8.14}$$

According to Pitacco et al. (2009), SMRs are around 50% for individual and group life insurance contracts before retirement age, and then decrease to reach 40% for individual policies and increase to 80% for group life policies for the Belgian population.

A life annuity purchaser is, with a high probability, a healthy person with a particularly low mortality in the first years of the life annuity payment and, generally, with an expected lifetime higher than average. In order to take selection into account, Delwarde et al. (2004) suggested a method for adjusting a reference life table to the experience of a given portfolio, based on non-linear regression models using local likelihood for inference.

Let us denote $\hat{m}_x^{HMD}(t)$ population death rates and $\hat{m}_x^{LIM}(t)$ life insurance market rates, then we define

$$\ln \hat{m}_x^{LIM}(t) = f(\ln \hat{m}_x^{HMD}(t)) + \varepsilon_{xt} \tag{8.15}$$

where ε_{xt} is Normally distributed with a mean 0 and a variance of σ^2 and $f()$ is an unknown smooth function. This model explains the link between population death rates and insurance market mortality. In Pitacco et al. (2009), the following model is suggested for ages above 60 years:

$$\ln \hat{m}_x^{LIM}(t) = f(x) + \ln \hat{m}_x^{HMD}(t) + \varepsilon_{xt} \tag{8.16}$$

This leads us to produce SMR in the form $e^{\hat{f}(x)}$, which can be used to adopt mortality projections to the insurance market. This approach will also be used to estimate Slovenian life insurance market rates.

8.6 The UK approach to building an insurance annuity table

8.6.1 Methodology

To build "00" Series tables, Continuous Mortality Investigation (CMI) used graduations of GM(r,s) class of models (the so-called Gompertz-Makeham class of models) in the form

$$\mu_x = \sum_{i=1}^{r-1} \alpha_i x^i + \exp\left[\sum_{j=1}^{s-1} \beta_j x^j\right] \tag{8.17}$$

They choose parameters by maximum likelihood, also taking account of the usual diagnostic tests (numbers of positive and negative deviations, runs, Kolmogorov-Smirnov, serial correlations and χ^2).

At the oldest ages, the values of μ_x for $x > a$, were blended into an arbitrary $\mu_{120} = 1$ using the formula:

$$\mu_x = \frac{(120-x)^c}{(120-a)^c}\mu_a + (1 - \frac{(120-x)^c}{(120-a)^c})\mu_{120} \qquad (8.18)$$

A CMI investigation covers data of immediate annuitants, retirement annuitants, personal pensioners, life office pensioners and widows of life office pensioners.

8.6.2 Observed data

"Immediate annuitants" cover (non-pension) purchased life annuities and they are among the longer running CMI mortality investigations. It consists of over 64,000 observed males and 142,000 observed females in the period from 1979 to 2002. This investigation is important because it measures the second type of the selection of immediate annuitants. Tables are published in the "I" series of life tables.

Table 14: Summary of immediate annuitant tables

Table name	Sex	Select period	Age range
IML00	male	None	60 – 120
IFL00	female	1 year	60 – 120

"Retirement annuitants" cover the mortality experience of the self-employed who have purchased retirement annuities. Data are gathered for both males and females, and are subdivided into two sections, deferred and vested, which together form the combined section. Tables are published in the "R" series of the "00" life tables.

"Personal pensioners" represent the mortality experience of holders of personal pension policies affected under Chapter IV of Part XIV of ICTA 1988. Data are gathered for both males and females, and are subdivided into two sections, deferred and vested, which together form the combined section. Tables are published in the "PP" series of the "00" life tables.

"Life office pensioners" cover the mortality experience of life office pensioners, i.e. retirements from occupational schemes where the benefits have been insured. It is carried out for both sexes on the basis of both lives and amounts, and is sub-divided into those who retired at or after the normal retirement age for their scheme (referred to as "Normal" retirements) and those who retired before their normal retirement age (referred to as "Early" retirements). Tables are published in the "PN", "PE" and "PC" series of the "00" life tables. Total numbers of those exposed to risk were around 1.6 million for males and 0.66 million for females (normal lives). Average amounts per life exposed to risk for 1999–2002 were GBP 2,388.61 for males and GBP 1,272.79 for females (normal lives).

The following GM models were used for the graduation of Normal pensioners' ultimate experience:

Male, Lives : GM(1,4)

Male, Amounts: GM(1,3)

Female, Lives : GM(2,2)

Female, Amounts: GM(2,2)

Table 15: Summary of life office pensioner tables

Table name	Category	Sex	Exposure	Age range
PNML00	Normal	male	Lives	20 – 120
PNMA00	Normal	male	Amounts	20 – 120
PNFL00	Normal	female	Lives	20 – 120
PNFA00	Normal	female	Amounts	20 – 120
PCML00	Combined	male	Lives	20 – 120
PCMA00	Combined	male	Amounts	20 – 120
PCFL00	Combined	female	Lives	20 – 120
PCFA00	Combined	female	Amounts	20 – 120
PEML00	Early	male	Lives	50 – 120
PEMA00	Early	male	Amounts	50 – 120
PEFL00	Early	female	Lives	50 – 120
PEFA00	Early	female	Amounts	50 – 120

8.6.3 Projections

It should be pointed out that there are no generally prescribed annuity tables in the UK, so each company makes its own projections according to the collected data described in the previous section. Projections are made using the standard exponential formula, the Lee-Carter methodology and projections with p-splines.

8.7 The German approach to building DAV 2004 R tables

8.7.1 Introduction

The German Actuarial Society (*Deutsche Aktuarvereinigung, DAV)* came to the conclusion that the post-2000 DAV 1994 R mortality improvement trend assumption did not appropriately reflect the mortality improvements in the last three decades of the 20th century. It was decided to develop a new table, called DAV 2004 R. For the first time it was also assumed that annuitants' mortality improvements exceed those of the general population (this is the main basis of their approach).

The DAV 2004 R tables are based on data that encompass 13.7 million years' exposure from 1995 and 2002. According to the data, the difference between the mortality rates of insured lives and the general population is slightly more pronounced than was assumed for DAV 1994 R.

The DAV 2004 R table consists of the following components:

• 2nd-order base table: a best estimate of insured lives' mortality rates in 1999;

• 1st-order base table: the 2nd-order base table reduced to take account of provisions for adverse deviations;

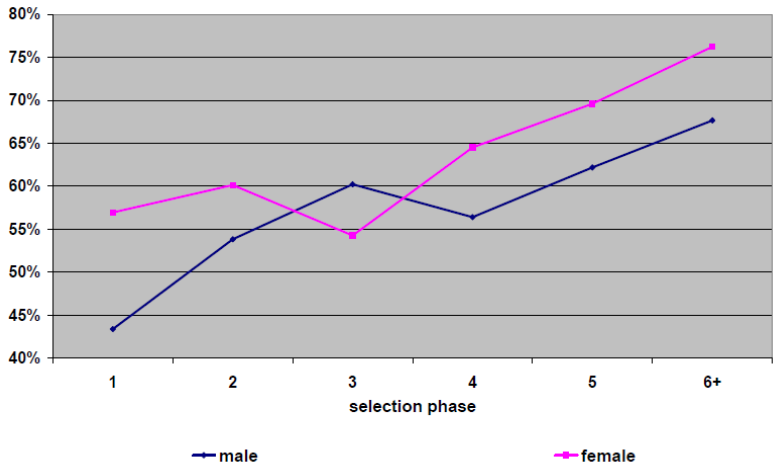
• 2nd-order mortality trend: the best estimate of future mortality improvements; and

• 1st-order mortality trend: the 2nd-order trend increased to take account of provisions for adverse deviations.

In order to incorporate the selection effect, DAV divided the annuity payment period into six selection phases by the number of years that have lapsed since the start of the benefit payment: 1st year,..., 5th year, 6th+ years ("ultimate").

Selection factors were derived by comparing the mortality rates in the different selection phases with the population mortality. The figure below shows the various ratios for the observed period. The initial comment by the working party is that the German selection factors do not differ much from the UK selection factors.

Figure 8-2 German selection factors



It was assumed that there is an ultimate mortality table for selection phase 6+ and that mortality in selection phases 1 to 5 is a factor of the ultimate mortality table. This factor depends not on age, but on gender. There is one factor for selection phase 1 and a common factor for selection phases 2 to 5.

The crude mortality rates for the observed insurance population were first graduated for the age band 60 to 99 using the weighted Whittaker-Henderson method. The selection factors are then defined as the ratio of the actual number of deceased persons in the respective selection phase to the number of deceased persons that would be expected if the graduated ultimate industry mortality rates were applied to the exposure of insured lives in the respective selection phase.

In this respect, DAV calculated the following selection ratios:

Table 16: DAV selection ratios

	Males	Females
Selection phase 1	0.670538	0.712823
Selection phases 2 to 5	0.876209	0.798230

Outside the age band 60 to 99, DAV extrapolated the ultimate mortality rates. For ages up to 59 they assume that the ratio of the ultimate mortality rate at age 60 to the 1999 population mortality rate at age 60 $q^{pop}_{x,1999}$ can be transferred to the younger ages as follows:

$$q^{6}_{x} = q^{pop}_{x,1999} \cdot \frac{q^{6}_{x}}{q^{pop}_{60,1999}} = q^{pop}_{x,1999} \cdot \begin{cases} 66.6\% & for \quad men \\ 85.2\% & for \quad women \end{cases}$$

For older ages, they use a Logistic model for extrapolating the ultimate mortality rates at ages 100 to 120.

8.7.2 Aggregate table for the deferment period

Since the data for deferred annuities did not show a strong selection effect in the deferment period, the aggregate table was derived for the deferment period, which is not graded in terms of the years that have lapsed since the commencement date.

In the first step, crude mortality rates were calculated both from the observation material relating to the deferment period and from the observation material relating to the benefit payment period. The obtained graduated rates for both data sets are then put together at age 65 in order to obtain aggregate mortality rates.

8.7.3 Safety margins

The safety margin was applied to face the volatility risk when the table is applied. The idea is to provide protection against a maximum loss at a defined confidence level. If 1-α=95%, DAV obtains a margin for volatility risk of 6.26% for males and 7.22% for females. In addition, a 10% flat-rate margin for level parameter risk is added, meaning that DAV implements a total deduction of 15.6% for males and 16.5% for females.

8.7.4 Mortality forecasting

For the purpose of forecasting, the traditional exponential formula (see Section 5.3) in the form

$$\frac{q_{x,t+1}}{q_{x,t}} = \exp(-F(x,t)) \tag{8.19}$$

was chosen where

$$F(x,t)=\begin{cases} F_1(x), & 1999 \le t \le 1999+T_1 \\ F_1(x)(1-\dfrac{t-1999-T_1}{T_2-T_1})+F_2(x)\dfrac{t-1999-T_1}{T_2-T_1}, & 1999+T_1 \le t \le 1999+T_2 \\ F_2(x), & t>1999+T_2 \end{cases} \quad (8.20)$$

They calculated age-dependent mortality improvements according to the traditional model (short-term, long-term and medium-term). In order to compare these trends, the arithmetic means of annual mortality improvement for ages from 60 to 89 are considered:

Table 17: DAV: Mortality improvement [%]

	Males	Females
Short-term trend	1.97	2.00
Medium-term trend	1.67	2.05
Long-term trend	0.62	1.04

The so-called initial trend $F_1(x)$ is used for the first years of the mortality projection. This initial trend is reduced linearly to the target trend in a transition period. The target trend is used after the transition period. The time 1999 corresponds to the start of the mortality projection.

The loading for insured persons of 0.2% annual mortality improvements is added to the initial trend. Finally, the trend is extrapolated for high ages to a level of a 1% annual mortality improvement and limited for low ages to the level of a 3% annual mortality improvement. This defines the initial trend for insured persons.

The target trend for insured persons was defined as follows: the annual mortality improvement of the target trend is 75% of the annual mortality improvement of the graduated (and for high ages extrapolated and for low ages limited) medium-term trend, which was increased by the loading for insured persons, but not by the medium difference between short- and medium-term trends for males.

8.7.5 The working party's comments

The main point when constructing the DAV 2004 R tables was that a loading for insured persons is needed as an adjustment for differences between the mortality improvement of the population and the mortality improvement of insured persons. The reason is that private annuities are mainly purchased by people belonging to upper socio-economic groups, which confirms that the mortality improvement of annuitants is greater than the mortality improvement of the population (based on the assumption that the mortality improvement of upper socio-economic groups is greater than the mortality improvement of lower socio-economic groups).

There is no theoretical justification to assume that the average yearly improvement of mortality for insured persons is different than the average yearly improvement of mortality for the general population. It is true that people belonging to upper socio-economic groups mainly purchase private annuities, and this fact can certainly explain the lower mortality level, in particular at old ages. However, with regards to improvements, is it reasonable that, starting from lower mortality levels,

the socio-economic standard can also justify stronger improvements? If so, the gap between mortality levels would increase in time. Can we accept this as a reasonable aspect of mortality trends? We strongly doubt this.

Nevertheless, a reason might be found in the fact that access to new medical and pharmacological treatments is restricted to high socio-economic classes, and/or that greater awareness of the benefits of appropriate eating, drinking, smoking etc. habits is also restricted to high socio-economic classes. If this is the case for Germany, it does not hold for Slovenia and we cannot make projections on this basis.

In any event, if we accept the "German approach", then we are implicitly working with scenario-based projections, which are not "bad" in themselves, but might be more or less controversial as they imply some judgments and hence some arbitrariness. In contrast with the scenario-based forecasting used in Section 0 and Section 6, the Lee-Carter methodology allows us to estimate the lower and upper bounds of the central tendency within a confidence interval.

The working party has some additional remarks regarding the German approach to DAV 2004 R:

1. The selection for observed ages was assumed to stay at the ultimate level, which is not the case in the UK data. The selection vanished at a very old age since people must eventually die of something.
2. The parameter risk margin (15%) is added arbitrarily.
3. DAV did not use stochastic methods for mortality forecasting.
4. The scenario-based projections are not in the context of modern mortality frameworks.

8.8 The construction of selection tables for Slovenia

8.8.1 Introduction

Slovenia mortality experience for annuity purchasers is not directly available and because the pension reform started only 10 years ago, there are no adequate statistical data to make a conclusion regarding the selection effect. In this respect, we chose an alternative solution introduced by ANIA (*Associazione Natzionale fra la Imprese Assicuratrici*, 2005), which was used to build the Italian insurance annuity tables.

The idea is to use SMR from another population that has similar characteristics as the population for which we want to introduce the selection effect. As we see from (8.16), SMR in general depends on age. Let us denote with $SMR_x^{RC}(t)$ the reference country standardised mortality ratio for particular year *t* between the insured and general population. Then we calculate life insurance market central death rates from

$$\hat{m}_x^{LIM}(t) = SMR_x^{RC}(t) \cdot \hat{m}_x^{HMD}(t) \tag{8.21}$$

8.8.2 The construction of annuity tables for deferred annuitants for Slovenia

To obtain $SMR_x^{RC}(t)$ we include an element of selection that emerges from the English data, where the annuity and pensions market income is, generally speaking, very well developed.

We used UK statistical data from 1999–2002 collected by the Continuous Mortality Investigation Bureau and published in Continuous Mortality Investigation Reports, Number 23 (2009), relating to the experience of the portfolios of immediate and deferred annuities. The 1999–2002 mortality investigation presents the so-called "00" Series base mortality tables adopted by the UK Actuarial Profession. The statistical base is extensive since it involves over 20 million lives exposed to risk.

In particular, we used the mortality investigation of life office pensioners (insured to deferred annuities) – PNM00 tables for men and PNF00 tables for women, which show the mortality rates for each age from 20 to 120 years, distinguished between "lives" (heads insured) and "amounts" (weighted by the benefit).

By comparing the mortality of UK insured lives with that of the general population of the United Kingdom (taken from English Life Table No. 16, 2000–2002), it was possible to quantify the increased survival of the insured population. To take account of the influence on the selection of the level of economic wealth of the insured, as confirmed by the experiences of other markets (for example, Germany), reference is made to mortality weighted by the size of an annuity.

We calculate:

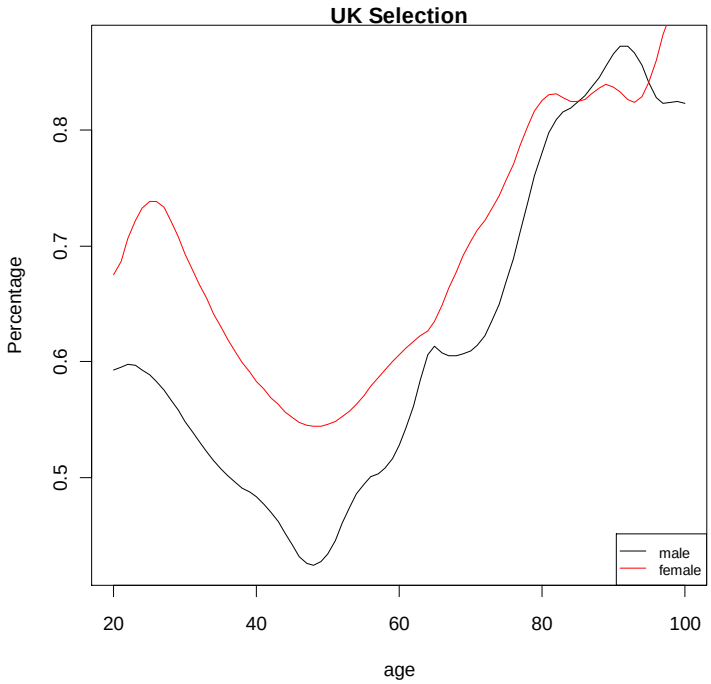
$$SMR_x^{RCM'} = \frac{q_x^{PNMA00}}{q_x^{ELT16M}} \tag{8.22}$$

for males and

$$SMR_x^{RCF'} = \frac{q_x^{PNFA00}}{q_x^{ELT16F}} \tag{8.23}$$

for females.

