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ZALOŽNIŠTVO

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Inventory Models with Uncertain Supply

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University of Ljubljana
FACULTY OF ECONOMICS
Publishing

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Publisher: Faculty of Economics Ljubljana, Publishing Office
For the publisher: Metka Tekavčič, dean

Ljubljana, 2018

The book is available online:
http://www.ef.uni-lj.si/zaloznistvo/raziskovalne_publikacije

Kataložni zapis o publikaciji (CIP) pripravili v Narodni in univerzitetni knjižnici v Ljubljani

COBISS.SI-ID=295514368

ISBN 978-961-240-338-6 (pdf)

Contents

List of Figures

List of Tables

Chapter 1

Introduction

Inventory and hence inventory management, plays a central role in the operational behavior of a production system or a supply chain. Due to complexities of modern production processes and the extent of supply chains, inventory appears in different forms at each stage of the supply chain. The fact is that on average 34% of the current assets and 90% of the working capital of a typical company in the United States are invested in inventories (Simchi-Levi et al., 2007). Every company, a party in a supply chain, needs to control its inventory levels by applying some sort of inventory control mechanism. An appropriate selection of this mechanism may have a significant impact on the customer service level and company's inventory cost, as well as supply chain systemwide cost.

Research on inventory management is focused on providing decision making tools, which improve the performance of inventory systems. Unfortunately, devising a good inventory control mechanism is difficult, since modern companies operate in a highly complex business environment, which exhibits complexities both within the company's own processes and due to interactions between the various levels in the supply chain as a whole. Multiple models have been developed with a desire to capture the complex interactions between supply chain parties, dealing with an efficient way of managing information and material flows through the supply chain. However, while a lot of the research tries to describe these complexities on one side, it fails to capture the dynamics and uncertainties, which are an integral part of the modern business environment, on the other.

It is demand uncertainties that attracted the attention first. One of the main objective of the companies is to provide good service to its customers. But changes in the customer's demand may cause difficulties in guaranteeing timely and reliable service, particularly if these changes are uncertain. Production companies or the retailers that provide the product to the end-customers often face seasonal and stochastic demand. A very partial listing that is due to Metters (1998) includes weather related industries (e.g. pharmaceutical products, lawnmowers, canned foods), back-to-school industries (e.g. pencils, clothing), and holiday related industries (e.g. toys, wrapping paper). These companies have to cope with difficulties in managing their inventories in comparison with ones that produce to satisfy stable stationary demand. The consequence of the seasonality is that the production capacities that are likely to be quite stable, are not aligned well with the demand (Fair, 1989; Krane and Braun, 1991). Typically in high demand season the capacity is not sufficient, and the companies have to resort to building up inventory in the low demand season in expectation of high demand later. In such a case it is not sufficient to devise a inventory policy that only seeks to satisfy short-term demand, but the company has to look into future and make the appropriate capacity allocation decisions between the products, and additional decisions on when to start with the inventory build-up to prepare for the high demand season. By having to anticipate far into the future, companies are now also facing higher uncertainties that make it even harder to devise good planning mechanisms.

Although capacity is frequently assumed stable, this does not need to hold in general. The aggregate capacity decisions are long-term strategic decisions, and they lack the flexibility in changing the capacity levels quickly, which would enable a company to align its capacities to the changing demand. Looking at production capacity in short-term reveals that the capacity allocated to produce a certain product may vary a lot through time, primarily due to multiple products sharing the same capacity. This may give some additional flexibility in terms of allocating the capacity to the product that needs the most in a certain period. However, for the company to be successful in determining the appropriate product mix in each period, it would need to use a highly evolved production/inventory control system. As it turns out, this is not the case in practice. Bush and Cooper (1988) and Metters (1997) report that firms typically do not use any formal analytic approach and often carry excess inventory. Other sources of uncertainty in capacity levels are related to changes in workforce (e.g. holiday leaves), machine breakdown and repair, preventive maintenance etc. Variable uncertain capacity is a thing that also can be observed in the supply of products to the end-customers. The retailers may face supply restrictions imposed by their suppliers. This can be due to supplier delivering its products to multiple retailers, or due to overall product unavailability at certain times.

Above we have pointed out the difficulties related to variable demand and supply capacity that the companies are facing in managing their inventories. Also, when one looks in prescriptive texts for production scheduling, it has been acknowledged in the past that the simple rules used under stable demands and unlimited capacity perform poorly, but acceptable practical rules are not given. However, this is changing in the last two decades. Companies have been extensively adopting and using new planning concepts, like Material Requirements Planning (MRP) and Manufacturing Resource Planning (MRP-II). This was supplemented by introduction of new business practices (e.g. electronic data interchange or EDI, point-of-sales or POS, internet sales) and the ways the companies started to cooperate (e.g. collaborative planning forecasting and replenishment or CPFR, vendor managed inventory or VMI) (Lee and Padmanabhan, 1997; McCullen and Towill, 2002). Bollapragada and Rao (2006) point out that particularly through the implementation of Enterprise Resource Planning (ERP), which provides timely access to accurate data, a fresh look at decision support and advanced production planning systems is warranted. This view is not directed only towards accepting more complex planning algorithms, but even more importantly at providing the relevant information that would reduce the uncertainties and stimulate the collaboration between the supply chain parties. The need to share the relevant information through the supply chains has also been widely recognized by the practitioners. Already more then a decade ago, the computer industry has been extensively sharing the information both on the demand side, as well as on the supply side, as reported by Austin et al. (1998) (Figure 1.1). The largest extent of information sharing is due to sharing capacity information, however due to the sensitivity of information and the rudimentary state of the information technology at the time the capacity information was not shared electronically.

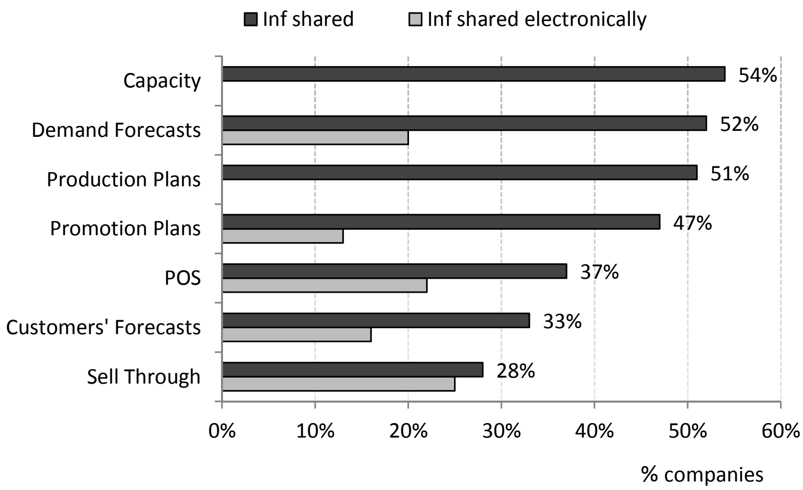


Figure 1.1 Extent of information sharing in supply chains.

Apart from sharing information, there are other possible ways to mitigate the undesirable effects of potential supply shortages, mainly related to production flexibility in the production setting, and multiple sourcing options and capacity reservations in the supply chain setting. By taking advantage of these options companies can alleviate the effect of supply shortages that are due to capacity constraints. The dynamic capacity investment or disinvestment problem has been investigated extensively, where we point out the works of Rocklin et al. (1984); Angelus and Porteus (2002); Gans and Zhou (2001) in establishing the optimal policies for managing capacity in a joint capacity and inventory management problem. These works were later extended by inclusion of other means of capacity flexibility, where Pinker and Larson (2003) and Tan and Alp (2005) discuss the option of hiring contingent labor upon the need to raise the capacity. This work is later complemented in Mincsovics et al. (2006), where they assume the lead time for capacity acquisitions. Ryan (2003) gives review of the literature on dynamic capacity expansions with lead times. Yang et al. (2005) introduce a model of a production/inventory system with uncertain capacity levels and the option of subcontracting. This extends the works concentrating on a single supplier setting to a dual or multiple supplier environment. Whittemore and Saunders (1977); Chiang and Gutierrez (1996, 1998); Tagaras and Vlachos (2001) all study a periodic review inventory system in which there are two modes of resupply, namely a regular mode and an emergency mode. Orders placed through the emergency channel have a shorter supply lead time but are subject to higher ordering costs compared to orders placed through the regular channel. In Vlachos and Tagaras (2001) they also impose capacity constraints. Minner (2003) presents a good overview of the multiple-supplier inventory models. Finally we point out the stream of modeling assuming the option of capacity reservations or supply quantity flexibility contracting. The capacity reservation is the company's ability to form an agreement with the supplier stating the extent of supply capacity that is to be reserved in advance and the associated costs (Tsay, 1999; Tsay and Lovejoy, 1999; Bassok and Anupindi, 1997; Jin and Wu, 2001; Cachon, 2004; Erkoc and Wu, 2005; Serel, 2007).

We feel that studying these models, complements the findings of our analysis in a sense that companies should explore all the available options to improve its inventory control. This can be either through improving the collaboration and coordination with their current suppliers or by establishing new alternative ways that give them additional flexibility in managing their inventories.

We have demonstrated above that companies are having troubles managing their inventories if they are working in a capacitated supply chain, where it is likely that the available supply may not cover the company's full needs in certain periods. Thus, our work is directed towards exploring two conceptual ways of improving the performance of inventory system facing uncertain supply capacity availability:

- ***Supply capacity information sharing***: sharing the information of the current or future supply capacity availability in a supply chain to improve the performance of the inventory management.
- ***Alternative supply options***: taking advantage of the alternative supply capabilities to mitigate the negative effect of supply shortages of the primary uncertain supply channel.

The remainder of this chapter is organized as follows. In Section 1.2, we describe the underlying stochastic capacitated inventory model, and proceed by giving the main model assumptions and parameters, and provide a short literature review of the research track leading to the formulation of the model. Then we give a short description of the four proposed inventory models that incorporate either capacity information sharing or alternative supply options in Section 1.3. We continue by formulating the relevant research objectives in Section 1.4, and give the outline of the rest of the monograph in Section 1.1.

1.1 Outline of the monograph

The remainder of this monograph is organized as follows. The monograph can be conceptually split into two parts, each consisting of two chapters. Chapters 3 and 4 constitute first part, where we study the role of sharing information on supply capacity availability on inventory management policies. In the second part we deal with an alternative replenishment option to improve the supply reliability: the dual sourcing setting in Chapter 5.

More specifically, in Chapter 3 we introduce the model with ACI on future supply capacity availability in detail and focus on establishing the optimal inventory policy and its properties. To quantify the value of ACI, we resort to numerical study, where we present the results for a broad selection of different scenarios, which enable us to establish the important managerial insights. We study the similarities between our ACI model and ADI models. For this purpose we formulate a special version of a capacitated ADI model. Parts of the contents of this chapter appeared in Jakšič and Rusjan (2009) and Jakšič et al. (2011).

In Chapter 4 we study a positive lead time variant of underlying stochastic capacitated inventory model, where advance information is available on the realization of the pipeline orders, denoted as ASI. In an addition to characterizing the optimal policy and its properties, we also study the behavior of the state-dependent myopic policy. In the numerical analysis we estimate the benefits obtained through sharing and integrating ASI into the inventory policy.

In Chapter 5 we study a dual sourcing inventory setting in which a slower reliable supplier is used to improve the reliability of sourcing through a faster stochastic capacitated supply supplier. Study of the optimal policy is complemented by the development of the near-optimal myopic policy, which proves to serve as a nearly perfect approximation for the optimal policy.

Finally, we summarize the main results of the research in Chapter 6.

1.2 Underlying stochastic capacitated inventory model

In this section, we describe the stochastic capacitated inventory model that was first introduced by Ciarallo et al. (1994). The model forms the base setting for the extensions proposed in this monograph, which are motivated in the Introduction and introduced in Section 1.3. We elaborate on the major model assumptions and introduce the relevant system parameters, and follow with a short literature review.

1.2.1 Underlying model description

We present the underlying stochastic capacitated inventory model in Figure 4.1. Below we list the major model assumptions and parameters:

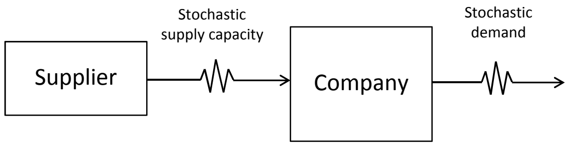


Figure 1.2 Schemes of the stochastic capacitated inventory model.

Single stage, single product: We focus on a single stage in a supply chain by considering an individual company or one stocking point, suggesting that the inventory is kept and reviewed at one location. This company can be either a retailer offering the products directly to the end-customers, or a manufacturing company producing the products on the production line. In both cases, we assume that demand from parties lower in a supply chain, and the supply availability of finished products or components needed in a production process, are exogenous to the company. This means, that we can not influence the demand and supply capacity through our actions, e.g. ordering decisions. We assume inventory control of a single item or product, which means that the product is treated independently of other products.