Figure 8-3: Selection in the UK population



In Figure 8-3, one can see the huge selection for both men and women from ages 40 to 60.

To correct some irregular patterns and to be closer to the German experience, we chose the following selection factors of mortality for males and females, respectively:

$$SMR_x^{RCM} = \begin{cases} 42,5\%, & x < 48 \\ SMR_x^{RCM'}, & 48 \le x \le 64 \\ 61,5\%, & 65 \le x \le 71 \\ SMR_x^{RCM'}, & 72 \le x \le 77 \\ SMR_{78}^{RCM'}, & 78 \le x \le 107 \\ SMR_{78}^{RCM'} + (1 - SMR_{78}^{RCM'}) \cdot \dfrac{x-108}{12}, & 108 \le x \le 120 \end{cases} \tag{8.24}$$

$$SMR_x^{RCF} = \begin{cases} 55\%, & x < 50 \\ SMR_x^{RCF'}, & 50 \le x \le 75 \\ SMR_{75}^{RCF'}, & 76 \le x < 111 \\ SMR_{75}^{RCF'} + (1 - SMR_{75}^{RCF'}) \cdot \dfrac{x-111}{9}, & 111 \le x \le 120 \end{cases} \tag{8.25}$$

Starting from the cohort life table of generation 1965, which we derived from historical data and stochastic projections, then applying the selection factors presented in this section, we obtain the projected and selected mortality table for the generation of insured persons born in 1965. To use the table for other generations, one should use the age shift tables described in Section 8.3. The

complete demographic tables are denoted **SDA65.**

It must be pointed out that the selection – which is an estimate of the differential mortality of the insured population which buys annuity insurance from the general population – also depends on the peculiarities of the products, the characteristics of the insured community to which they relate, and the business objectives the insurance company intends to pursue.

8.8.3 The construction of aggregate annuity tables for Slovenia

As mentioned in the previous paragraph, **SDA65** is structured to represent the mortality of the insured's deferred annuity or pension insurance. In other cases, such as immediate annuity or annuity conversion options further selectivity should be added.

In this regard, a comparison was made between mortality relative to insured owners of deferred annuities with immediate annuitants in the UK – the IML00 and IFL00 tables for men and women, respectively.

We applied a correction factor to the mortality rates of the **SDA65** table for delayed commitments which includes the increased expected survival of recipients of immediate annuities, namely:

$$q_x^{SIA65} = q_x^{SDA65} \cdot K_x \tag{8.26}$$

where

$$K_x^M = \begin{cases} 1, & x \le 63 \\ \dfrac{q_x^{\mathrm{IML00}}}{q_x^{PNMA00}}, & 64 \le x \le 86 \\ 1, & x \ge 87 \end{cases} \tag{8.27}$$

for men and

$$K_x^F = \begin{cases} 1, & x \le 54 \\ 1 - 0.02559(x - 54), & 55 \le x \le 59 \\ \dfrac{q_x^{\mathrm{IFL00}}}{q_x^{PNFA00}}, & 60 \le x \le 84 \\ 1, & x \ge 85 \end{cases} \tag{8.28}$$

for women.

We call this life table **SIA65.** As shown in Section 8.7.2, the mortality of deferred annuitants merges with the mortality of immediate annuitants after age 60, so **SIA65** is a table that may be considered an aggregate table recommended for use in the annuity business in Slovenia. The tables are presented in

Appendix 3. Selection factors used for aggregate annuity tables are shown in Figure 8-4.

Figure 8-4: Selection factors for aggregate tables

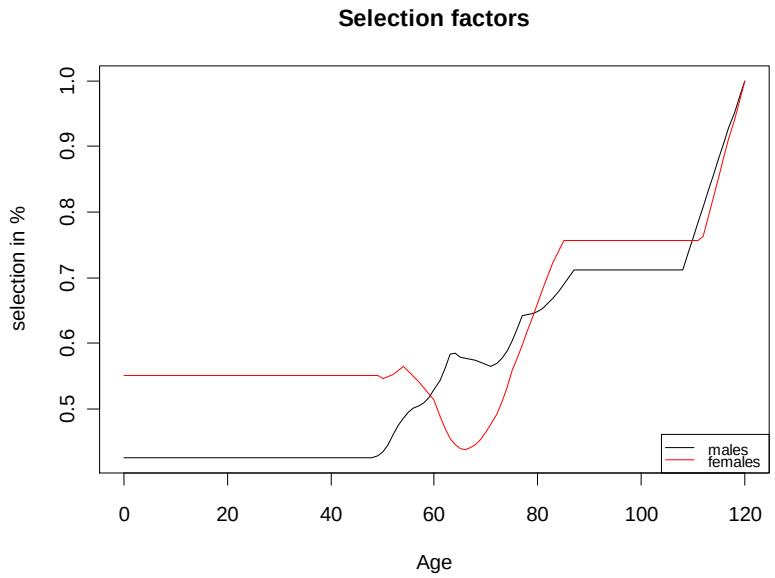
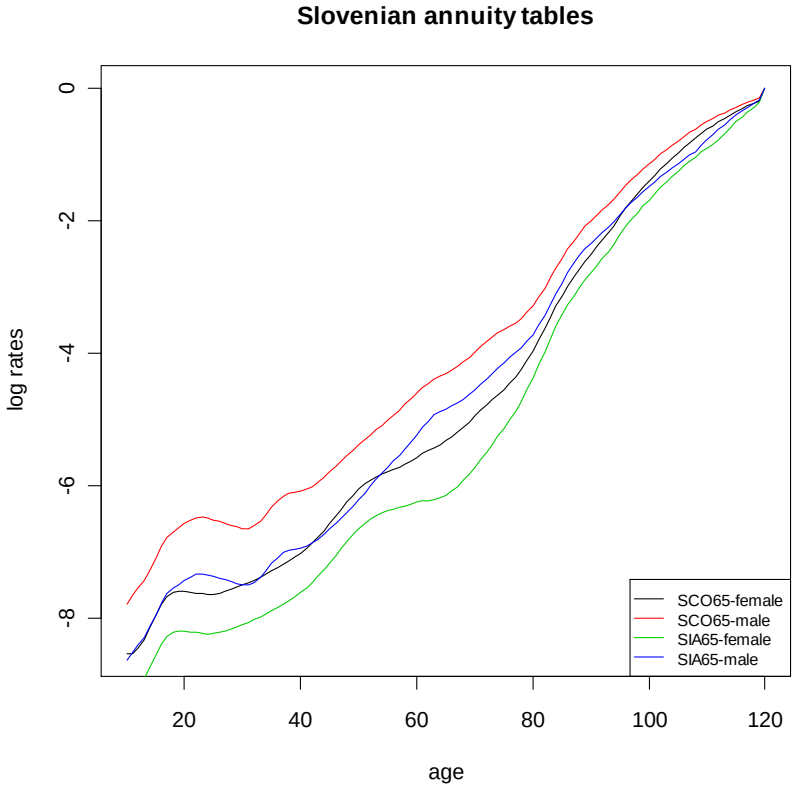


Figure 8-5: Slovenian gender-related annuity tables



8.8.4 International comparisons

Immediate annuity:

Age at issue 60 / birth year 1950 (annuity starts in 2010)

	R94	R04	IPS55	SIA65	SIA65-low mortality scenario	R94/ SIA65
net single premium male	18.104883	20.369429	18.991395	18.50437	18.98097	0.98
net single premium female	20.493777	22.000235	20.705882	20.98405	21.76160	0.97

The net single premium based on SIA65 tables is up to 12% lower than R04 single premium annuities. The reason for this gap is the 15% margin that is incorporated in the new German tables and the fact that SIA65 tables represent a best estimate of future annuitant mortality. In this respect those tables cannot be directly compared. Under the best estimate scenario, we determine future values of kappa by taking kappa to be equal to the expected value. We also calculated SIA65 under low mortality scenario. In this case, future values of kappa are obtained by assuming lower than expected values of kappa. The future values of kappa are obtained by taking $\kappa_{2008+t} = \kappa_{2008} + ct - 2\sigma_{\varepsilon}\sqrt{t}$. If we compare low mortality scenario SIA65 single premium with R04, we can observe only marginal differences in rates (from 1% to up to 7% higher single premium is observed in the case of R04). The comparisons also show that for current generations who are in front of their retirement, R94 tables underestimate best estimate annuity from 2% to 4%. We believe that SIA65 should be used for best estimate valuation of insurance liabilities within Solvency II Framework.

Age at issue 60 / birth year 1980 (annuity starts in 2040)

	R94	R04	IPS55	SIA65	SIA65-low mortality scenario	R94/ SIA65
net single premium male	19.720558	22.945504	19.856934	20. 46949	21.28829	0.96
net single premium female	22. 81581	24.426940	21.514670	22. 87369	23.58179	0.97

Age at issue 60 / birth year 2000 (annuity starts in 2060)

	R94	R04	IPS55	SIA65	SIA65-low mortality scenario	R94/ SIA65
net single premium male	20.87668	24.54484	19.856934	21.5960	22.37775	0.96
net single premium female	24.19244	25.64188	21.514670	23.56325	24.24350	1.03

Deferred annuity

Deferment period 20 years / birth year 1965 / age at issue 45

	R94	R04	IPS55	SIA65
net single premium male	9.052749	10.85729	9.573143	9.347653
net single premium female	11.08434	12.00785	10.72477	11.11152

Deferment period 35 years / birth year 1975 / age at issue 35

	R94	R04	IPS55	SIA65
net single premium male	5.334621	6.778083	5.486585	5.55777
net single premium female	6.776977	7.600209	6.320382	6.799525

Notation of tables are as follows:

R94 – DAV 1994 R annuity tables

R04 – DAV 2004 R annuity tables

IPS55 – the latest Italian annuity table (2005)

SIA65 – Slovenian aggregate annuity table (2010)

8.9 Unisex annuity tables

8.9.1 Introduction

The European Court of Justice (ECJ) passed new regulation on 1 March 2011 that bans gender-specific differentiation in insurance pricing. The ECJ ruling means that gender-neutral premiums and benefits for all new insurance policies taken out after 21 December 2012 should be

implemented. This decision will bring important changes to the development of new insurance products, especially in the life insurance line of business. In this respect, the unisex rates will introduce new risks of uncertainties arising from the gender mix of the portfolio. This is to a great extent true for the annuity business where there is still a substantial gap between the life expectancy of males and females.

An important consequence of the ECJ ruling is that the expected gender mix of an insurance portfolio will influence product pricing. Since the gender mix can vary significantly for different products, actuaries should pay special attention when making projections. Among others, the gender mix could vary depending on the sales channels mix, companies' orientation to target groups, the employment ratio of males/females and many more factors.

To be more concrete, when generating unisex annuity tables it is therefore essential that attention is paid not only to the historical gender-related portfolio mix (i.e. percentage of males and females who bought an annuity in past), but also to possible future changes of the portfolio arising from the new market situation. Another consequence of the ECJ ruling will be that the required single premium for an annuity paid by a female will in most cases be lower. Whether or not this will lead to a situation whereby the proportion of insured females will increase remains an open question.

Since circumstances are different from company to company, no universal unisex annuity tables can be recommended. Instead, the methodology will be explained (see the next section) which will lead to unisex annuity tables.

8.9.2 Unisex annuity tables

In general, we propose two approaches to building unisex annuity tables for Slovenia:

1.
- implement Lee-Carter on the total population
- implement the average (male and female) UK selection factor on Slovenian mortality data
- include an additional selection (gender selection). We can expect that females will more likely buy annuities since they expect to enjoy annuity payments for a longer time. The question is: How do we derive the sex selection factor?

2.
- use the already built Slovenian annuity tables for males and females SIA65
- take, as the unisex table, the average of the male table and the female table, with each one having a weight that should reflect the proportion of males and females, respectively, in future portfolios (100% would mean that we take female rates)
- calculate age shifts with the same weights

We suggest that actuaries apply approach 2, which uses already built tables. This will give pricing actuaries flexibility in scenario testing when determining the proper weights for males and females. We will continue our discussion based on the assumption of approach 2.

A question arising with approach 2 concerns how we can calculate the upper and lower boundaries of point estimates. In general, lower boundaries of the unisex table cannot be calculated as a

weighted average of the male/female lower boundaries since it is a matter of the properties of the mathematical functions through which the lower boundaries are constructed. If we denote with L(qM) and L(qF) the lower bounds for probabilities q for males and females, respectively (for a given age and future year), the weighted average of the lower bounds A(L(qM),L(qF)) is not necessarily equal to the lower bound of the weighted average L(A(qM,qF)) because A is a linear function, and the functional form of L is generally not known.

Nevertheless, if we take the lower bounds for males and females as our (prudential) projected mortality functions, namely if we "forget" that these functions are constructed as lower bounds, the weighted average clearly represents the average projected mortality function. This approach is applied whenever a weighted average of elements is used, namely for elements whose construction is unknown to us.

One more observation: if we take the weighted average of mortality rates, we implicitly assume that average annuity (in euros) will be the same for males and females.

In Table 18we made a calculation for a unisex single premium for an immediate annuity (birth year 1965), which one can compare with the male and female single premium immediate annuity of the same cohort. Weights were taken from 45% to up to 65% of the female rates. For example, if w = 45%, then we take 45% of the female rates and 55% of the male rates.

Table 18 Calculation of a unisex single annuity premium

			unisex single premium rates				
age	male	female	w=45%	w=50%	w=55%	w=60%	w=65%
45	24.67695	26.58489	25.536	25.631	25.726	25.822	25.917
50	23.02504	25.14922	23.981	24.087	24.193	24.300	24.406
55	21.22580	23.56325	22.278	22.395	22.511	22.628	22.745
60	19.29927	21.76982	20.411	20.535	20.658	20.782	20.905
65	17.29047	19.71976	18.384	18.505	18.627	18.748	18.870

In the tables below, we calculated the percentage increase of a single premium if unisex rates are used instead of male rates, and a percentage decrease when female rates are used.

Table 19: Average increase of single premium - male [%]

age	w=45%	w=50%	w=55%	w=60%	w=65%
45	3.48	3.87	4.25	4.64	5.03
50	4.15	4.61	5.07	5.54	6.00
55	4.96	5.51	6.06	6.61	7.16
60	5.76	6.40	7.04	7.68	8.32
65	6.32	7.02	7.73	8.43	9.13

Table 20: Average decrease of single premium – female [%]

age	w=45%	w=50%	w=55%	w=60%	w=65%
45	-3.95	-3.59	-3.23	-2.87	-2.51
50	-4.65	-4.22	-3.80	-3.38	-2.96
55	-5.46	-4.96	-4.46	-3.97	-3.47
60	-6.24	-5.67	-5.11	-4.54	-3.97
65	-6.78	-6.16	-5.54	-4.93	-4.31

At present, in Italy more than 50% of current annuitants are males. A similar situation is found in Slovenia, at least for the pension business. In this respect, for a balanced portfolio we suggest taking unisex rates of around w =60%.

9 SUMMARY

In 2010 the working group on mortality was established in order to develop the first annuity mortality tables for the Slovenian market. This document provides a report of the results of the group's work.

One of the most important parts of forecasting mortality accurately is to collect appropriate statistics. The Statistical Office of the Republic of Slovenia (SORS) provided the Slovenian population data. Data were provided for the time span from 1971 to 2008, for each age and separated for men and women. Data for the period before 1971 were collected from the Human Mortality Database (http://www.mortality.org/). Data for mortality by cause analysis were obtained from the Institute of Public Health of the Republic of Slovenia (IVZ – *Inštitut za varovanje zdravja*) database for the time span from 1971 to 2008, for five-year age groups, separately for men and women.

The Slovenian mortality data had some irregularities that needed to be adjusted before we could use the data for forecasting. We used the method proposed by Denuit and Goderniaux to extrapolate death rates at very old ages. Following this approach, the death rates for very old ages were estimated according to the logistic formula. Some death rates were 0 (meaning no deaths in the observed period), which, due to the small population involved, happens quite often at lower ages. Since in forecasting we observed log death rates, adjustment techniques were implemented to obtain positive values. In particular, we used interpolation techniques with neighbour central death rates to obtain the best estimate for such cases.

General approaches to projecting age-specific mortality rates can be categorised in various ways; for example, as process-based methods, explanatory methods, forecasting methods or a combination of these methods.

Forecasting is the process of projecting mortality based on historical trends. Forecasting methods include some element of subjective judgment, for example, the type of underlying function, the time series we take into account etc. Simple forecasting methods (for example, exponential formula) are only usable in the sense that the pattern of changing mortality in the past will continue in the future.

Extrapolation of a time series as an example of forecasting methods assumes that all we need to know is contained in the historical values of the time series being forecasted. The main shortcoming of a time-series extrapolation is the assumption that nothing else besides the prior values of a series is relevant. In this monograph we present two extrapolation methods to project population mortality. We have adjusted the standard UK exponential formula for forecasting and applied it to the Slovenian mortality data. The life expectancy derived from the extrapolated mortality data does not differ from that derived from the LC projections, yet it does not give the prediction interval of the variability of the prediction.