Discrete time, periodic review, finite horizon: We assume discrete time inventory control, and study a finite horizon inventory problem. We assume periodic review, where inventory is reviewed in regular fixed time intervals, and more importantly, the order decision is given at these prespecified points as well. Without the loss of generality, we set the length of the review period R to 1.

Lead time: The supply or replenishment lead time L imposes a time lag between the moment when order is placed with the supplier or manufacturing unit, and the time the products are actually received or produced. L is assumed to be nonnegative and constant, meaning that the delivery time is known with certainty. The entire order is delivered at the same time, where the order quantity may be restricted by the available supply capacity. The only restriction we impose is, that if L is longer than a review period, L needs to be a multiple of R.

Demand and supply capacity distribution, demand backorders and lost supply capacity: The demand and supply capacity are assumed to be nonnegative random variables with known probability distributions. We restrict ourselves to the case where both the demand and supply capacity distributions are independent over time periods, but may be non-stationary over time. Demand backordering case is assumed, meaning that any leftover inventory at the end of the period is retained and can be used to satisfy demand in the following period. In the base case, we assume that the unused supply capacity at the supplier is lost to the company.

Costs: We consider linear inventory holding cost and backorder costs, where per unit costs are assumed the same in all periods. We do not consider any fixed ordering costs. The objective of the inventory control policy is to minimize the discounted expected cost accumulating over a finite number of future periods. The costs incurred in future periods are discounted to their net present value.

1.2.2 Literature review

We proceed with the short literature review of the inventory models that share the basic modeling assumptions with the inventory models proposed in this monograph. We first briefly discuss the models with constant capacity and then focus on research track modeling the stochastic capacity.

In the context of our research, we are interested models that do not only recognize that the supply chain's demand side is facing uncertain market conditions, but also look at the risks of limited or even uncertain supply conditions. The researchers revisited the early stochastic demand models and extended them to incorporate the uncertainty on the supply side. Federgruen and Zipkin (1986a) first address the fixed capacity constraint for stationary inventory problems and prove the optimality of the modified base-stock policy. They have considered an infinite horizon case both under average and discounted cost criterion (Federgruen and Zipkin, 1986b). Tayur (1993) extends this work by developing an algorithm for the computation of the optimal policy parameters and cost. For a specific setting of stochastic seasonal demand and fixed production capacity Kapuscinski and Tayur (1998) and Aviv and Federgruen (1997) also show the optimality of a modified base-stock policy. Anticipation of future demand, due to its periodic nature, causes a corresponding increase or decrease in the base-stock level.

Later, Ciarallo et al. (1994) were the first to capture the uncertainty in supply capacity, by analyzing limited stochastic production capacity model. Particularly the inclusion of limited stochastic capacity is of interest to us, as it is closely related to the work presented in this monograph and we refer to it throughout the monograph. In their view, the random capacity assumption is appropriate for systems where there is uncertainty about which resources within the process will be available, resulting in limits on the ability to produce in any period. As such, stochastic capacity is a very general way of representing a variety of internal uncertainties. As we have argued before, this notion can be generalized even further by including the aspect of varying availability of supply, due to uncertainties in the supply capacity of the supplier.

They start their analysis by studying a single period problem, where they show that variable capacity does not affect the order policy. The myopic policy of newsvendor type is optimal. Meaning that the decision maker has no incentive to try to produce more than it is dictated by the demand and the costs, and simply has to hope that the capacity is sufficient to produce to the optimal amount. However, in a multiple period situation, one can respond to the possible capacity unavailability by building up inventories in advance.

Due to capacity uncertainty the cost function is somewhat complicated, having a quasi-convex form. The available capacity is likely to limit the actual production and as such effectively prevents high inventory accumulation. The consequence of capacity shortage is that the cost function levels out for high order sizes, making it nonconvex. In the finite horizon stationary case they show that the optimal policy remains to be a base-stock policy, where the optimal base-stock level is increased to account for the uncertainty in capacity. More precisely, the additional inventory, above the level needed to cope with uncertain demand, is there to cover the possible capacity shortfalls in future periods. They extend this work by introducing a notion of extended myopic policies, where they show that these policies are optimal if the decision maker considers appropriately defined review periods.

For the same setting, Iida (2002) obtains upper and lower bounds of the optimal base-stock levels, and shows that for an infinite horizon problem the upper and the lower bounds of the optimal base-stock levels for the finite horizon counterparts converge as the planning horizons considered get longer. This allows him to establish minimum planning horizons over which the solution of the finite horizon problem is close enough to the infinite horizon case. Thus, it is possible to obtain a policy sufficiently close to the optimal one by solving finite horizon problems in a rolling horizon manner. Khang and Fujiwara (2000) present sufficient conditions for the myopic policies to be optimal under stochastic supply, however, the demand is assumed deterministic. Güllü et al. (1997) model supply uncertainty in a different way, by introducing a notion of partial availability, meaning if an order is placed above the available supply capacity there is a positive probability that only the capacity restricted amount will be delivered. Again, it is shown that the optimal policy is a base-stock policy, but they were also able to develop a simple newsvendor like formula to compute the optimal base-stock levels under a more restrictive assumption of two-point stationary supply availability (supply is either fully available or completely fails).

To summarize, the models capturing the effects of limited capacity all show that carrying extra inventory is needed in comparison with the case of an uncapacitated supply chain. Such strategy would guarantee the optimal level of performance. This suggests that implementing the inventory policies proposed by the classical uncapacitated inventory theory in practical situations leads to a significant deviation from the optimal performance.

1.3 Stochastic capacitated inventory models under study

In this section we provide a description of the inventory models studied in this monograph. Proposed inventory models can be characterized as a variants of the stochastic capacitated inventory model introduced in Section 1.2. We represent the proposed inventory models in Figure 1.3.

Our focus is on a single party in a supply chain, denoted as a *company*. The role of a company is to serve the stochastic customer demand. This is done through replenishment of products by placing orders with the supplier. However, the supply availability may be limited due to stochastic capacity availability at the supplier. In the introduction we have formulated two conceptual approaches to tackle the uncertainty in supply capacity availability: supply capacity information sharing and alternative supply options.

In line with the first approach, companies should exploit the potential of sharing the relevant information about the supply conditions to reduce the supply uncertainty. Here, our contribution lies in developing models that would incorporate this information in the inventory policy that would capture these potential benefits. However, the company that wants to establish means of necessary information sharing with its supplier, has to find a way to communicate these benefits. We propose two different ways in which supply capacity information is communicated to the company by its supplier. The difference is due to whether the supply capacity information is revealed before the order is placed, or later, during the time the order is being processed at the supplier. In both cases the information is revealed before the order is replenished. Therefore we denote this information as *advance information*.

The capacity availability may vary substantially depending on the current inventory or/and production status at the supplier, and the future production plans. It is reasonable to assume that the supplier has an insight into the current inventory position, status of accepted orders, and the capacity availability for a number of future periods. This allows the supplier to identify empty production capacity slots, quote reliable lead times to new customers' orders, and thus improve the service to the customers. We denote the first way of sharing advance information as *advance capacity information* (ACI), as the supplier reveals the information about the future capacity availability to the company. The ordering process at the company can be optimize to avoid the supply shortages by accumulating inventories upfront in the periods with adequate capacity. The second way to share advance information, is that supplier's provides the information on the current order status after the decision maker in the company has already placed the order with the supplier. We denote this way of information sharing as sharing emphadvance supply information (ASI).

The second approach is targeted at increasing supply availability through exercising alternative supply options. We are looking for a way to effectively decrease the utilization of the

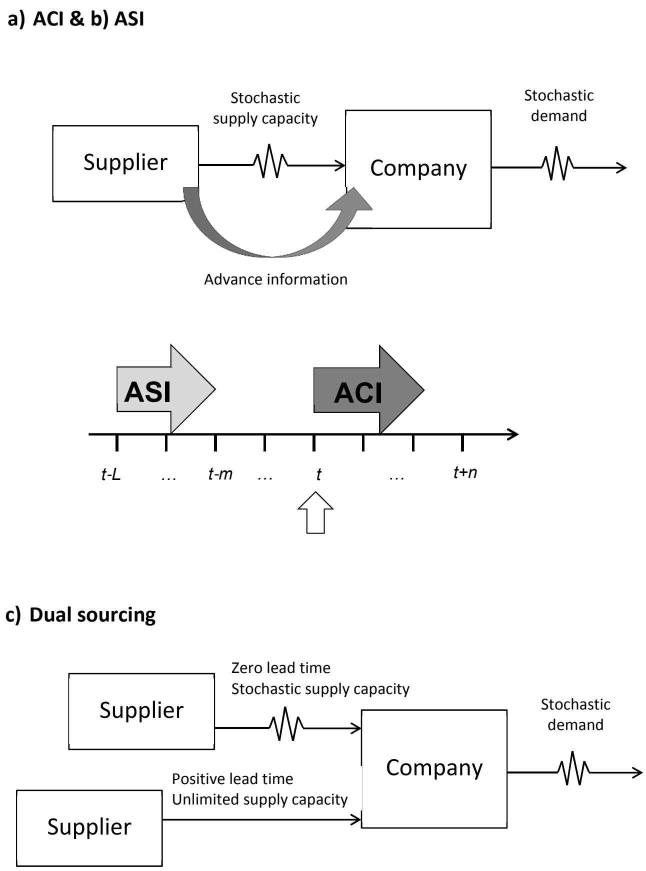


Figure 1.3 Schemes of the models under study.

primary stochastic capacitated supply source, and thus improving its supply availability. We proposse the option targeted at expanding the supply base through an alternative supplier in addition to a regular supplier with uncertain supply capacity availability. The alternative supplier is modeled as a fully reliable supplier, but its replenishment lead time is longer. We denote this option as *dual sourcing* option. Therefore, we seek to develop a dual sourcing inventory policy that would successfully split the order between the two supply sources, taking advantage of fast replenishment of the regular supply channel, and at the same time decreasing the possibility of the supply shortage by exercising a reliable replenishment option available at the slower supplier.

To summarize, we propose the following inventory models, where each of the models is analyzed in the succeeding chapters of this monograph:

- ***ACI model*** (Chapter 3): inventory model with *advance capacity information* on future supply capacity availability, limiting the orders placed in the near-future periods.
- ***ASI model*** (Chapter 4): inventory model with *advance supply information* on supply capacity available for replenishment of the orders already placed, but are currently still in the pipeline.
- ***Dual sourcing model*** (Chapter 5): inventory model where an alternative reliable yet slower supplier is used to cope with the supply capacity uncertainty of the primary supplier.

1.4 Research objectives

The main goal of this monograph is to develop quantitative inventory models that capture the stochastic nature of the demand and supply process, and possible ways of either reducing the uncertainty of supply or taking advantage of alternative supply options, in an integrated manner. The motivation behind this is twofold. Firstly, we would like to enrich the existing capacitated stochastic inventory research literature by developing the new models, and characterizing the resulting optimal or near-optimal inventory policies. Secondly, taking a practitioners' point of view, we aim to show the potential reduction of inventory costs in comparison to the underlying stochastic capacitated inventory system presented in 1.2, and provide the relevant managerial insights and decision policies that would allow the decision maker to achieve these benefits.

In accordance with the above, we formulate two groups of research questions that are relevant to all chapters of the monograph. The first group of research questions deals with the modeling perspective and the structural analysis of the inventory policies:

Q1. *How can we incorporate supply capacity information and alternative supply options into the underlying stochastic capacitated inventory model?*

Q2. *What is the structure of the optimal ordering policy and can we derive its properties?*

Addressing Q1 is relevant due to the fact that the models presented have received none or very limited attention in the inventory control literature. When exploring the inventory problems, the important objective is to characterize the control mechanism that guarantees the optimal behavior of the inventory system. We wish to answer Q2 by explicitly describing the control mechanism behind the optimal inventory control policy. Here, it is helpful if one can show that the optimal policy has a particular structure. Problems similar to the ones studied in this monograph have been known to have a structure of the base-stock policy, thus we wish to confirm if this also holds in our case. As the complexity of models under study is larger then the complexity of the underlying stochastic capacitated model, similar is expected for the optimal policy structure. The additional complexity is captured by a more comprehensive system's state description, which will affect the optimal policy parameters.

The motivation behind developing new inventory control policies lies in the potential benefits that can be achieved through their application. By answering the next group of questions, we wish to evaluate these benefits and provide the relevant managerial insights, which will serve as guidelines for better ordering decisions:

Q3. *What is the value of supply capacity information and alternative supply options in terms of inventory cost reduction?*

Q4. *Which are the determining factors of the magnitude of the expected cost benefits, or more specifically, what is the influence of the relevant system parameters?*

By answering Q3 we want to show that the proposed inventory policies lead to improved performance of the stochastic capacitated inventory system. However, the magnitude of the observed benefits may vary a lot depending on the particular system setting. Further elaboration of the influence of the system's parameters on the inventory cost reduction is needed to characterize these settings, which we address with Q4.