Explanatory-based methods use econometric techniques based on variables such as economic or environmental factors. We have employed these techniques to forecast mortality trends by cause. We have found that by far the most important causes of death in Slovenia in 2008 were "diseases of the circulatory system" (39.5% of all deaths) and "neoplasms", i.e. cancer (31.4% of all deaths). By also including the third largest group – deaths caused by external causes (8.3% of all deaths) – about

80% of all deaths are included in the analysis. The trends of these three cause groups of death differ. Diseases of the circulatory system exhibit a strong decline in the 2071–2008 period: from over 900 to less than 400 deaths per 100,000 population (using a standard population from 2008). Representing a high share among all causes of death, this group is a main driving force in the decline of total mortality in the 1971–2008 period in Slovenia. Although the share of external causes of morbidity and mortality is much smaller and the negative trend is less pronounced, they contribute substantially to the total mortality decline as well. In contrast, neoplasms do not show any clear trend. If the trends for these two groups of diseases were to continue, neoplasms would become the no. 1 cause of death in the near future.

By taking into account the trends in mortality by the three main causes of death groups in the past, we project the future development of longevity. In a baseline scenario we assume gradual convergence from initial rates of mortality improvement (average annual growth rates in 2001–2008 period) to the 1% long-term rate of mortality improvement. Under these assumptions life expectancy at birth for males would increase from 76.3 years in 2010 to 83.9 years in 2060 whereas for females it would increase from 83.1 years in 2010 to 90.0 years in 2060.

We have also implemented different stochastic approaches (basic Lee-Carter, Poisson log-bilinear and APC model) to model longevity in Slovenia. Stochastic methods are a powerful approach to mortality projections and combine a demographic model with a time-series model. In the stochastic framework, the results of the Lee-Carter (LC) projections consist of point estimates and interval estimates. In this respect, the LC method allows uncertainty in forecasts, which is not the case with deterministic projections. The LC method is the current standard for the actuarial modelling of mortality. Due to the small Slovenian population and since stochastic methods give a prediction interval, it is very important to know the boundaries of projections as well as the standard error. In this respect, due to a small population, deterministic approaches could be arbitrary.

Results of the Lee-Carter, Poisson log-bilinear and APC models are carefully examined in the document. The results were compared for the Lee-Carter model vs. the Poisson log-bilinear model based on non-smoothed data because both models are three-parameter models and are thus easier to compare graphically/visually, whereas APC is a five-factor model and we therefore analyse the result in a separate subsection.

The main reason to test the APC model was to see whether the mortality cohort effect exists in the Slovenian data similar to that which was detected in the UK data. The projected data showed that in the Slovenian mortality data there is no significant cohort effect.

Based on a back-testing analysis we concluded that the Poisson log bilinear model best fits the past observed central death rates. It was then a natural conclusion that we would use a Poisson log bilinear model for forecasting mortality in the future.

Actuaries usually employ cohort life tables to calculate annuities since cohort tables better explain improvements of mortality over time. The birth year $\bar{\tau}$ =1965 was used for the fundamental cohort year to generate the Slovenian base cohort population mortality table. We then used a smoothing procedure in order to reduce the variability in the data. The age cohort life table based on the birth year 1965 is denoted **SCO65** (reference population mortality table).

The annuity calculated with a cohort life table not only depends on age at entry, but also on the individual birth year of the insured person. This would lead to the construction of a cohort life table for every generation, which would involve the use of a large series of tables. This is, of course, impractical and cannot be used in everyday actuarial calculation. Therefore a simplified solution, the so-called Rueff method, has been adopted, which is also used by several European countries. Age shifts for the **SCO65** reference population mortality table were calculated separately for males and females.

A life annuity purchaser is, with a high probability, a healthy person with particularly low mortality in the first years of the life annuity payment and, generally, with a higher life expectancy than the average person. Slovenian mortality experience for annuity purchasers is not directly available and, because the pension reform only started 10 years ago, there are no adequate statistical data to draw conclusions regarding the selection effect. In this respect, we chose an alternative solution that is also used by ANIA (*Associazione Nazionale fra la Imprese Assicuratrici*).

We used UK statistical data from 1999–2002 collected by the Continuous Mortality Investigation Bureau and published in Continuous Mortality Investigation Reports, relating to the experience of the portfolios of immediate and deferred annuities. By comparing the mortality of UK insured lives with that of the general population of the United Kingdom (taken from English Life Table No. 16, 2000–2002), it was possible to quantify the increased survival of the insured population (so-called selection). To take into account the influence of the level of economic wealth of the insured on the selection, reference is made to mortality weighted by the size of an annuity. This type of selection has been observed in many Western markets (for example, Germany).

Starting from the cohort life table of generation 1965, which we derived from historical data and stochastic projections, and then by applying the selection factors, we obtain the projected and selected mortality table for the generation of insured persons born in 1965. The complete demographic tables are denoted **SDA65.**

The **SDA65** tables are structured to represent the mortality of the insured's deferred annuity or pension insurance. In other cases, such as immediate annuity or annuity conversion options, further selectivity should be added. In this regard, a comparison was made between the mortality relative to insured owners of deferred annuities with immediate annuitants in the UK – the IML00 and IFL00 tables, respectively, for men and women.

We applied a correction factor to the mortality rates of the **SDA65** table for delayed commitments, which includes the increased expected survival of recipients of immediate annuities. We denote this life table **SIA65.** As in the German case, the mortality of deferred annuitants merges with the mortality of immediate annuitants after age 60, therefore Table **SIA65** is considered an **aggregate** table which is recommend for use in the annuity business in Slovenia.

It is important to stress that the UK selection factors used in projections do not differ from the German selection factors from 50 to 70 years of age. We did not directly use the German selection factors because of the smaller sample involved.

The net single premium based on SIA65 tables is up to 12% lower than R04 single premium annuities. The reason for a gap is the 15% margin that is incorporated in the new German tables and the fact that SIA65 tables represent a best estimate of future annuitant mortality. In this respect these tables cannot be directly compared. We also calculated SIA65 under low mortality scenario. In this case, future values of kappa are obtained by assuming lower than expected values of kappa. If we compare low mortality scenario with R04, we can observe only marginal differences in rates (from 1% to up to 7% higher single premium is observed in the case of R04). The comparisons also show that for current generations who are just before their retirement, R94 tables underestimate the best estimate annuity between 2% and 4%. In this respect it should be stressed that SIA65 annuity tables should be used for best estimate valuation of annuity liabilities within Solvency II framework.

After 21 December 2012 only unisex tables will be allowed for actuarial pricing. This will also hold for annuities. Therefore it would be reasonable to include unisex annuity within this project. An important consequence of the ECJ unisex approach ruling is that the expected gender mix of an insurance company's portfolio will have an influence on product pricing. Since the gender mix can vary significantly for different products and portfolios, actuaries should pay special attention not only to the historical gender mix but also to possible future changes. Since circumstances are different from company to company, no universal unisex annuity tables can be recommended. Instead, the methodology that leads to appropriate unisex annuity tables is explained in this document.

Finally, it should be noted that the method based on Lee-Carter does not attempt to incorporate assumptions about future extreme events caused by wars, epidemics, pollution, natural catastrophes or, on the other hand, the discovery of a drug that would cure cancer, for example. In this respect, it lies within the discretion of the actuary to include such "extreme" events in calculations, if appropriate.

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11 APPENDIX

Appendix 1: Slovenia 2010 reference population mortality table SCO65

Table 21: SCO65 Male

age	qx	lx	dx	ex	ax	Nx	Dx	age	qx	lx	dx	ex	ax	Nx	Dx
0	0.04334	1000000.0	43343.7	74.7	30.8	30790772.6	1000000.0	61	0.01086	817962.6	8881.7	23.1	16.9	2638268.9	156329.8
1	0.00350	956656.3	3344.6	77.1	32.0	29790772.6	931052.4	62	0.01161	809080.9	9391.1	22.3	16.5	2481939.1	150493.8
2	0.00164	953311.7	1560.3	76.4	32.0	28859720.2	902965.7	63	0.01228	799689.9	9816.8	21.6	16.1	2331445.3	144765.9
3	0.00115	951751.4	1097.2	75.5	31.9	27956754.5	877360.4	64	0.01288	789873.0	10172.5	20.8	15.7	2186679.4	139161.8
4	0.00100	950654.1	947.0	74.6	31.7	27079394.2	852894.3	65	0.01347	779700.6	10500.9	20.1	15.3	2047517.6	133693.1
5	0.00081	949707.1	772.2	73.6	31.6	26226499.9	829240.6	66	0.01412	769199.6	10861.5	19.4	14.9	1913824.5	128362.5
6	0.00063	948934.9	598.4	72.7	31.5	25397259.3	806390.6	67	0.01492	758338.1	11311.4	18.6	14.5	1785462.0	123163.0
7	0.00049	948336.5	461.1	71.7	31.4	24590868.7	784313.4	68	0.01592	747026.8	11889.4	17.9	14.1	1662299.0	118078.7
8	0.00040	947875.4	379.5	70.8	31.2	23806555.3	762950.9	69	0.01714	735137.3	12603.7	17.2	13.7	1544220.3	113089.5
9	0.00038	947495.9	361.5	69.8	31.0	23043604.4	742234.0	70	0.01858	722533.6	13425.6	16.5	13.2	1431130.8	108175.8
10	0.00041	947134.4	393.0	68.8	30.9	22301370.4	722093.3	71	0.02016	709108.0	14292.2	15.8	12.8	1322955.0	103324.3
11	0.00047	946741.4	444.9	67.9	30.7	21579277.1	702475.6	72	0.02176	694815.8	15121.5	15.1	12.4	1219630.7	98532.1
12	0.00052	946296.5	495.6	66.9	30.6	20876801.6	683353.2	73	0.02330	679694.4	15839.5	14.4	12.0	1121098.6	93808.0
13	0.00059	945800.8	554.6	65.9	30.4	20193448.3	664715.6	74	0.02472	663854.8	16411.0	13.7	11.5	1027290.6	89169.8
14	0.00068	945246.3	647.2	65.0	30.2	19528732.7	646545.9	75	0.02604	647443.9	16858.5	13.1	11.1	938120.8	84637.9
15	0.00083	944599.1	782.5	64.0	30.0	18882186.8	628810.9	76	0.02737	630585.4	17261.9	12.4	10.6	853482.9	80227.8
16	0.00099	943816.5	936.3	63.1	29.9	18253376.0	611474.4	77	0.02892	613323.5	17738.5	11.8	10.2	773255.1	75943.2
17	0.00113	942880.2	1067.7	62.1	29.7	17641901.6	594518.5	78	0.03092	595585.0	18418.1	11.1	9.7	697312.0	71773.0
18	0.00123	941812.4	1160.0	61.2	29.5	17047383.0	577951.6	79	0.03365	577166.9	19420.1	10.4	9.2	625539.0	67691.9
19	0.00131	940652.4	1231.1	60.3	29.3	16469431.4	561790.5	80	0.03736	557746.9	20835.5	9.8	8.8	557847.1	63663.5
20	0.00139	939421.3	1302.1	59.4	29.1	15907640.9	546039.2	81	0.04230	536911.4	22709.7	9.1	8.3	494183.5	59645.0
21	0.00146	938119.2	1372.1	58.4	28.9	15361601.8	530688.4	82	0.04866	514201.7	25021.0	8.5	7.8	434538.5	55593.4
22	0.00152	936747.1	1421.3	57.5	28.8	14830913.4	515729.6	83	0.05654	489180.7	27657.3	7.9	7.4	378945.1	51472.8
23	0.00153	935325.8	1433.5	56.6	28.6	14315183.7	501165.1	84	0.06588	461523.4	30404.2	7.4	6.9	327472.3	47262.9
24	0.00151	933892.3	1412.6	55.7	28.4	13814018.6	487004.4	85	0.07645	431119.2	32961.0	6.9	6.5	280209.5	42967.7
25	0.00148	932479.7	1376.2	54.8	28.2	13327014.2	473253.3	86	0.08790	398158.2	34996.6	6.4	6.1	237241.8	38620.5
26	0.00144	931103.5	1338.3	53.9	27.9	12853761.0	459907.4	87	0.09977	363161.6	36232.7	5.9	5.8	198621.3	34283.1
27	0.00140	929765.2	1301.9	52.9	27.7	12393853.6	446955.1	88	0.11170	326928.8	36518.5	5.5	5.5	164338.1	30036.7
28	0.00136	928463.3	1263.9	52.0	27.5	11946898.5	434383.7	89	0.12348	290410.4	35860.8	5.2	5.2	134301.4	25967.4
29	0.00132	927199.5	1225.5	51.1	27.3	11512514.8	422182.4	90	0.13513	254549.6	34397.8	4.8	4.9	108334.0	22151.7
30	0.00129	925974.0	1197.6	50.1	27.0	11090332.5	410340.0	91	0.14688	220151.8	32335.2	4.5	4.6	86182.3	18645.6
31	0.00129	924776.4	1197.2	49.2	26.8	10679992.5	398841.2	92	0.15910	187816.6	29881.4	4.2	4.4	67536.7	15481.2
32	0.00134	923579.2	1240.0	48.3	26.5	10281151.3	387664.1	93	0.17225	157935.2	27203.8	3.9	4.1	52055.5	12669.8
33	0.00145	922339.2	1334.2	47.3	26.3	9893487.2	376782.1	94	0.18676	130731.4	24415.4	3.6	3.9	39385.7	10206.8
34	0.00160	921005.0	1475.1	46.4	26.0	9516705.1	366167.5	95	0.21029	106316.0	22356.9	3.3	3.6	29178.9	8078.4
35	0.00179	919529.9	1642.3	45.5	25.7	9150537.6	355796.6	96	0.22959	83959.0	19276.3	3.1	3.4	21100.5	6208.9
36	0.00196	917887.6	1803.0	44.6	25.4	8794741.0	345655.6	97	0.25003	64682.7	16172.7	2.9	3.2	14891.7	4655.3
37	0.00210	916084.7	1926.1	43.6	25.2	8449085.4	335743.7	98	0.27160	48510.0	13175.2	2.7	3.0	10236.3	3397.9
38	0.00219	914158.5	1999.9	42.7	24.9	8113341.7	326070.8	99	0.29428	35334.7	10398.1	2.5	2.8	6838.4	2408.8
39	0.00223	912158.6	2037.6	41.8	24.6	7787270.9	316649.6	100	0.31804	24936.6	7930.7	2.3	2.7	4429.6	1654.5
40	0.00227	910121.0	2067.4	40.9	24.3	7470621.3	307486.4	101	0.34284	17005.9	5830.3	2.1	2.5	2775.2	1098.1
41	0.00233	908053.7	2117.6	40.0	24.0	7163134.8	298577.1	102	0.36864	11175.6	4119.8	2.0	2.4	1677.1	702.3
42	0.00244	905936.0	2206.9	39.1	23.7	6864557.8	289908.3	103	0.39537	7055.8	2789.7	1.8	2.3	974.8	431.5
43	0.00259	903729.1	2341.8	38.2	23.4	6574649.5	281461.9	104	0.42296	4266.2	1804.4	1.7	2.1	543.3	253.9
44	0.00279	901387.3	2518.8	37.3	23.0	6293187.6	273219.0	105	0.45133	2461.7	1111.1	1.6	2.0	289.3	142.6
45	0.00304	898868.5	2728.5	36.4	22.7	6019968.6	265163.5	106	0.48038	1350.7	648.8	1.5	1.9	146.7	76.2
46	0.00330	896140.0	2960.5	35.5	22.4	5754805.1	257283.3	107	0.51000	701.8	357.9	1.4	1.8	70.6	38.5
47	0.00359	893179.5	3207.5	34.6	22.0	5497521.8	249570.2	108	0.54006	343.9	185.7	1.3	1.7	32.1	18.4
48	0.00390	889972.0	3467.2	33.7	21.7	5247951.6	242018.5	109	0.57044	158.2	90.2	1.2	1.7	13.7	8.2
49	0.00422	886504.8	3740.6	32.9	21.3	5005933.1	234623.4	110	0.60100	67.9	40.8	1.1	1.6	5.5	3.4
50	0.00456	882764.2	4028.6	32.0	21.0	4771309.7	227380.5	111	0.63159	27.1	17.1	1.0	1.5	2.0	1.3

51	0.00493	878735.5	4330.7	31.2	20.6	4543929.2	220285.0	112	0.66205	10.0	6.6	1.0	1.5	0.7	0.5
52	0.00531	874404.9	4644.9	30.3	20.3	4323644.3	213332.7	113	0.69221	3.4	2.3	0.9	1.4	0.2	0.2
53	0.00572	869760.0	4971.2	29.5	19.9	4110311.6	206520.1	114	0.72191	1.0	0.7	0.9	1.4	0.1	0.0
54	0.00615	864788.8	5315.1	28.6	19.5	3903791.4	199844.0	115	0.75096	0.3	0.2	0.8	1.3	0.0	0.0
55	0.00662	859473.6	5688.4	27.8	19.2	3703947.4	193300.0	116	0.77920	0.1	0.1	0.8	1.3	0.0	0.0
56	0.00715	853785.2	6106.6	27.0	18.8	3510647.4	186881.4	117	0.80645	0.0	0.0	0.7	1.2	0.0	0.0
57	0.00777	847678.6	6583.0	26.2	18.4	3323766.0	180578.8	118	0.83252	0.0	0.0	0.7	1.2	0.0	0.0
58	0.00847	841095.6	7120.5	25.4	18.0	3143187.1	174381.0	119	0.85726	0.0	0.0	0.6	1.1	0.0	0.0
59	0.00924	833975.1	7705.6	24.6	17.6	2968806.1	168277.1	120	1.00000	0.0	0.0	0.5	1.0	0.0	0.0
60	0.01005	826269.5	8306.9	23.8	17.3	2800529.0	162260.2								