From the methodological perspective, the analysis of the structural properties of the proposed inventory models, is complemented by the numerical analysis to evaluate the extent and the influence of the system parameters on the benefits of information sharing and using alternative supply options.

Chapter 2

Preliminaries

In this chapter, we give some preliminaries on the concepts we will be exploring in further depth later in this monograph. The main objective of this chapter is twofold, we: (1)present a general way in which stochastic inventory models are developed and analyzed, and (2) give some of the fundamental results stemming from the analysis of both the uncapacitated and the capacitated inventory model. Throughout the chapter we remain within the model context presented in Chapter 1, that is single stage, single product, periodic review inventory model.

We first present a discrete-time model formulation of a general stochastic inventory model. The systems dynamics are described, identification of the control and state variables, and determination of the functional relationships, which describe the evolution of the state variables. We proceed by closer inspection of the relevant cost structure. A short description of the cost parameters is given, and based on those single period inventory costs are assessed. We then give a short review of a single period inventory problem in which we already capture the importance of finding an optimum balance between the relevant costs, namely inventory holding and backorder costs. The focus of this chapter is on derivation of a multi-period discounted expected optimal cost function by means of a dynamic programming technique. We first give a short review of the fundamental results of the uncapacitated inventory model. Showing the idea of the state space reduction, the optimality of the base stock policy and the near-optimal behavior of the myopic policy. This part relies strongly on Zipkin (2000) and Porteus (2002)[1], to whom we also refer in some of the proofs. We conclude this section by giving a review of the optimal policy characterization in the case of capacitated inventory problem, where we show the monotonicity results describing the dependency between the optimal base stock level and the capacity limit size.

2.1 Discrete-time model formulation

Throughout the monograph we deal with the notion of discrete time inventory control, where all important events occur at prespecified time points. We are concentrating on the stream of research assuming periodic review, which means that the inventory position is reviewed in

[1] Also, we advise the reader more interested into fundamentals of stochastic inventory control to review these two works.

every time period t, where $t = 0, 1, \ldots, T$. The planning horizon T may in general be either finite or infinite, where we limit ourselves to the finite horizon case, but also touch upon the known results from the infinite horizon case upon need. After review, the ordering decision z_t is made to raise the inventory position, which is then later used to satisfy the demand d_t. The demand is assumed to be a nonnegative random variable, where we resort to the case where d_t are independent over t. Due to the multi-period setting, it holds that any leftover inventory at the end of one period is retained and can be used to satisfy demand in the following period. This fact distinguishes the model from a simple single period newsvendor model. The consequence of the inherently stochastic nature of the demand process is that we ran out of stock in the situation where demand exceeds the available inventory on hand. In our model we assume backorders rather than lost sales, which means that any supply shortage carries over to the next period and remains to be satisfied.

In general, we assume that there is a certain positive supply lead time L needed for the order to be delivered. To make effective ordering decisions we must have insight into the available inventory. That is inventory already on hand or net inventory $\hat{x}_t$, and the pipeline inventory, which constitutes of orders already given but not yet received. Both together constitute an important state variable, inventory position before ordering x_t. Due to the supply lead time, each of the orders remains in the pipeline for L periods. Therefore we can express the inventory position before ordering x_t as the sum of net inventory and pipeline inventory.

$$x_t = \hat{x}_t + \sum_{s=t-L}^{t-1} z_s. \tag{2.1}$$

Note that in the special case of zero lead time, $L = 0$, there is no pipeline inventory as there are no outstanding orders. (2.1) is then simplified to $x_t = \hat{x}_t$, which means that the inventory position before ordering corresponds to the inventory on hand.

At the beginning of period t, the starting inventory position x_t is reviewed and correspondingly the ordering decision z_t is made to increase the inventory position to a desired level, by adding the order quantity to x_t, $y_t = x_t + z_t$. The order placed L periods ago is received, and used together with available net inventory to cover the period's demand d_t. Moving from period t to $t + 1$ the system dynamics is described by the following equation:

$$\begin{aligned} x_{t+1} &= x_t + z_t - d_t \\ &= y_t - d_t. \end{aligned} \tag{2.2}$$

The inventory position after ordering y_t is set at the level that is assumed sufficient to cover the lead time demand. That is demand, which is realized in the time interval $(t, t + L)$, and can be expressed as:

$$D_t^L = \sum_{s=t}^{t+L} D_s.$$

Based on y_t and a certain realization of lead time demand D_t^L, we can determine the end-of-period net inventory in period $t+L$, as the difference between the two:

$$\hat{x}_{t+L+1} = y_t - D_t^L. \tag{2.3}$$

This corresponds to period $t+L$ being the first future period we can affect by our ordering decision at time t, since the actual delivery of order z_t will occur L periods later in period $t+L$. End-of-period net inventory gives us the extent of excess inventory or backorders at the end of period $t+L$. Note above that the end-of-period net inventory in period $t+L$ is by definition equal to starting net inventory $\hat{x}_{t+L+1}$.

The potential excess inventory or backorders are already indicators of the appropriateness of ordering decisions. In an ideal situation we want to avoid either. While in a deterministic setting we could decide on future orders all at once due to the perfect information about future, this is not possible in a stochastic environment. It makes no sense to specify z_t early, since through time we obtain relevant demand information and the system evolves accordingly. It is better to react when additional information becomes available. Our next goal is therefore to determine the best ordering policy, given the economics of a situation. Rather than working with physical performance measures (e.g. inventory, backorder levels), we proceed by evaluating those by introducing the relevant costs associated with inventory, backorders, etc.

2.2 Cost parameters

In the majority of the backorder inventory models the following cost parameters are assumed[2]:

h	:	inventory holding cost per unit per period
b	:	backorder cost per unit per period
c	:	unit variable order/production cost
k	:	fixed order/production cost

The inventory holding cost h is charged per each unit of end-of-period inventory on hand after the current period's demand is satisfied. The inventory holding cost represents all the costs associated with the storage of the inventory until it is sold or used. Here, not only

[2]For a general review of the role and type of inventories, the inventory costs etc., look at any of Nahmias (1993); Chopra and Meindl (2001); Cachon (2004).

a mere financial holding cost (opportunity cost of capital) is important, but many other reasons that could make the inventory holding cost substantially higher: obsolescence of inventory, inventory might physically perish, inventory requires handling, storage space and other overhead cost (insurance, security, etc).

On the contrary, when there is too little inventory to cover the demand, backorder or penalty cost b is charged per each unit of backorder. This backorder cost may incorporate the lost opportunity value of the delayed revenue, the additional cost required to place an expedited order, and a more vague concept of lost goodwill, which is intended to account for any resulting reduction of future demands.