Table 22: SCO65 Female

age	qx	lx	dx	ex	ax	Nx	Dx	age	qx	lx	dx	ex	ax	Nx	Dx
0	0.03382	1000000.0	33818.9	83.7	32.2	32243440.4	1000000.0	61	0.00399	900547.0	3591.9	28.9	19.9	3431116.5	172113.4
1	0.00268	966181.1	2585.8	85.6	33.2	31243440.4	940322.2	62	0.00419	896955.1	3756.3	28.0	19.5	3259003.1	166838.9
2	0.00147	963595.2	1412.2	84.8	33.2	30303118.2	912706.2	63	0.00439	893198.8	3922.4	27.1	19.1	3092164.2	161693.6
3	0.00095	962183.1	914.8	84.0	33.1	29390412.1	886976.7	64	0.00461	889276.4	4099.1	26.2	18.7	2930470.6	156675.0
4	0.00069	961268.3	663.4	83.1	33.1	28503435.4	862417.0	65	0.00486	885177.3	4300.7	25.3	18.3	2773795.7	151778.9
5	0.00052	960604.9	502.0	82.1	33.0	27641018.4	838756.0	66	0.00516	880876.6	4543.3	24.5	17.8	2622016.8	146999.0
6	0.00042	960102.9	398.7	81.2	32.9	26802262.4	815880.9	67	0.00552	876333.3	4840.4	23.6	17.4	2475017.8	142326.8
7	0.00033	959704.2	318.5	80.2	32.7	25986381.4	793715.0	68	0.00596	871492.9	5198.4	22.7	16.9	2332691.0	137752.5
8	0.00026	959385.7	250.8	79.2	32.6	25192666.5	772215.6	69	0.00648	866294.5	5614.9	21.9	16.5	2194938.6	133266.0
9	0.00021	959134.8	206.0	78.2	32.5	24420450.8	751351.6	70	0.00706	860679.7	6077.8	21.0	16.0	2061672.6	128858.6
10	0.00019	958928.9	186.7	77.2	32.4	23669099.3	731085.4	71	0.00769	854601.9	6569.3	20.1	15.5	1932814.0	124524.2
11	0.00020	958742.2	187.2	76.3	32.2	22938013.9	711380.1	72	0.00834	848032.5	7072.8	19.3	15.0	1808289.8	120259.9
12	0.00021	958555.0	202.5	75.3	32.1	22226633.8	692205.5	73	0.00902	840959.7	7582.4	18.4	14.5	1688029.9	116065.1
13	0.00024	958352.5	231.8	74.3	32.0	21534428.3	673537.0	74	0.00973	833377.3	8110.4	17.6	14.0	1571964.8	111940.2
14	0.00029	958120.7	276.7	73.3	31.8	20860891.2	655351.9	75	0.01053	825266.9	8690.8	16.8	13.5	1460024.6	107884.0
15	0.00035	957844.0	334.6	72.3	31.7	20205539.3	637627.9	76	0.01148	816576.1	9377.9	16.0	13.0	1352140.6	103890.9
16	0.00041	957509.4	395.1	71.4	31.5	19567911.4	620345.7	77	0.01269	807198.2	10240.4	15.1	12.5	1248249.7	99949.2
17	0.00046	957114.3	443.0	70.4	31.4	18947565.7	603493.6	78	0.01425	796957.7	11355.2	14.3	12.0	1148300.5	96040.1
18	0.00049	956671.3	469.9	69.4	31.2	18344072.1	587069.9	79	0.01629	785602.5	12799.1	13.5	11.4	1052260.4	92137.9
19	0.00050	956201.4	478.5	68.5	31.1	17757002.2	571076.9	80	0.01894	772803.4	14637.8	12.7	10.9	960122.5	88211.0
20	0.00050	955722.8	477.1	67.5	30.9	17185925.3	555514.4	81	0.02230	758165.7	16910.5	12.0	10.4	871911.6	84224.0
21	0.00049	955245.7	471.8	66.5	30.8	16630410.9	540376.8	82	0.02646	741255.2	19610.7	11.2	9.8	787687.6	80141.5
22	0.00049	954773.9	465.5	65.6	30.6	16090034.1	525654.4	83	0.03141	721644.5	22669.4	10.5	9.3	707546.0	75933.1
23	0.00048	954308.4	459.7	64.6	30.4	15564379.8	511336.3	84	0.03713	698975.1	25950.0	9.8	8.8	631612.9	71579.4
24	0.00048	953848.7	456.2	63.6	30.3	15053043.5	497411.2	85	0.04348	673025.1	29264.7	9.2	8.3	560033.5	67077.3
25	0.00048	953392.5	457.4	62.6	30.1	14555632.2	483867.0	86	0.05035	643760.4	32411.0	8.6	7.9	492956.2	62443.4
26	0.00049	952935.1	464.4	61.7	29.9	14071765.3	470690.8	87	0.05760	611349.4	35215.4	8.0	7.5	430512.8	57712.5
27	0.00050	952470.7	476.6	60.7	29.7	13601074.5	457870.0	88	0.06520	576134.0	37565.5	7.5	7.0	372800.2	52932.5
28	0.00052	951994.1	492.1	59.7	29.5	13143204.4	445392.6	89	0.07319	538568.5	39415.2	7.0	6.6	319867.7	48156.8
29	0.00053	951502.1	508.7	58.8	29.3	12697811.8	433248.1	90	0.08167	499153.3	40767.5	6.5	6.3	271710.9	43437.9
30	0.00055	950993.3	525.6	57.8	29.1	12264563.7	421427.2	91	0.09085	458385.7	41642.6	6.0	5.9	228272.9	38822.6
31	0.00057	950467.8	543.1	56.8	28.9	11843136.5	409921.5	92	0.10090	416743.2	42050.2	5.6	5.5	189450.4	34351.1
32	0.00059	949924.6	562.9	55.9	28.7	11433215.1	398722.3	93	0.11202	374693.0	41974.4	5.2	5.2	155099.3	30058.4
33	0.00062	949361.7	586.1	54.9	28.5	11034492.7	387821.0	94	0.12435	332718.6	41375.2	4.7	4.8	125040.9	25976.8
34	0.00065	948775.6	613.1	53.9	28.2	10646671.8	377208.3	95	0.14722	291343.4	42890.7	4.3	4.5	99064.2	22137.6
35	0.00068	948162.6	643.2	53.0	28.0	10269463.4	366875.5	96	0.16399	248452.7	40744.4	4.0	4.2	76926.5	18373.3
36	0.00071	947519.4	675.6	52.0	27.8	9902587.9	356814.3	97	0.18211	207708.3	37825.6	3.7	3.9	58553.2	14949.1
37	0.00075	946843.7	710.2	51.0	27.5	9545773.6	347016.9	98	0.20160	169882.7	34247.6	3.4	3.7	43604.1	11899.5
38	0.00079	946133.6	747.8	50.1	27.3	9198756.8	337476.0	99	0.22247	135635.1	30174.8	3.1	3.4	31704.5	9246.4
39	0.00084	945385.8	790.9	49.1	27.0	8861280.8	328184.2	100	0.24474	105460.3	25810.3	2.9	3.2	22458.2	6996.9
40	0.00089	944594.9	842.1	48.2	26.7	8533096.6	319133.5	101	0.26840	79650.0	21377.8	2.7	3.0	15461.3	5143.1
41	0.00096	943752.7	904.4	47.2	26.5	8213963.1	310315.3	102	0.29342	58272.2	17098.4	2.4	2.8	10318.2	3662.0
42	0.00104	942848.3	980.2	46.2	26.2	7903647.8	301720.6	103	0.31978	41173.8	13166.5	2.3	2.6	6656.2	2518.2

43	0.00114	941868.1	1071.8	45.3	25.9	7601927.2	293340.1	104	0.34742	28007.3	9730.2	2.1	2.5	4138.0	1667.1
44	0.00126	940796.3	1181.2	44.3	25.6	7308587.1	285164.2	105	0.37626	18277.1	6877.0	1.9	2.3	2470.9	1058.8
45	0.00139	939615.1	1310.1	43.4	25.3	7023422.9	277183.7	106	0.40623	11400.2	4631.1	1.8	2.2	1412.1	642.7
46	0.00156	938305.1	1459.5	42.5	25.0	6746239.2	269389.0	107	0.43721	6769.1	2959.5	1.6	2.1	769.4	371.4
47	0.00174	936845.5	1628.6	41.5	24.7	6476850.2	261771.3	108	0.46909	3809.6	1787.0	1.5	2.0	398.0	203.4
48	0.00194	935217.0	1812.8	40.6	24.4	6215079.0	254322.3	109	0.50172	2022.5	1014.7	1.4	1.9	194.5	105.1
49	0.00215	933404.1	2003.8	39.7	24.1	5960756.6	247035.9	110	0.53495	1007.8	539.1	1.3	1.8	89.4	51.0
50	0.00235	931400.3	2189.4	38.8	23.8	5713720.8	239908.1	111	0.56859	468.7	266.5	1.2	1.7	38.4	23.1
51	0.00254	929210.9	2357.3	37.8	23.5	5473812.7	232938.3	112	0.60247	202.2	121.8	1.1	1.6	15.4	9.7
52	0.00270	926853.6	2498.8	36.9	23.2	5240874.4	226128.8	113	0.63636	80.4	51.1	1.0	1.5	5.7	3.7
53	0.00283	924354.7	2613.1	36.0	22.8	5014745.6	219483.4	114	0.67007	29.2	19.6	1.0	1.4	1.9	1.3
54	0.00294	921741.7	2707.0	35.1	22.5	4795262.2	213005.3	115	0.70336	9.6	6.8	0.9	1.4	0.6	0.4
55	0.00304	919034.6	2793.2	34.2	22.2	4582256.9	206695.6	116	0.73599	2.9	2.1	0.8	1.3	0.2	0.1
56	0.00315	916241.5	2885.0	33.3	21.8	4375561.3	200552.2	117	0.76774	0.8	0.6	0.8	1.3	0.0	0.0
57	0.00328	913356.5	2992.8	32.4	21.5	4175009.1	194570.0	118	0.79835	0.2	0.1	0.7	1.2	0.0	0.0
58	0.00343	910363.7	3121.0	31.6	21.1	3980439.1	188742.1	119	0.82759	0.0	0.0	0.7	1.2	0.0	0.0
59	0.00360	907242.7	3268.1	30.7	20.7	3791697.0	183060.8	120	1.00000	0.0	0.0	0.5	1.0	0.0	0.0
60	0.00379	903974.6	3427.6	29.8	20.3	3608636.1	177519.6								

Appendix 2: Age shifts for population mortality – fundamental cohort 1965

Table 23: Age shifts

Birth year	Male	Female	Birth year	Male	Female	Birth year	Male	Female
up to1950	2	2	1976	-2	-2	2002	-6	-5
1951	2	2	1977	-2	-2	2003	-6	-5
1952	2	2	1978	-3	-2	2004	-6	-5
1953	2	1	1979	-3	-2	2005	-6	-6
1954	1	1	1980	-3	-3	2006	-7	-6
1955	1	1	1981	-3	-3	2007	-7	-6
1956	1	1	1982	-3	-3	2008	-7	-6
1957	1	1	1983	-3	-3	2009	-7	-6
1958	1	1	1984	-4	-3	2010	-7	-6
1959	1	0	1985	-4	-3	2011	-7	-6
1960	0	0	1986	-4	-3	2012	-8	-6
1961	0	0	1987	-4	-4	2013	-8	-6
1962	0	0	1988	-4	-4	2014	-8	-6
1963	0	0	1989	-4	-4	2015	-8	-6
1964	0	0	1990	-4	-4	2016	-8	-6
1965	0	0	1991	-5	-4	2017	-8	-7
1966	-1	-1	1992	-5	-4	2018	-8	-7
1967	-1	-1	1993	-5	-4	2019	-8	-7
1968	-1	-1	1994	-5	-4	2020	-8	-7
1969	-1	-1	1995	-5	-4			
1970	-1	-1	1996	-5	-5			
1971	-2	-1	1997	-6	-5			
1972	-2	-2	1998	-6	-5			
1973	-2	-2	1999	-6	-5			
1974	-2	-2	2000	-6	-5			
1975	-2	-2	2001	-6	-5			

Appendix 3: First Slovenian gender-specific annuity tables

Table 24: Slovenia gender-related annuity tables

age	SCO65_male	SMR male	SDA65_male	Kx male	SIA65_male	SCO65_female	SMR female	SDA65_female	Kx female	SIA65_female
0	0.04334	0.425	0.01842	1.000	0.01842	0.03382	0.550	0.01860	1.000	0.01860
1	0.00350	0.425	0.00149	1.000	0.00149	0.00268	0.550	0.00147	1.000	0.00147
2	0.00164	0.425	0.00070	1.000	0.00070	0.00147	0.550	0.00081	1.000	0.00081
3	0.00115	0.425	0.00049	1.000	0.00049	0.00095	0.550	0.00052	1.000	0.00052
4	0.00100	0.425	0.00042	1.000	0.00042	0.00069	0.550	0.00038	1.000	0.00038
5	0.00081	0.425	0.00035	1.000	0.00035	0.00052	0.550	0.00029	1.000	0.00029
6	0.00063	0.425	0.00027	1.000	0.00027	0.00042	0.550	0.00023	1.000	0.00023
7	0.00049	0.425	0.00021	1.000	0.00021	0.00033	0.550	0.00018	1.000	0.00018
8	0.00040	0.425	0.00017	1.000	0.00017	0.00026	0.550	0.00014	1.000	0.00014
9	0.00038	0.425	0.00016	1.000	0.00016	0.00021	0.550	0.00012	1.000	0.00012
10	0.00041	0.425	0.00018	1.000	0.00018	0.00019	0.550	0.00011	1.000	0.00011
11	0.00047	0.425	0.00020	1.000	0.00020	0.00020	0.550	0.00011	1.000	0.00011
12	0.00052	0.425	0.00022	1.000	0.00022	0.00021	0.550	0.00012	1.000	0.00012
13	0.00059	0.425	0.00025	1.000	0.00025	0.00024	0.550	0.00013	1.000	0.00013
14	0.00068	0.425	0.00029	1.000	0.00029	0.00029	0.550	0.00016	1.000	0.00016
15	0.00083	0.425	0.00035	1.000	0.00035	0.00035	0.550	0.00019	1.000	0.00019
16	0.00099	0.425	0.00042	1.000	0.00042	0.00041	0.550	0.00023	1.000	0.00023
17	0.00113	0.425	0.00048	1.000	0.00048	0.00046	0.550	0.00025	1.000	0.00025
18	0.00123	0.425	0.00052	1.000	0.00052	0.00049	0.550	0.00027	1.000	0.00027
19	0.00131	0.425	0.00056	1.000	0.00056	0.00050	0.550	0.00028	1.000	0.00028
20	0.00139	0.425	0.00059	1.000	0.00059	0.00050	0.550	0.00027	1.000	0.00027
21	0.00146	0.425	0.00062	1.000	0.00062	0.00049	0.550	0.00027	1.000	0.00027
22	0.00152	0.425	0.00064	1.000	0.00064	0.00049	0.550	0.00027	1.000	0.00027
23	0.00153	0.425	0.00065	1.000	0.00065	0.00048	0.550	0.00026	1.000	0.00026
24	0.00151	0.425	0.00064	1.000	0.00064	0.00048	0.550	0.00026	1.000	0.00026
25	0.00148	0.425	0.00063	1.000	0.00063	0.00048	0.550	0.00026	1.000	0.00026
26	0.00144	0.425	0.00061	1.000	0.00061	0.00049	0.550	0.00027	1.000	0.00027
27	0.00140	0.425	0.00060	1.000	0.00060	0.00050	0.550	0.00028	1.000	0.00028
28	0.00136	0.425	0.00058	1.000	0.00058	0.00052	0.550	0.00028	1.000	0.00028
29	0.00132	0.425	0.00056	1.000	0.00056	0.00053	0.550	0.00029	1.000	0.00029
30	0.00129	0.425	0.00055	1.000	0.00055	0.00055	0.550	0.00030	1.000	0.00030
31	0.00129	0.425	0.00055	1.000	0.00055	0.00057	0.550	0.00031	1.000	0.00031
32	0.00134	0.425	0.00057	1.000	0.00057	0.00059	0.550	0.00033	1.000	0.00033
33	0.00145	0.425	0.00061	1.000	0.00061	0.00062	0.550	0.00034	1.000	0.00034
34	0.00160	0.425	0.00068	1.000	0.00068	0.00065	0.550	0.00036	1.000	0.00036
35	0.00179	0.425	0.00076	1.000	0.00076	0.00068	0.550	0.00037	1.000	0.00037
36	0.00196	0.425	0.00083	1.000	0.00083	0.00071	0.550	0.00039	1.000	0.00039
37	0.00210	0.425	0.00089	1.000	0.00089	0.00075	0.550	0.00041	1.000	0.00041
38	0.00219	0.425	0.00093	1.000	0.00093	0.00079	0.550	0.00043	1.000	0.00043
39	0.00223	0.425	0.00095	1.000	0.00095	0.00084	0.550	0.00046	1.000	0.00046
40	0.00227	0.425	0.00097	1.000	0.00097	0.00089	0.550	0.00049	1.000	0.00049
41	0.00233	0.425	0.00099	1.000	0.00099	0.00096	0.550	0.00053	1.000	0.00053
42	0.00244	0.425	0.00104	1.000	0.00104	0.00104	0.550	0.00057	1.000	0.00057
43	0.00259	0.425	0.00110	1.000	0.00110	0.00114	0.550	0.00063	1.000	0.00063
44	0.00279	0.425	0.00119	1.000	0.00119	0.00126	0.550	0.00069	1.000	0.00069
45	0.00304	0.425	0.00129	1.000	0.00129	0.00139	0.550	0.00077	1.000	0.00077
46	0.00330	0.425	0.00140	1.000	0.00140	0.00156	0.550	0.00086	1.000	0.00086
47	0.00359	0.425	0.00153	1.000	0.00153	0.00174	0.550	0.00096	1.000	0.00096
48	0.00390	0.425	0.00166	1.000	0.00166	0.00194	0.550	0.00107	1.000	0.00107
49	0.00422	0.428	0.00180	1.000	0.00180	0.00215	0.550	0.00118	1.000	0.00118
50	0.00456	0.435	0.00198	1.000	0.00198	0.00235	0.546	0.00128	1.000	0.00128
51	0.00493	0.446	0.00220	1.000	0.00220	0.00254	0.549	0.00139	1.000	0.00139
52	0.00531	0.461	0.00245	1.000	0.00245	0.00270	0.553	0.00149	1.000	0.00149
53	0.00572	0.475	0.00271	1.000	0.00271	0.00283	0.558	0.00158	1.000	0.00158
54	0.00615	0.486	0.00299	1.000	0.00299	0.00294	0.564	0.00166	1.000	0.00166
55	0.00662	0.495	0.00328	1.000	0.00328	0.00304	0.571	0.00174	0.974	0.00169
56	0.00715	0.501	0.00358	1.000	0.00358	0.00315	0.579	0.00182	0.949	0.00173
57	0.00777	0.504	0.00391	1.000	0.00391	0.00328	0.586	0.00192	0.923	0.00177
58	0.00847	0.509	0.00431	1.000	0.00431	0.00343	0.593	0.00203	0.898	0.00183
59	0.00924	0.517	0.00477	1.000	0.00477	0.00360	0.600	0.00216	0.872	0.00189
60	0.01005	0.528	0.00531	1.000	0.00531	0.00379	0.606	0.00230	0.846	0.00195
61	0.01086	0.543	0.00590	1.000	0.00590	0.00399	0.612	0.00244	0.798	0.00195
62	0.01161	0.562	0.00652	1.000	0.00652	0.00419	0.618	0.00259	0.760	0.00197