When ordering, we also incur variable and fixed ordering cost. When ordering or producing a product, a certain cost is incurred per unit of product. This variable cost can be denoted as the purchase price or a production cost. The fixed ordering cost is incurred if an order is placed, regardless of its size. The cost can be due to incremental time of the buyer placing the extra order, fixed part of the transportation or delivery cost, receiving costs like administration work related to updating the inventory records, and other. In a production setting, the equivalent of the order quantity is the batch size or a production run. The fixed cost associated with the production of a batch is due to setting up the equipment. It can include direct costs, the opportunity cost of time it takes to carry out the setup, and the implicit cost of initiating a production run because of learning and inefficiencies at the beginning of the run.

The above cost parameters need not to be stationary, their values can change through time, h_t, b_t, etc. Although we make no explicit assumption on cost stationarity throughout the monograph and keep the derivation of the inventory models general in this manner, we consider the cost parameters to be stationary in the numerical studies. Therefore, we suppress the subscript t in writing the cost parameters, from now on.

Note that, for the purpose of deriving the ordering policies, it is safe to ignore sales revenues and rather concentrate on minimizing the expected cost of running the inventory system (Silver and Peterson, 1985). This is possible since over a long run the total demand is set and also satisfied. The ordering decisions might affect the number of products sold in a certain period (and thus revenue) and change the sales pattern. But these effects can be captured through the cost itself. For instance, a delayed revenue is the consequence of incurring backorders, and the effect of this lateness can be captured through backorder cost itself.

Based on the proposed cost structure, we can develop a single period cost function, which allows us to determine the cost we incur in each period. The total cost associated with ordering at time t is $k\delta(z_t) + cz_t$, where we use $\delta(z_t)$ to denote a Heaviside function (1 when

$z_t > 0$, 0 otherwise). Meaning that the fixed part of ordering cost is only incurred in the case of positive order size. If no order is placed, the ordering cost is not incurred. In establishing inventory holding and backorder cost in period t, we again proceed with assumption of constant, positive lead time L. Thus, these costs are actually charged at the end of period $t + L$ when the order given at time t arrives and is eventually used to satisfy demand in a corresponding period. Based on (2.3), we write a single period holding and backorder cost function:

$$\hat{C}_{t+L}(\hat{x}_{t+L+1}) = h(y_t - D_t^L)^+ + b(D_t^L - y_t)^+. \tag{2.4}$$

We are now ready to evaluate the costs of running the inventory system over multiple time periods. Our focus lies primarily in establishing an optimal or at least near-optimal decision rule, which, when utilized will guarantee low costs. From here on we will only concentrate on inventory problems in which fixed ordering cost is not substantial ($k = 0$), and can therefore be neglected. In a periodic review setting this actually means that there is no incentive not to order, except if the current inventory position is already sufficiently high. In case of $k > 0$, the decision to order a small order might not be optimal due to fixed ordering costs associated with placing the order.

2.3 Optimizing cost in a single period problem

Before we proceed with optimizing a multi-period inventory problem, it is worth to look at a simpler variant of stochastic inventory problems - a single period problem. The problem applies in the analysis of items that perish quickly, thus, they cannot be stored in stock for future periods. Practical situations can be observed in setting the initial sizes for high fashion items, ordering policies for food products that become obsolete quickly, or determining run sizes for items with short useful lifetimes, such as newspapers (Silver et al., 1998). Because of this last application, the single-period stochastic inventory model has come to be known as the *newsvendor model*.

The importance of a single period problem analysis for our work is twofold:

- the newsvendor model is the simplest problem solved by a critical number, or a single base stock level, and
- the relationship of a single period problem analysis to the analysis of the myopic behavior of the multi-period optimal policy.

When the decision horizon $T = 1$ the decision maker is facing a single order decision at time 1. Note, that the notion of a single period corresponds to starting in period 1 and ending in the beginning of period 2. During this period a random demand occurs and the order arrives (when talking about a single period problem it is rational to assume a zero lead time, $L = 0$, without loss of generality). The system variables and the equations describing the system dynamics as given in (2.2), simplify to $x_1 = \hat{x}_1$, $y_1 = \hat{x}_1 + z_1$, where at the end of the period the relevant costs are incurred based on the net inventory $\hat{x}_2$:

$$\begin{aligned} \hat{x}_2 &= y_1 - d_1 \\ &= \hat{x}_1 + z_1 - d_1. \end{aligned} \tag{2.5}$$

To chose the order quantity, the decision maker aims to balance the order cost, inventory holding and backorder cost, plus some terminal cost. This terminal cost is called the *salvage value*. The salvage cost equals the order (purchase) cost of leftover inventory, which at the end of period 1 amounts to $-c_2\hat{x}_2$. This case arises when we can obtain reimbursement of the ordering cost for each leftover unit. This corresponds to selling the leftover stock at price equivalent to ordering cost.

To determine the optimal policy, that is the optimal order size, we write the optimization problem. We simplify the notation further by suppressing the subscripts in $x_1 = \hat{x}_1 = x$, $y_1 = y$, $d_1 = d$, and by considering stationary costs also $c = c_1 = c_2$. The inventory holding and backorder costs are assessed based on the simplified form of (2.4), where after taking the expectation over possible realizations of random demand D, we have $C(y) = E_D\hat{C}(y - D)$. The minimum expected cost function f^s, as a function of starting inventory x and inventory after order y, is expressed in the following form:

$$f^s(x, y) = \min_{y \geq x}\{c(y - x) + C(y) - \gamma c(y - E(D])\}. \tag{2.6}$$

The first term is the ordering cost, the second term represents the holding/backorder cost, and the last the salvage value. The fixed order cost charge is irrelevant here, since we have only one ordering opportunity that we have to make use of anyway. We solve (2.6) to find the optimal y, by first expressing the optimum cost $f^s(y) = f^s(x, y) + cx$ as a function of single decision variable y:

$$f^s(y) = \min_{y \geq 0}\{cy(1 - \gamma) + C(y) - \gamma cE[D]\}. \tag{2.7}$$

Lemma 2.1 *Let $\hat{y}^s$ be the smallest minimizer of function f^s. The following holds for all t:*

1. *The optimal ordering policy is a base stock policy with the optimal base stock level $\hat{y}^s$.*
2. *Under the optimal policy, the inventory position after ordering y is given by*

$$y = \begin{cases} x, & \hat{y}^s \leq x, \\ \hat{y}^s, & x < \hat{y}^s. \end{cases} \tag{2.8}$$

The ordering policy instructs to order only if the initial inventory x is below the target inventory level $\hat{y}^s$. Corresponding order size z is equivalent to the difference $\hat{y}^s - x$. If we start off with a high inventory level above $\hat{y}^s$, the order should not be placed. This is precisely a characterization of a *base stock policy* with a base stock level $\hat{y}^s$.