63	0.01228	0.584	0.00717	1.000	0.00717	0.00439	0.622	0.00273	0.731	0.00200
64	0.01288	0.606	0.00781	0.964	0.00753	0.00461	0.627	0.00289	0.710	0.00205
65	0.01347	0.615	0.00828	0.940	0.00779	0.00486	0.635	0.00309	0.692	0.00214
66	0.01412	0.615	0.00868	0.939	0.00816	0.00516	0.649	0.00335	0.675	0.00226
67	0.01492	0.615	0.00917	0.937	0.00859	0.00552	0.664	0.00366	0.663	0.00243
68	0.01592	0.615	0.00979	0.933	0.00913	0.00596	0.678	0.00404	0.657	0.00266
69	0.01714	0.615	0.01054	0.928	0.00979	0.00648	0.692	0.00448	0.656	0.00294
70	0.01858	0.615	0.01143	0.923	0.01055	0.00706	0.704	0.00497	0.660	0.00328
71	0.02016	0.615	0.01240	0.918	0.01138	0.00769	0.714	0.00549	0.668	0.00367
72	0.02176	0.623	0.01355	0.913	0.01238	0.00834	0.722	0.00602	0.681	0.00410
73	0.02330	0.635	0.01479	0.909	0.01345	0.00902	0.732	0.00660	0.697	0.00460
74	0.02472	0.650	0.01607	0.905	0.01455	0.00973	0.743	0.00723	0.717	0.00518
75	0.02604	0.668	0.01740	0.903	0.01571	0.01053	0.756	0.00796	0.739	0.00588
76	0.02737	0.689	0.01887	0.901	0.01701	0.01148	0.756	0.00868	0.764	0.00663
77	0.02892	0.712	0.02060	0.901	0.01857	0.01269	0.756	0.00959	0.790	0.00758
78	0.03092	0.712	0.02203	0.903	0.01989	0.01425	0.756	0.01077	0.817	0.00881
79	0.03365	0.712	0.02397	0.906	0.02172	0.01629	0.756	0.01232	0.846	0.01042
80	0.03736	0.712	0.02661	0.911	0.02424	0.01894	0.756	0.01432	0.874	0.01252
81	0.04230	0.712	0.03013	0.917	0.02764	0.02230	0.756	0.01686	0.903	0.01522
82	0.04866	0.712	0.03467	0.927	0.03213	0.02646	0.756	0.02000	0.930	0.01860
83	0.05654	0.712	0.04028	0.939	0.03782	0.03141	0.756	0.02375	0.956	0.02272
84	0.06588	0.712	0.04693	0.953	0.04471	0.03713	0.756	0.02807	0.981	0.02754
85	0.07645	0.712	0.05447	0.968	0.05270	0.04348	0.756	0.03287	1.000	0.03287
86	0.08790	0.712	0.06262	0.984	0.06160	0.05035	0.756	0.03806	1.000	0.03806
87	0.09977	0.712	0.07108	1.000	0.07108	0.05760	0.756	0.04355	1.000	0.04355
88	0.11170	0.712	0.07958	1.000	0.07958	0.06520	0.756	0.04930	1.000	0.04930
89	0.12348	0.712	0.08797	1.000	0.08797	0.07319	0.756	0.05533	1.000	0.05533
90	0.13513	0.712	0.09627	1.000	0.09627	0.08167	0.756	0.06175	1.000	0.06175
91	0.14688	0.712	0.10464	1.000	0.10464	0.09085	0.756	0.06868	1.000	0.06868
92	0.15910	0.712	0.11334	1.000	0.11334	0.10090	0.756	0.07628	1.000	0.07628
93	0.17225	0.712	0.12271	1.000	0.12271	0.11202	0.756	0.08469	1.000	0.08469
94	0.18676	0.712	0.13305	1.000	0.13305	0.12435	0.756	0.09402	1.000	0.09402
95	0.21029	0.712	0.14981	1.000	0.14981	0.14722	0.756	0.11130	1.000	0.11130
96	0.22959	0.712	0.16356	1.000	0.16356	0.16399	0.756	0.12398	1.000	0.12398
97	0.25003	0.712	0.17813	1.000	0.17813	0.18211	0.756	0.13768	1.000	0.13768
98	0.27160	0.712	0.19349	1.000	0.19349	0.20160	0.756	0.15241	1.000	0.15241
99	0.29428	0.712	0.20965	1.000	0.20965	0.22247	0.756	0.16819	1.000	0.16819
100	0.31804	0.712	0.22657	1.000	0.22657	0.24474	0.756	0.18503	1.000	0.18503
101	0.34284	0.712	0.24424	1.000	0.24424	0.26840	0.756	0.20292	1.000	0.20292
102	0.36864	0.712	0.26262	1.000	0.26262	0.29342	0.756	0.22184	1.000	0.22184
103	0.39537	0.712	0.28167	1.000	0.28167	0.31978	0.756	0.24176	1.000	0.24176
104	0.42296	0.712	0.30132	1.000	0.30132	0.34742	0.756	0.26266	1.000	0.26266
105	0.45133	0.712	0.32153	1.000	0.32153	0.37626	0.756	0.28446	1.000	0.28446
106	0.48038	0.712	0.34223	1.000	0.34223	0.40623	0.756	0.30712	1.000	0.30712
107	0.51000	0.712	0.36333	1.000	0.36333	0.43721	0.756	0.33055	1.000	0.33055
108	0.54006	0.712	0.38475	1.000	0.38475	0.46909	0.756	0.35465	1.000	0.35465
109	0.57044	0.736	0.42006	1.000	0.42006	0.50172	0.756	0.37932	1.000	0.37932
110	0.60100	0.760	0.45697	1.000	0.45697	0.53495	0.756	0.40444	1.000	0.40444
111	0.63159	0.784	0.49536	1.000	0.49536	0.56859	0.756	0.42987	1.000	0.42987
112	0.66205	0.808	0.53512	1.000	0.53512	0.60247	0.763	0.45947	1.000	0.45947
113	0.69221	0.832	0.57609	1.000	0.57609	0.63636	0.792	0.50420	1.000	0.50420
114	0.72191	0.856	0.61810	1.000	0.61810	0.67007	0.822	0.55079	1.000	0.55079
115	0.75096	0.880	0.66098	1.000	0.66098	0.70336	0.852	0.59902	1.000	0.59902
116	0.77920	0.904	0.70451	1.000	0.70451	0.73599	0.881	0.64865	1.000	0.64865
117	0.80645	0.928	0.74847	1.000	0.74847	0.76774	0.911	0.69940	1.000	0.69940
118	0.83252	0.952	0.79262	1.000	0.79262	0.79835	0.941	0.75098	1.000	0.75098
119	0.85726	0.976	0.83671	1.000	0.83671	0.82759	0.970	0.80304	1.000	0.80304
120	1.00000	1.000	1.00000	1.000	1.00000	1.00000	1.000	1.00000	1.000	1.00000

Appendix 4: Residuals from modelling kappa

Table 25. LC model residuals for kappa modelling – females

	AC	PAC	Q-Stat	Prob
1	-0.210	-0.210	1.7712	0.183
2	-0.089	-0.139	2.0984	0.350
3	0.052	0.001	2.2112	0.530
4	-0.075	-0.080	2.4579	0.652
5	0.304	0.295	6.6169	0.251
6	-0.122	-0.009	7.3086	0.293
7	0.015	0.071	7.3194	0.396
8	-0.213	-0.284	9.5842	0.295
9	0.034	-0.025	9.6439	0.380
10	0.201	0.054	11.807	0.298
11	-0.214	-0.094	14.338	0.215
12	0.179	0.159	16.178	0.183
13	-0.193	-0.073	18.426	0.142
14	0.001	-0.016	18.426	0.188
15	-0.010	-0.211	18.433	0.241
16	0.070	0.151	18.770	0.281

Table 26. Poisson log-bilinear residuals for kappa modelling – males

	AC	PAC	Q-Stat	Prob
1	-0.255	-0.255	2.6156	0.106
2	-0.118	-0.196	3.1852	0.203
3	-0.089	-0.195	3.5248	0.318
4	-0.029	-0.161	3.5618	0.469
5	0.261	0.181	6.6234	0.250
6	-0.225	-0.153	8.9865	0.174
7	-0.003	-0.060	8.9868	0.254
8	-0.177	-0.257	10.540	0.229
9	0.160	-0.003	11.866	0.221
10	0.048	-0.076	11.990	0.286
11	-0.230	-0.238	14.929	0.186
12	0.310	0.213	20.473	0.059
13	-0.156	-0.042	21.929	0.056
14	0.046	-0.084	22.063	0.077
15	-0.025	-0.028	22.104	0.105
16	-0.038	-0.016	22.201	0.137

Table 27. APC model residuals for kappa modelling – females

	AC	PAC	Q-Stat	Prob
1	-0.025	-0.025	0.0242	0.876
2	-0.076	-0.077	0.2587	0.879
3	-0.160	-0.165	1.3137	0.726
4	-0.331	-0.360	6.0064	0.199
5	0.031	-0.050	6.0479	0.302
6	0.068	-0.031	6.2615	0.395
7	-0.000	-0.137	6.2615	0.510
8	0.122	-0.013	6.9867	0.538
9	-0.032	-0.038	7.0376	0.633
10	0.080	0.096	7.3740	0.690
11	-0.061	-0.077	7.5807	0.750
12	0.026	0.088	7.6203	0.814
13	-0.048	-0.030	7.7580	0.859
14	-0.100	-0.070	8.3845	0.868
15	0.032	-0.003	8.4503	0.904
16	-0.001	-0.012	8.4503	0.934

Table 28. Poisson log-bilinear residuals for kappa modelling – females

	AC	PAC	Q-Stat	Prob
1	-0.072	-0.072	0.2090	0.648
2	-0.161	-0.167	1.2718	0.529
3	-0.050	-0.079	1.3799	0.710
4	-0.013	-0.053	1.3870	0.846
5	0.020	-0.008	1.4055	0.924
6	-0.094	-0.113	1.8171	0.936
7	0.130	0.114	2.6304	0.917
8	0.035	0.024	2.6912	0.952
9	-0.154	-0.127	3.9196	0.917
10	0.128	0.134	4.7928	0.905
11	0.155	0.160	6.1273	0.865
12	0.152	0.214	7.4576	0.826
13	-0.251	-0.155	11.245	0.590
14	-0.087	-0.047	11.717	0.629
15	0.133	0.075	12.877	0.612
16	-0.064	-0.038	13.157	0.661

Table 29. LC model residuals for kappa modelling – females

	AC	PAC	Q-Stat	Prob
1	0.105	0.105	0.4340	0.510
2	0.063	0.053	0.5952	0.743
3	-0.331	-0.348	5.1487	0.161
4	-0.093	-0.027	5.5211	0.238
5	0.191	0.295	7.1360	0.211
6	0.098	-0.073	7.5702	0.271
7	0.108	-0.006	8.1237	0.322
8	-0.359	-0.272	14.420	0.071
9	-0.043	0.094	14.513	0.105
10	0.108	0.266	15.128	0.127
11	0.408	0.219	24.232	0.012
12	0.055	-0.234	24.407	0.018
13	-0.038	0.097	24.493	0.027
14	-0.227	0.035	27.701	0.016
15	-0.134	-0.122	28.874	0.017
16	0.070	-0.142	29.206	0.023

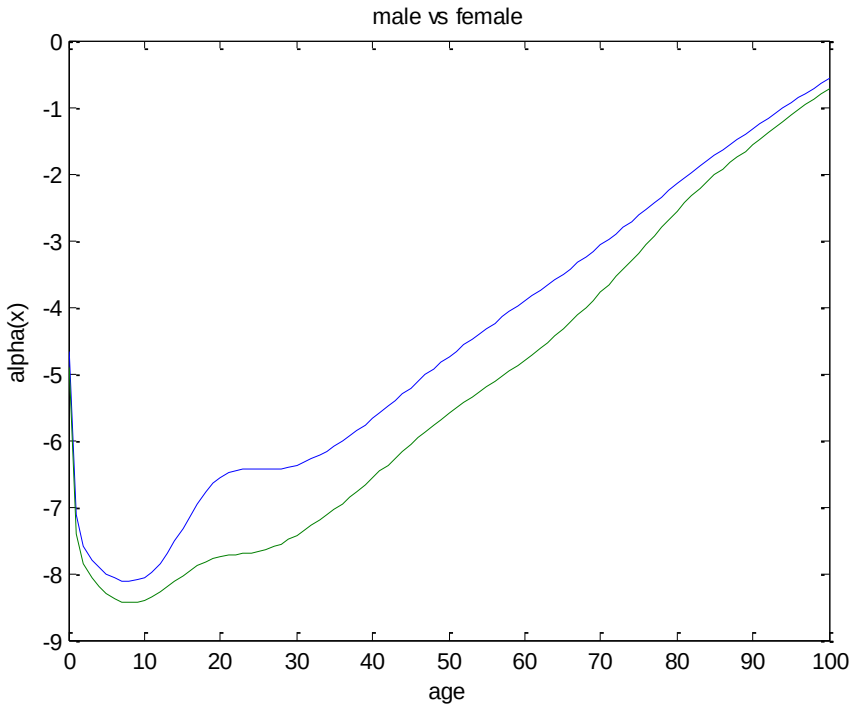
Table 30. APC model residuals for kappa modelling – females

	AC	PAC	Q-Stat	Prob
1	-0.108	-0.108	0.4578	0.499
2	-0.240	-0.255	2.7745	0.250
3	-0.096	-0.169	3.1544	0.368
4	0.049	-0.059	3.2579	0.516
5	0.314	0.275	7.6128	0.179
6	-0.227	-0.172	9.9628	0.126
7	-0.119	-0.037	10.636	0.155
8	-0.028	-0.102	10.675	0.221
9	0.104	0.017	11.220	0.261
10	-0.028	-0.169	11.263	0.337
11	-0.154	-0.064	12.569	0.322
12	0.186	0.166	14.536	0.268
13	-0.088	-0.127	14.997	0.308
14	-0.075	-0.137	15.344	0.355
15	-0.062	-0.077	15.598	0.409
16	0.145	0.146	17.028	0.384

Appendix 5: LC with smoothed data

In this appendix we compare the results of the LC model for both males and females using smoothed data as explained in Section 4.2.

Figure 11-1: Alpha(x) as a function of age (females vs. males), the LC model; Blue line males, Green line females



When comparing the results of alpha for the smoothed population, one can see that the average mortality over the past 40 years favoured women to a greater extent than men. Besides lower mortality in the case of women, we do not observe a mortality hump around the age of 20, which could be attributable to the "testosterone" effect. The values of beta show that (given that the difference in values of kappa between the beginning and end of the period is almost identical for both sexes) young males (with the exception of new-borns) and middle-aged men (aged between 30 and 50) fared better mortality-wise than females. On the other hand, women in the age group between 20 and 30 and women aged above 55 observed decreases in mortality above decreases for men of a comparable age. The dynamics of kappa again exhibit a downward trend with mild oscillations around the trend.

Figure 11-2: Beta(x) as a function of year (females vs. males), the Poisson model

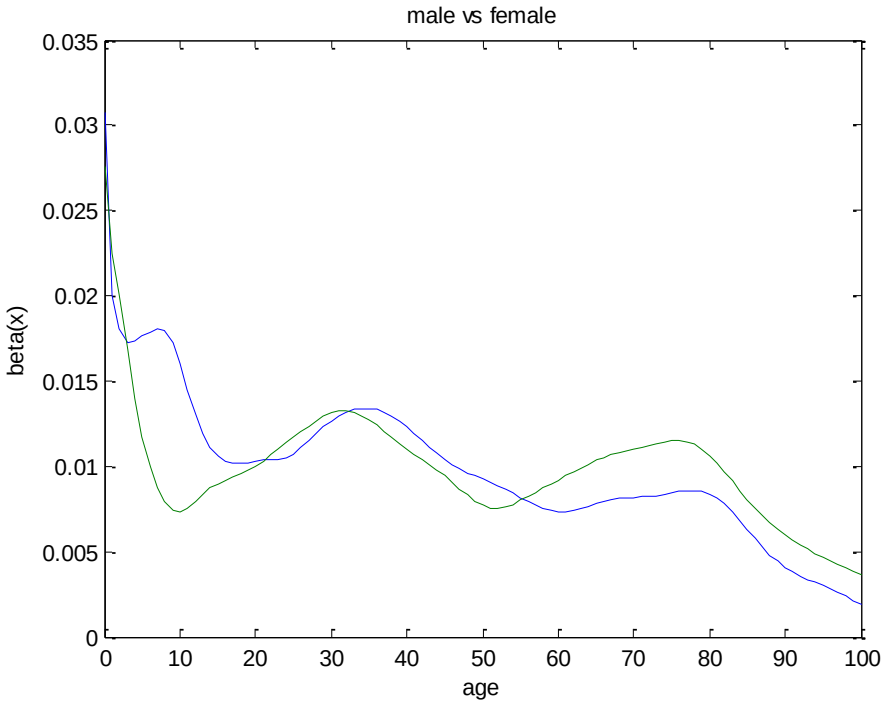
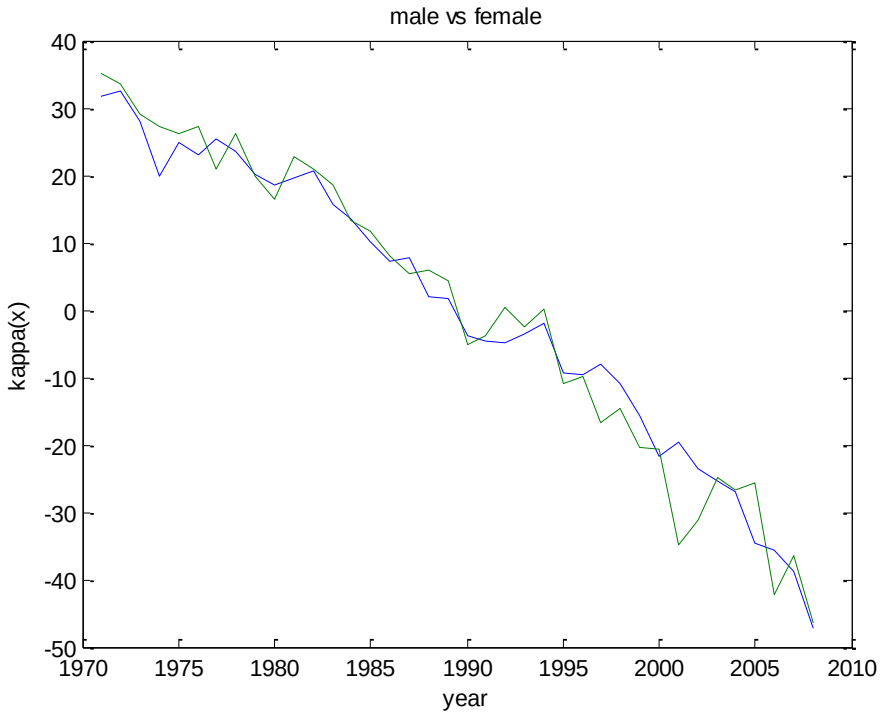
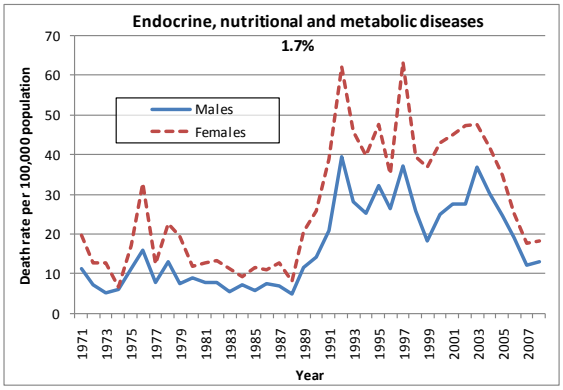
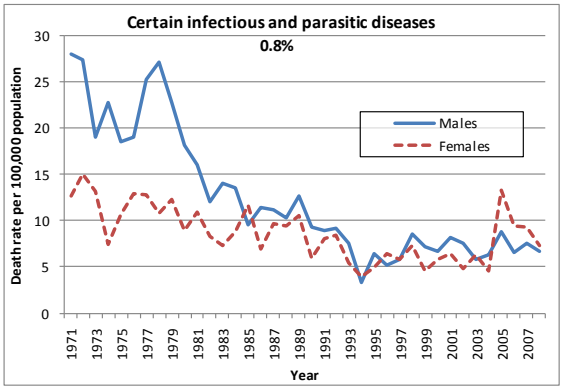


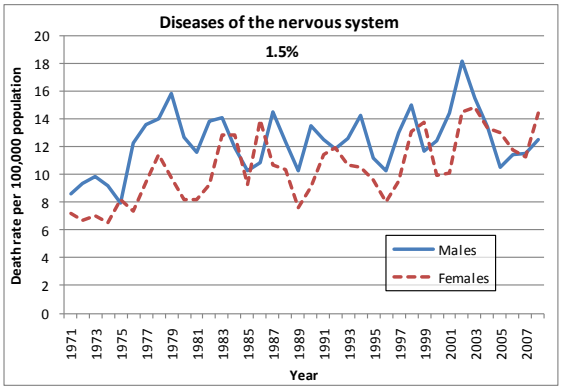
Figure 11-3: Kappa as a function of year (females), the Poisson vs. the LC model

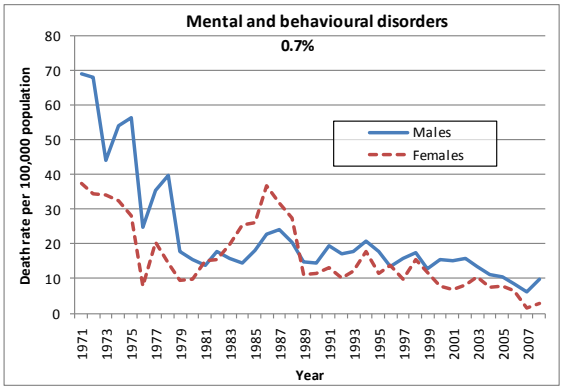


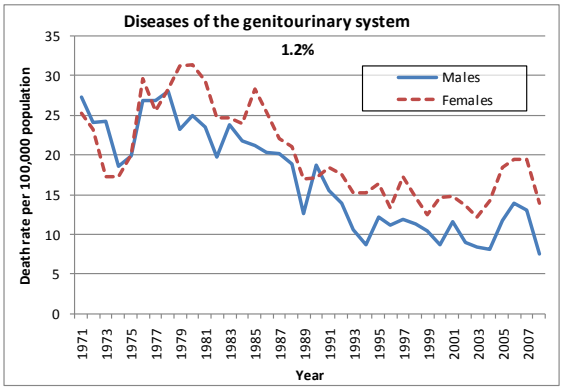
Appendix 6: Age-standardised death rates by cause of death (remaining major cause groups)

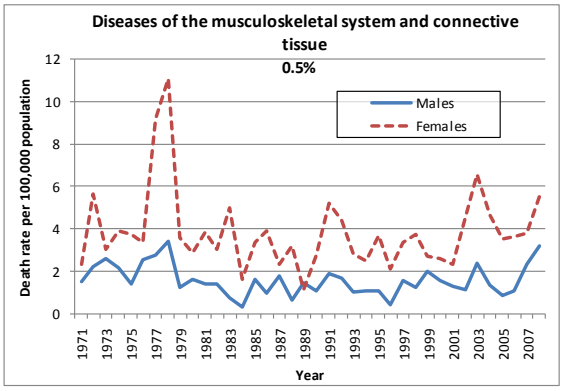


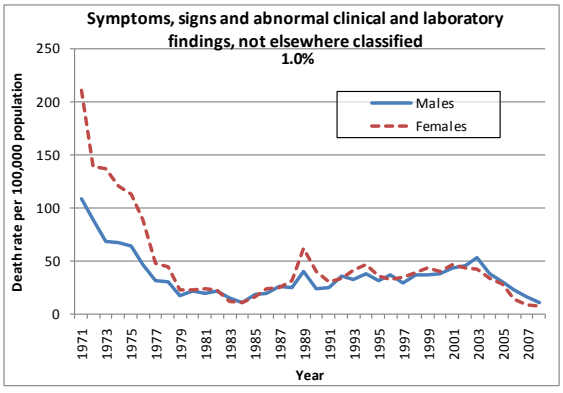


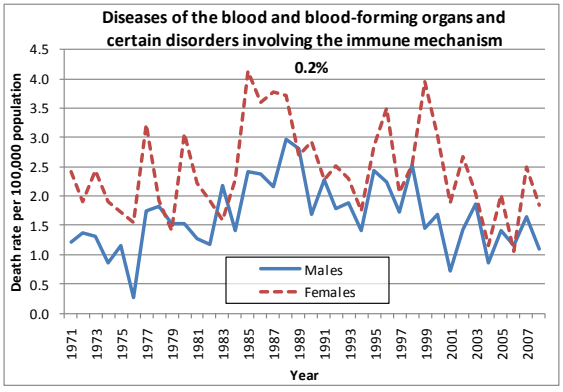


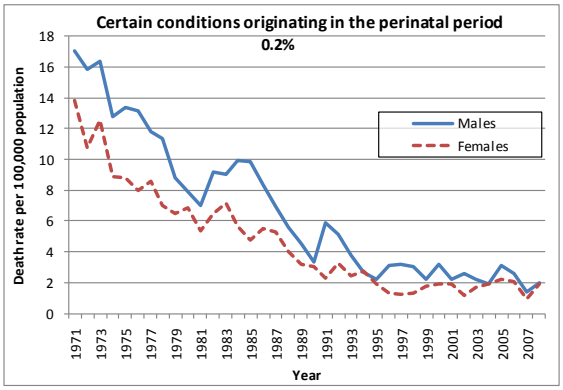


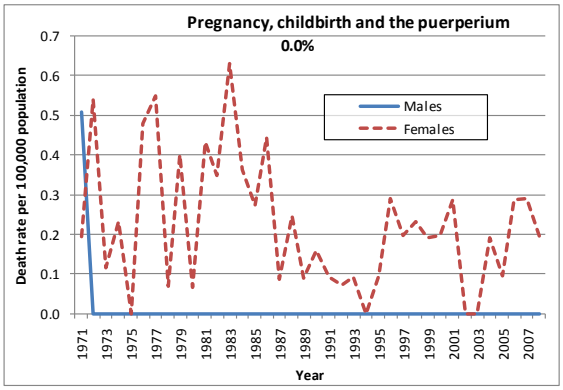


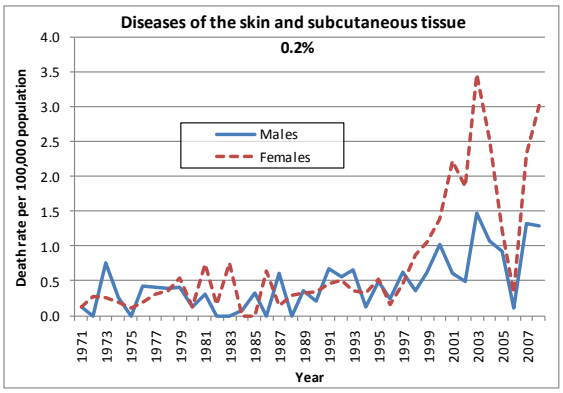


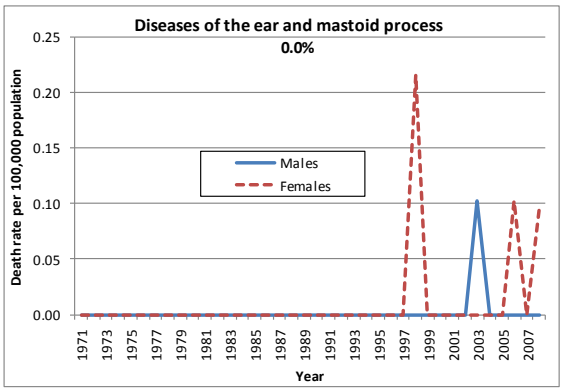


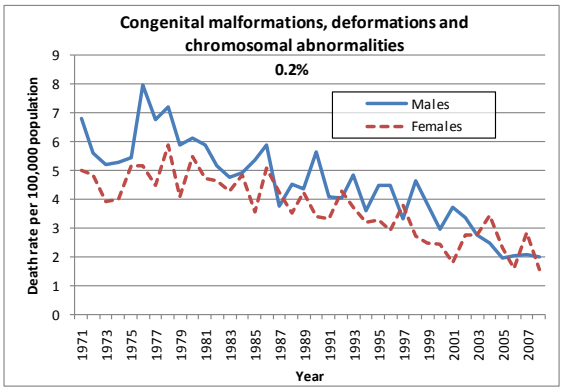


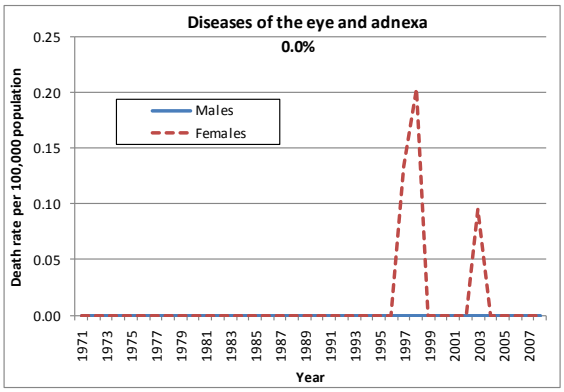








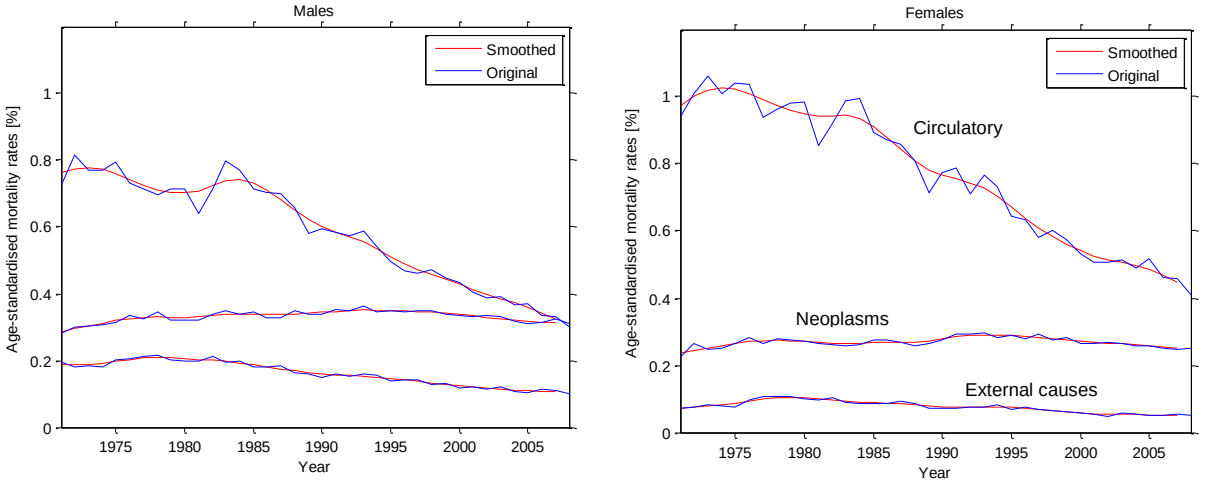




Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia; authors' calculations

Appendix 7: Original and smoothed age-standardised death rates by cause of death (three major cause groups with the biggest share among total deaths)

Figure 11-4: Original and smoothed age-standardised death rates by cause of death (three main cause groups with the biggest share among total deaths)



Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia; authors' calculations

Circulatory

Appendix 8: Number of deaths in 2008 by cause groups of death and age groups

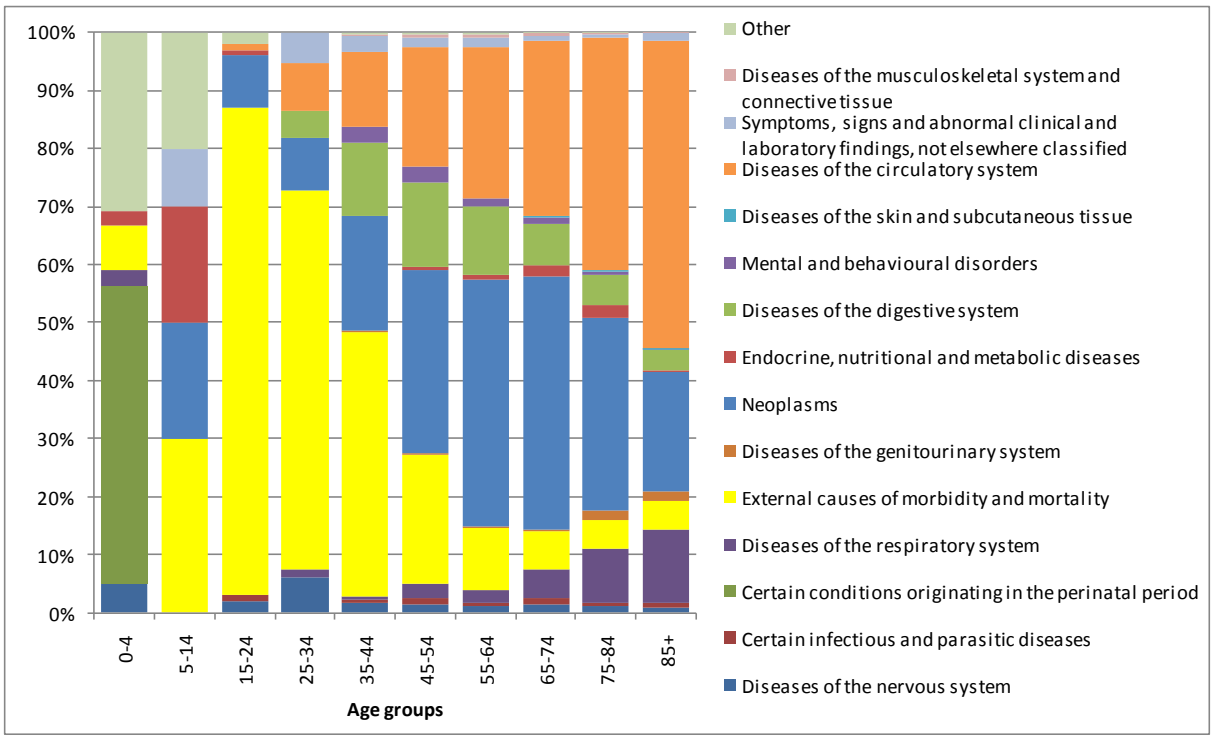
Neoplasms **Table 31: Number of deaths in 2008 by cause groups of death and age groups**

External causes	0–4	5–14	15–24	25–34	35–44	45–54	55–64	65–74	75–84	85+	TOTAL
Diseases of the nervous system	3	3	3	12	12	19	32	62	91	38	**275**
Certain infectious and parasitic diseases	0	0	1	0	3	11	10	34	56	27	**142**
Certain conditions originating in the perinatal period	40	0	0	0	0	0	0	0	0	0	**40**
Diseases of the respiratory system	1	1	1	2	1	27	51	172	480	406	**1142**
External causes of morbidity and mortality	3	6	101	105	151	221	206	209	283	230	**1515**
Diseases of the genitourinary system	0	0	0	0	1	3	3	28	104	80	**219**
Neoplasms	1	5	10	26	108	497	1007	1577	1823	695	**5749**
Endocrine, nutritional and metabolic diseases	1	3	2	0	1	9	20	75	153	52	**316**
Diseases of the digestive system	0	0	0	9	46	158	237	233	329	161	**1173**
Mental and behavioural disorders	0	0	0	1	9	24	32	36	18	9	**129**
Diseases of the skin and subcutaneous tissue	0	0	0	0	0	0	0	8	24	12	**44**
Diseases of the circulatory system	0	0	2	12	50	221	513	1065	2853	2509	**7225**
Symptoms, signs and abnormal clinical and laboratory findings, not elsewhere classified	0	1	0	8	10	18	31	32	29	52	**181**
Diseases of the musculoskeletal system and connective tissue	0	0	0	0	1	5	19	27	29	8	**89**
Other	14	4	4	4	4	4	6	10	11	8	**69**
TOTAL	**63**	**23**	**124**	**179**	**397**	**1217**	**2167**	**3568**	**6283**	**4287**	**18308**

Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia

Appendix 9: Gender-specific analysis of major cause groups of death

Figure 11-5: Age-standardised death rates by cause of death (five major cause groups with the biggest share among total deaths); males



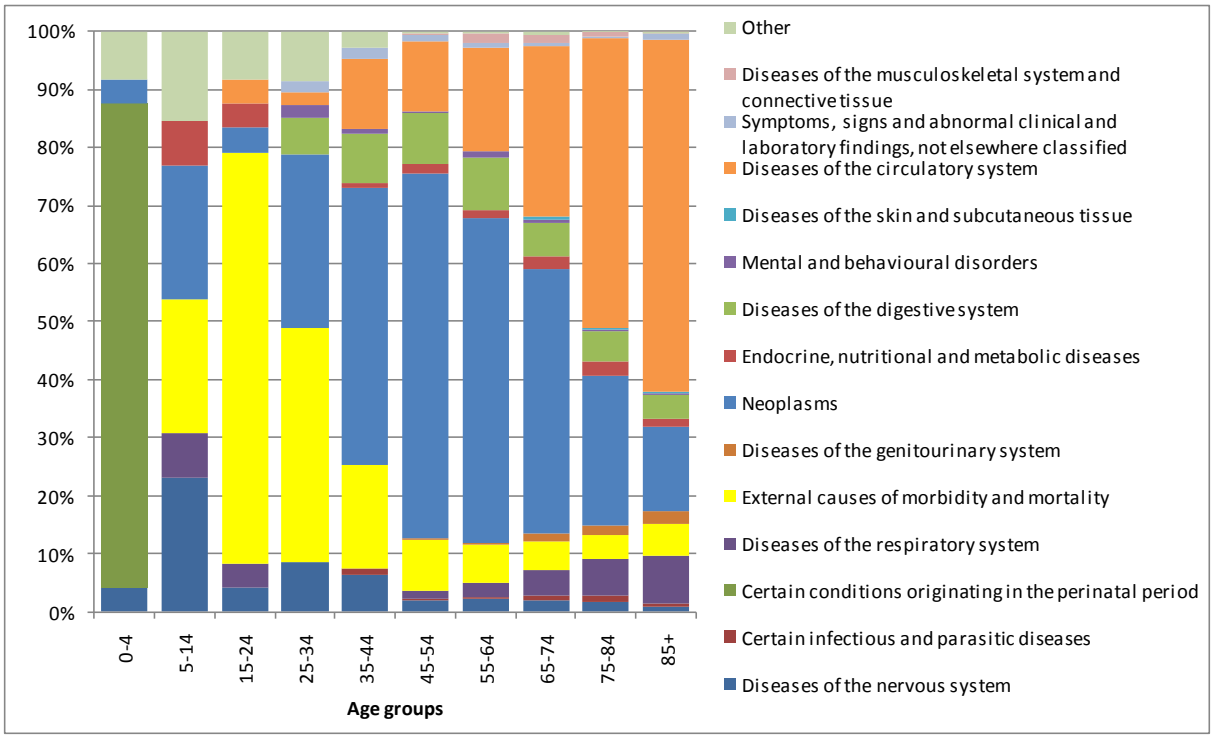
Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia; authors' calculations

Table 32: Number of deaths in 2008 by cause of death and age groups; males

	0–4	5–14	15–24	25–34	35–44	45–54	55–64	65–74	75–84	85+	TOTAL
Diseases of the nervous system	2	0	2	8	5	12	18	34	34	12	**127**
Certain infectious and parasitic diseases	0	0	1	0	2	10	8	23	16	7	**67**
Certain conditions originating in the perinatal period	20	0	0	0	0	0	0	0	0	0	**20**
Diseases of the respiratory system	1	0	0	2	1	22	36	115	256	146	**579**
External causes of morbidity and mortality	3	3	84	86	132	189	165	145	145	56	**1008**
Diseases of the genitourinary system	0	0	0	0	1	2	2	10	44	17	**76**
Neoplasms	0	2	9	12	57	270	661	983	927	236	**3157**
Endocrine, nutritional and metabolic diseases	1	2	1	0	0	4	12	45	60	5	**130**
Diseases of the digestive system	0	0	0	6	37	126	181	157	146	38	**691**
Mental and behavioural disorders	0	0	0	0	8	23	25	29	15	1	**101**
Diseases of the skin and subcutaneous tissue	0	0	0	0	0	0	0	1	10	2	**13**
Diseases of the circulatory system	0	0	1	11	37	177	403	682	1117	605	**3033**
Symptoms, signs and abnormal clinical and laboratory findings, not elsewhere classified	0	1	0	7	8	14	26	23	14	16	**109**
Diseases of the musculoskeletal system and connective tissue	0	0	0	0	1	4	9	9	7	2	**32**
Other	12	2	2	0	1	3	4	3	4	0	**31**
TOTAL	**39**	**10**	**100**	**132**	**290**	**856**	**1550**	**2259**	**2795**	**1143**	**9174**

Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia; authors' calculations

Figure 11-6: Age-standardised death rates by cause of death (five major cause groups with the biggest share among total deaths); females



Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia; authors' calculations

Table 33: Number of deaths in 2008 by cause of death and age groups; females

	0–4	5–14	15–24	25–34	35–44	45–54	55–64	65–74	75–84	85+	TOTAL
Diseases of the nervous system	1	3	1	4	7	7	14	28	57	26	**148**
Certain infectious and parasitic diseases	0	0	0	0	1	1	2	11	40	20	**75**
Certain conditions originating in the perinatal period	20	0	0	0	0	0	0	0	0	0	**20**
Diseases of the respiratory system	0	1	1	0	0	5	15	57	224	260	**563**
External causes of morbidity and mortality	0	3	17	19	19	32	41	64	138	174	**507**
Diseases of the genitourinary system	0	0	0	0	0	1	1	18	60	63	**143**
Neoplasms	1	3	1	14	51	227	346	594	896	459	**2592**
Endocrine, nutritional and metabolic diseases	0	1	1	0	1	5	8	30	93	47	**186**
Diseases of the digestive system	0	0	0	3	9	32	56	76	183	123	**482**
Mental and behavioural disorders	0	0	0	1	1	1	7	7	3	8	**28**
Diseases of the skin and subcutaneous tissue	0	0	0	0	0	0	0	7	14	10	**31**
Diseases of the circulatory system	0	0	1	1	13	44	110	383	1736	1904	**4192**
Symptoms, signs and abnormal clinical and laboratory findings, not elsewhere classified	0	0	0	1	2	4	5	9	15	36	**72**
Diseases of the musculoskeletal system and connective tissue	0	0	0	0	0	1	10	18	22	6	**57**
Other	2	2	2	4	3	1	2	7	7	8	**38**
TOTAL	**24**	**13**	**24**	**47**	**107**	**361**	**617**	**1309**	**3488**	**3144**	**9134**

Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia; authors' calculations

Table 34: Differences between shares of death for males and females

	0–4	5–14	15–24	25–34	35–44	45–54	55–64	65–74	75–84	85+	TOTAL
Diseases of the nervous system	-1.0	23.1	2.2	2.5	4.8	0.5	1.1	0.6	0.4	-0.2	**0.2**
Certain infectious and parasitic diseases	0.0	0.0	-1.0	0.0	0.2	-0.9	-0.2	-0.2	0.6	0.0	**0.1**
Certain conditions originating in the perinatal period	32.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	**0.0**
Diseases of the respiratory system	-2.6	7.7	4.2	-1.5	-0.3	-1.2	0.1	-0.7	-2.7	-4.5	**-0.1**
External causes of morbidity and mortality	-7.7	-6.9	-13.2	-24.7	-27.8	-13.2	-4.0	-1.5	-1.2	0.6	**-5.4**
Diseases of the genitourinary system	0.0	0.0	0.0	0.0	-0.3	0.0	0.0	0.9	0.1	0.5	**0.7**
Neoplasms	4.2	3.1	-4.8	20.7	28.0	31.3	13.4	1.9	-7.5	-6.0	**-6.0**
Endocrine, nutritional and metabolic diseases	-2.6	-12.3	3.2	0.0	0.9	0.9	0.5	0.3	0.5	1.1	**0.6**
Diseases of the digestive system	0.0	0.0	0.0	1.8	-4.3	-5.9	-2.6	-1.1	0.0	0.6	**-2.3**
Mental and behavioural disorders	0.0	0.0	0.0	2.1	-1.8	-2.4	-0.5	-0.7	-0.5	0.2	**-0.8**
Diseases of the skin and subcutaneous tissue	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.5	0.0	0.1	**0.2**
Diseases of the circulatory system	0.0	0.0	3.2	-6.2	-0.6	-8.5	-8.2	-0.9	9.8	7.6	**12.8**
Symptoms, signs and abnormal clinical and laboratory findings, not elsewhere classified	0.0	-10.0	0.0	-3.2	-0.9	-0.5	-0.9	-0.3	-0.1	-0.3	**-0.4**
Diseases of the musculoskeletal system and connective tissue	0.0	0.0	0.0	0.0	-0.3	-0.2	1.0	1.0	0.4	0.0	**0.3**
Other	-22.4	-4.6	6.3	8.5	2.5	-0.1	0.1	0.4	0.1	0.3	**0.1**

Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia; authors' calculations

Appendix 10: Life expectancy by age and gender whereby the rate of mortality improvement is calculated using total mortality rates – not as the sum of individual cause groups of death

Table 35: Life expectancy for males, by age; Scenario 1, mortality rates not being calculated as the sum of assumed mortality rates by cause groups of death but as a total

	2010	2015	2020	2025	2030	2035	2040	2045	2050	2055	2060
0	76.36	77.85	78.91	79.70	80.33	80.96	81.60	82.25	82.91	83.59	84.28
1–4	75.56	77.03	78.08	78.86	79.49	80.11	80.74	81.39	82.04	82.71	83.40
5–9	71.62	73.09	74.13	74.92	75.54	76.16	76.79	77.43	78.09	78.76	79.44
10–14	66.63	68.10	69.14	69.92	70.55	71.17	71.79	72.44	73.09	73.76	74.45
15–19	61.68	63.14	64.19	64.97	65.59	66.21	66.83	67.47	68.13	68.80	69.48
20–24	56.87	58.31	59.35	60.12	60.74	61.35	61.97	62.61	63.25	63.92	64.60
25–29	52.08	53.51	54.53	55.30	55.91	56.52	57.13	57.76	58.40	59.06	59.73
30–34	47.27	48.68	49.70	50.46	51.07	51.66	52.27	52.90	53.53	54.18	54.85
35–39	42.43	43.82	44.83	45.59	46.19	46.79	47.39	48.01	48.64	49.29	49.96
40–44	37.68	39.03	40.04	40.79	41.39	41.97	42.57	43.18	43.81	44.45	45.11
45–49	33.05	34.38	35.38	36.12	36.70	37.28	37.86	38.46	39.08	39.71	40.36
50–54	28.67	29.95	30.93	31.66	32.23	32.78	33.35	33.94	34.54	35.16	35.79
55–59	24.47	25.72	26.67	27.38	27.92	28.46	29.01	29.57	30.15	30.75	31.37
60–64	20.65	21.83	22.74	23.42	23.93	24.43	24.95	25.49	26.04	26.62	27.21
65–69	16.95	18.03	18.86	19.48	19.95	20.43	20.92	21.43	21.96	22.51	23.08
70–74	13.36	14.23	14.93	15.48	15.93	16.38	16.85	17.34	17.84	18.37	18.92
75–79	10.34	11.02	11.59	12.05	12.46	12.89	13.33	13.79	14.27	14.77	15.29
80–84	7.71	8.21	8.65	9.03	9.41	9.80	10.22	10.65	11.11	11.59	12.09
85+	5.87	6.31	6.69	7.03	7.40	7.78	8.18	8.60	9.04	9.51	10.00

Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia; authors' calculations

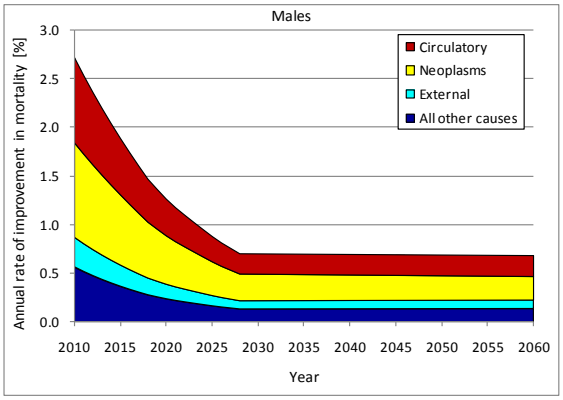
Table 36: Life expectancy for females, by age; Scenario 1, mortality rates not being calculated as the sum of assumed mortality rates by cause groups of death but as a total