To compute $\hat{y}^s$, we must solve $(f^s)' = 0$. Taking the first derivative of $C(y)$ gives $C'(y) = (h+b)\Phi(y) - b$, where Φ represents a cumulative distribution function of demand. We have

$$(f^s)'(y) = c(1-\gamma) + (h+b)\Phi(y) - b,$$

and $\hat{y}^s$ is finally given by

$$\Phi(\hat{y}^s) = \frac{b - c(1-\gamma)}{h+b}. \tag{2.9}$$

Some additional assumption have to be made for (2.9). First, $b > (1-\gamma)c$ ensures that it is not optimal to not order anything and merely incur backorder cost. We also assume that $h + (1-\gamma)c > 0$, which holds in a realistic case of $h > 0$. This also guarantees that $(f^s)'$ is not strictly decreasing, which would lead to the optimal base stock level of $\hat{y}^s = \infty$.

2.4 Optimizing cost in a finite-horizon problem

In this section we present some results of the classical stochastic inventory theory based on Karlin and Scarf (1958); Karlin (1960a); Veinott (1963) that are highly relevant for our work. The idea is to derive the ordering policy, which would enable the decision maker to make optimal ordering decisions in a multi-period non-stationary stochastic demand environment. We continue from the finite horizon inventory model description given in Sections 2.1 and 2.2. At the end we give some insights into capacitated multi-period inventory problem, based on Federgruen and Zipkin (1986a).

Due to constant nonzero lead time the decision maker should protect the system against lead time demand, $D_t^L = \sum_{s=t}^{t+L} D_s$, which is demand realized in the time interval $(t, t+L)$. Since the current order z_t affects the net inventory at time $t+L$, and no later order does so, it makes sense to reassign the corresponding inventory holding and backorder cost to period t. The expected cost charged to period t is based on the net inventory at the end of the period $t+L$ and we can write it in the following form by using the definition of a single period cost function $\hat{C}_{t+L}(y_t)$ given in (2.4):

$$C_t(y_t) = \gamma^L E_{D_t^L} \hat{C}_{t+L}(y_t - D_t^L). \tag{2.10}$$

The reassigning of cost is done through discounting with a discount factor γ over a relevant lead time period. When thinking of cost that are accrued in the finite horizon setting (from

t onward to the end of horizon T), we see that the cost in time interval $[t, t+L)$ cannot be influenced. However, there are also cost that are incurred after T up to period $T+L$, through (2.10) these are a consequence of ordering decisions made in the last L periods before T. Thus, we seek to minimize the total discounted expected cost, which are the consequence of ordering decisions made in $[t, T)$.

We proceed by developing the *dynamic programming formulation*, which is a term describing the optimization of the performance of a multi-period stochastic system. The goal is to develop a recursive formulation of the minimum cost function as a function of the system variables. When thinking about the adequate representation of the state space at time t, we should include all the relevant information currently available. To assess the inventory cost through (2.10), we need to keep track of net inventory $\hat{x}_t$ and the information about outstanding orders. These are pipeline orders already given in past L periods, but are not yet delivered. We define the vector of outstanding orders, $\vec{z}_t = (z_{t-L}, z_{t-L+1}, \ldots, z_{t-2}, z_{t-1})$, given in periods $t-L, \ldots, t-1$. The state space gets updated when we move from t to $t+1$ in the following way:

$$\hat{x}_{t+1} = \hat{x}_t + z_{t-L} - d_t, \tag{2.11}$$

$$\vec{z}_{t+1} = (z_{t-L+1}, z_{t-L+2}, \ldots, z_{t-1}, z_t). \tag{2.12}$$

Observe that the order z_t is given based on the available information described by the state space variables $(\hat{x}_t, \vec{z}_t)$, then $\vec{z}_t$ is updated with z_t and z_{t-L} component is purged out due to order arrival.

Before writing the optimum cost formulation, we define the cost function representing all holding and backorder cost in time interval $[t+l-1, t+l)$:

$$\tilde{C}_t^l(y_t) = \gamma^l E_{D_t^l} \hat{C}_{t+s}(y_t - D_t^l), \tag{2.13}$$

where $D_t^l = \sum_{s=t}^{t+l} D_s$. The full holding and backorder cost in time interval $[t, t+L)$ can be expressed as a sum of (2.13) for $l = 0, \ldots, L$:

$$\tilde{C}_t(\hat{x}_t, \vec{z}_t) = \sum_{l=0}^{L} \tilde{C}_t^l(\hat{x}_t + \sum_{s=t-L}^{l+t-L-1} z_s).$$

To account for the cost in the remaining periods, we write the minimal discounted expected cost function, optimizing the total cost over a finite planning horizon T from time t onward and starting in the initial state $(\hat{x}_t, \vec{z}_t)$, as:

$$\tilde{f}_t(\hat{x}_t, z_t, \vec{z}_t) = \min_{z_t \geq 0} \{ c z_t + \tilde{C}_t^0(\hat{x}_t) + \gamma E_{D_t} \tilde{f}_{t+1}(\hat{x}_t + z_{t-L} - D_t, z_{t+1}, \vec{z}_{t+1}) \}, \text{ if } \; t \leq T+L, \tag{2.14}$$

where the remaining costs from T onward are given by:

$$\tilde{f}_{T+1}(\hat{x}_{T+1}, z_{T+1}, \vec{z}_{T+1}) = \tilde{C}_{T+1}(\hat{x}_{T+1}, \vec{z}_{T+1}). \tag{2.15}$$

At time $T+1$, there are no further orders to be placed, but costs continue to accrue until time $T+L$. Alternatively, we can set $\tilde{f}_{T+L}(\cdot) = 0$ and apply (2.14) for all $t < T+L+1$, which again leads to (2.15). This is possible since placing orders beyond T is not rational $z_{t>T} = 0$, doing so would raise the ordering cost due to $c \geq 0$. Although we do not consider ordering cost explicitly later in formulations of the proposed models, it is also reasonable to assume this when operating with holding and backorder cost only.