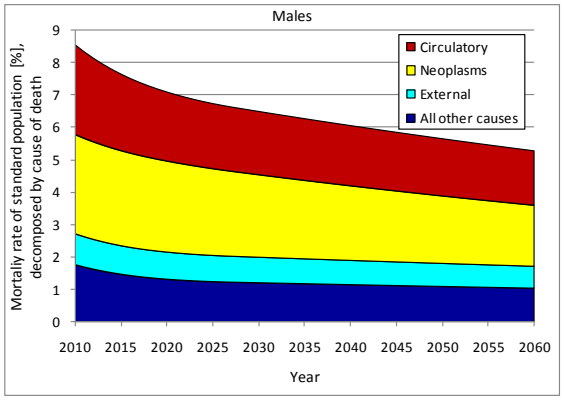
	2010	2015	2020	2025	2030	2035	2040	2045	2050	2055	2060
0	83.13	84.33	85.21	85.89	86.49	87.10	87.72	88.37	89.03	89.72	90.43
1–4	82.30	83.47	84.35	85.03	85.62	86.22	86.84	87.48	88.14	88.82	89.52
5–9	78.32	79.49	80.36	81.04	81.63	82.23	82.85	83.49	84.15	84.83	85.53
10–14	73.39	74.55	75.42	76.10	76.69	77.29	77.90	78.54	79.19	79.87	80.58
15–19	68.42	69.58	70.44	71.12	71.71	72.31	72.92	73.56	74.21	74.89	75.59
20–24	63.49	64.64	65.50	66.18	66.77	67.36	67.97	68.61	69.26	69.94	70.64
25–29	58.54	59.68	60.54	61.22	61.81	62.40	63.01	63.64	64.29	64.97	65.67
30–34	53.61	54.75	55.60	56.28	56.86	57.45	58.06	58.69	59.34	60.01	60.71
35–39	48.70	49.83	50.68	51.35	51.93	52.52	53.13	53.75	54.40	55.07	55.77
40–44	43.80	44.92	45.77	46.44	47.02	47.60	48.20	48.83	49.47	50.14	50.83
45–49	38.98	40.08	40.92	41.58	42.15	42.73	43.33	43.95	44.59	45.26	45.94
50–54	34.23	35.29	36.12	36.78	37.34	37.92	38.51	39.12	39.76	40.41	41.10
55–59	29.72	30.75	31.56	32.21	32.76	33.32	33.89	34.49	35.12	35.76	36.44
60–64	25.24	26.23	27.02	27.64	28.18	28.72	29.29	29.88	30.49	31.12	31.79
65–69	20.91	21.84	22.58	23.17	23.69	24.22	24.78	25.35	25.95	26.57	27.22
70–74	16.73	17.56	18.24	18.80	19.30	19.81	20.35	20.91	21.49	22.10	22.74
75–79	12.86	13.58	14.18	14.68	15.16	15.66	16.18	16.72	17.29	17.88	18.50
80–84	9.57	10.18	10.70	11.15	11.61	12.08	12.58	13.10	13.65	14.23	14.83
85+	7.06	7.63	8.11	8.52	8.96	9.43	9.91	10.42	10.96	11.52	12.12

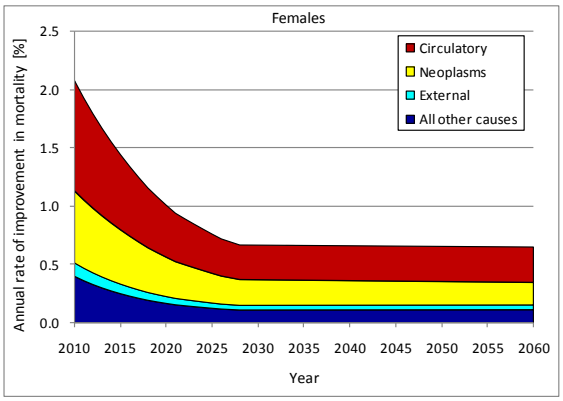
Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia; authors' calculations

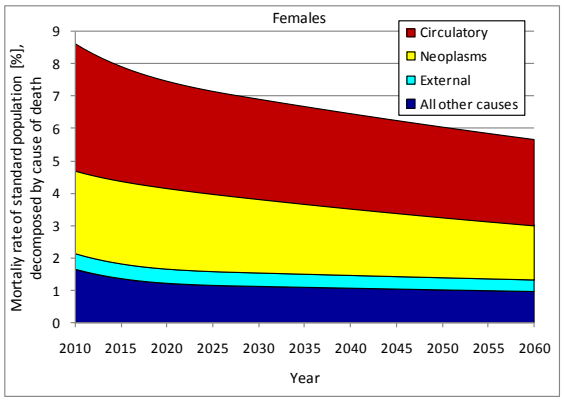
Appendix 11: Results of Scenarios 2 and 3 – Annual rate of improvement in mortality [%], mortality rate of standard population [%] and life expectancy

Figure 11-7: Annual rate of improvement in mortality [%] and mortality rate of standard population [%] in 2010–2060 projection period, decomposed by cause of death; Scenario 2









Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia

Table 37: Life expectancy for males, by age; Scenario 2, mortality rates are calculated as a sum of assumed mortality rates by cause groups of death

	2010	2015	2020	2025	2030	2035	2040	2045	2050	2055	2060
0	76.31	77.60	78.42	78.99	79.42	79.83	80.24	80.65	81.07	81.49	81.91
1–4	75.50	76.76	77.58	78.15	78.57	78.98	79.39	79.80	80.21	80.63	81.04
5–9	71.56	72.82	73.64	74.20	74.63	75.03	75.44	75.85	76.26	76.68	77.09
10–14	66.57	67.83	68.64	69.21	69.63	70.04	70.45	70.86	71.27	71.68	72.10
15–19	61.62	62.88	63.69	64.26	64.68	65.08	65.49	65.90	66.31	66.72	67.14
20–24	56.81	58.05	58.86	59.42	59.84	60.25	60.65	61.05	61.46	61.87	62.28
25–29	52.03	53.26	54.07	54.62	55.04	55.44	55.84	56.24	56.64	57.05	57.46
30–34	47.21	48.43	49.23	49.79	50.20	50.59	50.99	51.39	51.79	52.19	52.60
35–39	42.38	43.59	44.39	44.94	45.35	45.74	46.14	46.53	46.93	47.33	47.73
40–44	37.63	38.81	39.61	40.16	40.56	40.95	41.34	41.73	42.13	42.52	42.92
45–49	33.02	34.18	34.97	35.51	35.91	36.29	36.67	37.06	37.45	37.84	38.23
50–54	28.63	29.76	30.54	31.08	31.46	31.83	32.21	32.58	32.96	33.34	33.72
55–59	24.44	25.52	26.29	26.81	27.18	27.54	27.89	28.25	28.62	28.98	29.35
60–64	20.61	21.64	22.37	22.86	23.21	23.54	23.88	24.21	24.55	24.90	25.25
65–69	16.92	17.86	18.52	18.97	19.29	19.60	19.91	20.22	20.54	20.86	21.19
70–74	13.33	14.07	14.61	15.00	15.29	15.58	15.87	16.16	16.46	16.76	17.07
75–79	10.32	10.89	11.32	11.63	11.89	12.14	12.40	12.67	12.94	13.22	13.50
80–84	7.69	8.10	8.41	8.65	8.88	9.11	9.34	9.59	9.83	10.09	10.35
85+	5.84	6.18	6.43	6.63	6.84	7.05	7.27	7.49	7.72	7.95	8.20

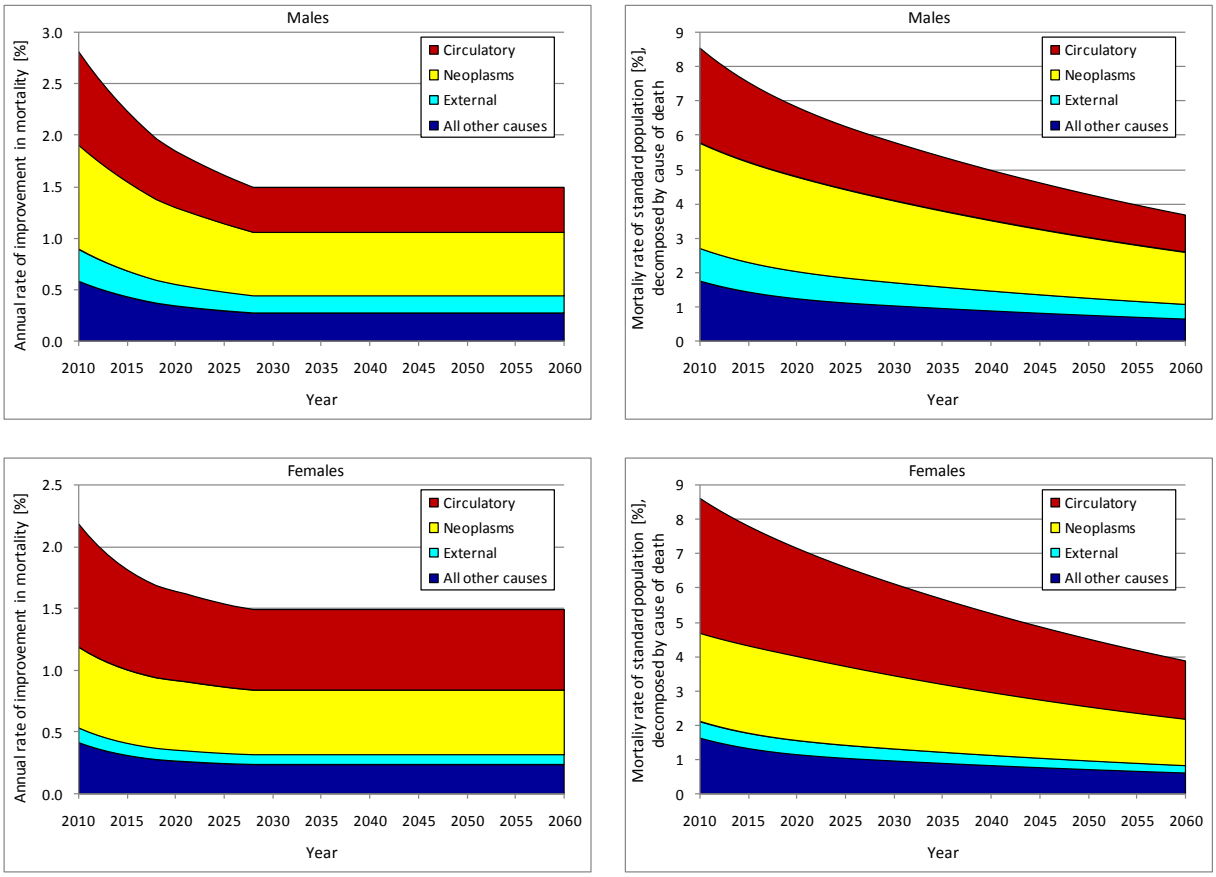
Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia; authors' calculations

Table 38: Life expectancy for females, by age; Scenario 2, mortality rates are calculated as a sum of assumed mortality rates by cause groups of death

	2010	2015	2020	2025	2030	2035	2040	2045	2050	2055	2060
0	83.05	83.99	84.63	85.09	85.48	85.86	86.24	86.63	87.02	87.41	87.81
1–4	82.23	83.17	83.80	84.27	84.65	85.03	85.41	85.79	86.18	86.57	86.96
5–9	78.25	79.19	79.82	80.28	80.66	81.04	81.42	81.80	82.19	82.58	82.97
10–14	73.32	74.26	74.89	75.35	75.73	76.11	76.49	76.87	77.25	77.64	78.03
15–19	68.35	69.29	69.92	70.38	70.76	71.13	71.51	71.89	72.28	72.66	73.05
20–24	63.43	64.36	64.98	65.44	65.82	66.20	66.57	66.95	67.33	67.72	68.11
25–29	58.48	59.40	60.03	60.49	60.86	61.24	61.61	61.99	62.37	62.76	63.14
30–34	53.55	54.47	55.10	55.55	55.93	56.30	56.67	57.05	57.43	57.81	58.20
35–39	48.64	49.56	50.18	50.63	51.01	51.38	51.75	52.12	52.50	52.88	53.27
40–44	43.75	44.65	45.27	45.72	46.09	46.46	46.83	47.20	47.58	47.96	48.34
45–49	38.92	39.81	40.42	40.87	41.24	41.60	41.96	42.33	42.71	43.08	43.46
50–54	34.18	35.03	35.64	36.08	36.44	36.80	37.16	37.52	37.89	38.26	38.63
55–59	29.67	30.50	31.09	31.52	31.87	32.21	32.56	32.91	33.26	33.62	33.98
60–64	25.19	25.99	26.56	26.97	27.31	27.64	27.97	28.31	28.65	29.00	29.35
65–69	20.87	21.61	22.14	22.52	22.84	23.16	23.47	23.80	24.13	24.46	24.80
70–74	16.69	17.35	17.82	18.18	18.47	18.77	19.08	19.38	19.70	20.02	20.34
75–79	12.83	13.40	13.81	14.12	14.40	14.68	14.96	15.25	15.55	15.85	16.16
80–84	9.54	10.02	10.36	10.63	10.88	11.14	11.40	11.67	11.95	12.23	12.52
85+	7.03	7.44	7.73	7.96	8.20	8.44	8.70	8.95	9.22	9.49	9.77

Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia; authors' calculations

Figure 11-8: Annual rate of improvement in mortality [%] and mortality rate of standard population [%] in 2010–2060 projection period, decomposed by cause of death; Scenario 3



Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia; authors' calculations

Table 39: Life expectancy for males, by age; Scenario 3, mortality rates are calculated as a sum of assumed mortality rates by cause groups of death

	2010	2015	2020	2025	2030	2035	2040	2045	2050	2055	2060
0	76.33	77.79	78.95	79.98	80.94	81.90	82.90	83.94	85.01	86.13	87.29
1–4	75.52	76.95	78.10	79.12	80.07	81.03	82.02	83.05	84.11	85.22	86.38
5–9	71.58	73.01	74.15	75.17	76.11	77.07	78.06	79.08	80.15	81.26	82.41
10–14	66.59	68.01	69.16	70.18	71.12	72.08	73.07	74.09	75.15	76.26	77.42
15–19	61.64	63.06	64.21	65.22	66.16	67.11	68.10	69.12	70.18	71.29	72.44
20–24	56.83	58.23	59.37	60.37	61.30	62.25	63.23	64.24	65.30	66.39	67.54
25–29	52.05	53.44	54.56	55.55	56.47	57.41	58.38	59.38	60.43	61.52	62.66
30–34	47.23	48.60	49.71	50.70	51.61	52.54	53.50	54.50	55.54	56.62	57.76
35–39	42.40	43.76	44.86	45.84	46.74	47.66	48.62	49.61	50.64	51.72	52.86
40–44	37.65	38.98	40.07	41.03	41.93	42.84	43.79	44.77	45.79	46.87	47.99
45–49	33.03	34.33	35.41	36.35	37.23	38.13	39.06	40.03	41.04	42.10	43.21
50–54	28.65	29.90	30.95	31.87	32.72	33.60	34.51	35.45	36.44	37.48	38.58
55–59	24.45	25.65	26.66	27.56	28.38	29.22	30.10	31.02	31.98	33.00	34.07
60–64	20.62	21.76	22.72	23.57	24.35	25.15	25.99	26.87	27.80	28.78	29.82
65–69	16.93	17.98	18.86	19.65	20.38	21.14	21.94	22.78	23.68	24.63	25.64
70–74	13.34	14.19	14.95	15.66	16.35	17.07	17.84	18.65	19.52	20.44	21.42
75–79	10.33	11.01	11.64	12.28	12.92	13.60	14.32	15.09	15.92	16.81	17.76
80–84	7.70	8.21	8.74	9.30	9.90	10.54	11.23	11.97	12.76	13.62	14.54
85+	5.85	6.30	6.79	7.32	7.89	8.51	9.18	9.90	10.68	11.52	12.42

Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia; authors' calculations

Table 40: Life expectancy for females, by age; Scenario 3, mortality rates are calculated as a sum of assumed mortality rates by cause groups of death

	2010	2015	2020	2025	2030	2035	2040	2045	2050	2055	2060
0	83.07	84.17	85.13	86.05	86.97	87.93	88.93	89.99	91.09	92.27	93.51
1–4	82.25	83.35	84.29	85.21	86.12	87.07	88.06	89.10	90.21	91.37	92.61
5–9	78.27	79.36	80.31	81.22	82.13	83.08	84.07	85.12	86.22	87.38	88.61
10–14	73.34	74.43	75.38	76.29	77.19	78.14	79.12	80.17	81.26	82.42	83.66
15–19	68.37	69.46	70.40	71.31	72.22	73.16	74.14	75.18	76.28	77.44	78.67
20–24	63.45	64.53	65.47	66.37	67.27	68.21	69.19	70.23	71.32	72.48	73.71
25–29	58.49	59.57	60.51	61.41	62.31	63.24	64.22	65.26	66.35	67.51	68.73
30–34	53.57	54.64	55.57	56.47	57.36	58.30	59.27	60.31	61.40	62.55	63.77
35–39	48.66	49.72	50.65	51.54	52.43	53.36	54.34	55.36	56.45	57.60	58.82
40–44	43.77	44.81	45.74	46.62	47.51	48.43	49.41	50.43	51.51	52.66	53.88
45–49	38.94	39.97	40.88	41.76	42.64	43.56	44.52	45.54	46.62	47.76	48.97
50–54	34.19	35.19	36.09	36.96	37.82	38.73	39.69	40.70	41.77	42.90	44.11
55–59	29.69	30.66	31.53	32.38	33.23	34.11	35.05	36.04	37.10	38.22	39.41
60–64	25.21	26.14	26.99	27.82	28.64	29.51	30.43	31.41	32.44	33.55	34.73
65–69	20.88	21.76	22.56	23.36	24.16	25.01	25.91	26.86	27.88	28.97	30.13
70–74	16.70	17.49	18.24	19.00	19.78	20.60	21.48	22.41	23.41	24.48	25.63
75–79	12.84	13.54	14.23	14.94	15.69	16.48	17.33	18.25	19.22	20.27	21.39
80–84	9.56	10.16	10.78	11.45	12.16	12.93	13.75	14.63	15.59	16.61	17.71
85+	7.04	7.59	8.17	8.81	9.50	10.25	11.05	11.92	12.85	13.86	14.95

Source: Institute of Public Health of the Republic of Slovenia, Statistical Office of the Republic of Slovenia; authors' calculations