By solving (2.14), the optimal order quantities z_t can be computed. However, the presented dynamic program is a complex one due to the nontrivial state representation. It can be shown that an equivalent simpler version exists with a single state variable x_t, as defined in (2.1). The following Theorem is due to Zipkin (2000); Theorem 9.6.1.:

Theorem 2.1 *The following holds for all t:*

1. *The relationship between the original dynamic program given by* (2.14) *and the new formulation $f_t(x_t)$ is expressed as:*

$$\tilde{f}_t(\hat{x}_t, z_t, \vec{z}_t) = \tilde{C}_t(\hat{x}_t, \vec{z}_t) + f_t(x_t), \ \textit{if} \ \ 1 \leq t \leq T.$$

2. *The optimal order z_t in* (2.14) *is the same one that achieves the minimum in $f_t(x_t)$.*

The simplified dynamic programming formulation has the following form:

$$f_t(x_t) = \min_{y_t \geq x_t} \{c(y_t - x_t) + C_t(y_t) + \gamma E_{D_t} f_{t+1}(y_t - D_t)\}, \text{ if } \ 1 \leq t \leq T, \tag{2.16}$$

where $f_{T+1}(\cdot) = 0$. $f_t(x_t)$ represents the optimal cost under a revised accounting scheme, excluding the holding and backorder costs before time $t+L$.

Although the problem complexity is now significantly reduced, it still poses a great difficulty when trying to obtain the optimum order quantities. In general, x_t and y_t are continuous variables, and so an infinite number of expectations and infinite number of minimizations have to be taken in (2.16) for each t. To solve functional equations of this kind typically requires numerical approximation techniques. In our numerical calculations later in this monograph we discretize and truncate the state space. This means that through an appropriate selection x_t and y_t values are restricted to a finite set of values. This effectively reduces the number of possible system states. In a similar fashion the restrictions are imposed on possible demand d_t and supply capacity realizations z_t^+. Obviously, care has to be taken in the way we restrict the state space, not to omit the possible directions in which the system can evolve.

The solution, the best y_t for every x_t for each t, written out in a form of table, would be hard to understand and tedious to implement. Similarly as in single period problem (Lemma 2.1), the optimal policy has a characteristic structure. Again, it fits into the scope of base stock policies. This is proven through the following Theorem, which is due to Porteus (2002); Lemma 4.3 and Theorem 4.2.:

Theorem 2.2 *The following holds for all t:*

1. *The cost-to-go function g_t, defined as $g_t(y_t) = c(y_t) + C_t(y_t) + \gamma E_{D_t} f_{t+1}(y_t - D_t)$, is a convex function of y_t.*
2. *Optimal ordering policy is a base stock policy. The optimal base stock level $\hat{y}_t$ is the smallest value y_t minimizing $g_t(y_t)$.*
3. *$f_t(x_t)$ is a convex function of x_t.*

The structure of the optimal policy is intuitive in the sense that one only orders if the starting inventory position x_t does not exceed the target inventory level $\hat{y}_t$. Otherwise, it is not rational to place the order. However, knowing the optimal policy structure is not sufficient to determine the optimal base stock levels. We still must perform the recursive calculations using numerical techniques.

It is worth to compare the optimum cost formulation (2.16) with its single period equivalent given in (2.6). Consider $f^s(x, y)$ now as a function of x only, we see that $f_t(x_t)$ plays the same role as $f^s(x)$ in a single period problem. In fact we can show that the smallest value of y that minimizes somewhat transformed $C_t^+(y_t)$, based on $C_t(y_t)$ as defined in (2.10) (transformation is due to setting $f_t^+(x_t) = cx_t + f_t(x_t)$), exactly matches the solution of a single period problem $\hat{y}^s$ from (2.9). Thus, the myopic base stock level $\hat{y}_t^M = \hat{y}^s$. The only change in (2.9) is due to the lead time L, which we have assumed in a multi-period setting. This means that a single period cumulative demand distribution Φ is replaced by Φ_{t,D^L}, representing the cumulative distribution of lead time demand $D[t, t+L)$. Interestingly, the righthand side of (2.9) is not affected, as it does not depend on L:

$$\Phi_{t,D^L}(\hat{y}_t^M) = \frac{b - c(1-\gamma)}{h+b}. \tag{2.17}$$

The corresponding base stock policy minimizes only the current period cost while ignoring the future, so it is called *myopic policy*.

The myopic policy is obviously much simpler that the optimal one, since it only optimizes costs in period-per-period manner, ignoring the future consequences. However, both are closely related, which is beneficial in certain settings, where a myopic policy exhibits near-optimal behavior. This means that the decision maker would not be worse off using the myopic policy when deciding on order quantities, rather than the optimal one. The following Theorem due to Zipkin (2000); Theorem 9.4.2, gives some of the policies interrelations:

Theorem 2.3 *The following holds for all t:*

1. $\hat{y}_t \leq \hat{y}_t^M$.
2. *If* $\hat{y}_t^M \leq \hat{y}_{t+1}$, *then* $\hat{y}_t = \hat{y}_t^M$.
3. $\hat{y}_t \geq \min_{u \geq t} \hat{y}_u^M$.

The first part tells us that the optimal base stock level is always at or below the myopic level, never above (Veinott, 1963). This means that taking the future into account can only reduce the base stock level. Part 2 states that both levels are equal, unless, $\hat{y}_t^M > \hat{y}_{t+1}$. The only reason we might choose $\hat{y}_t < \hat{y}_t^M$ is to reduce the chance that $x_{t+1} > \hat{y}_{t+1}$ in the next period, and this concern is only relevant when $\hat{y}_{t+1}$ is small. By part 3, $\hat{y}_t$ is bounded below by the smallest subsequent myopic level. These results together imply that, if the $\hat{y}_t^M$ are nondecreasing in t, then $\hat{y}_t = \hat{y}_t^M$ for all t. In such a setting the myopic policy is optimal, which was first shown by Karlin (1960b).

Although the analysis of an infinite horizon problem is not within the scope of this monograph, it is appropriate to mention the following result, which further enhances the importance of studying the myopic behavior. The stationary base stock policy with a base stock level $\hat{y}_t$ is optimal, where $\hat{y}_t = \hat{y}_t^M$ for all t. This means the myopic policy is optimal. However, in an infinite horizon setting the assumption that not only cost parameters, but also the demands are stationary in time, is required. Due to this $\hat{y}_t$ remains constant in time, $\hat{y}_t = \hat{y}$, and the policy can be described simply as a *demand replacement rule.* Meaning that each new order just replaces the prior period's demand.

Finally, we give a comment on the cost transformation proposed by Veinott (1963, 1965), which allows us to exclude the presence of variable ordering cost in the functional equations presented above. Through the cost transformation we establish a new optimum cost formulation $f_t^*(x_t)$, derived from $f_t(x_t)$ defined in (2.16), by making the substitution $f_t^*(x_t) = cx_t + f_t(x_t)$. The variable ordering cost term $c(y_t - x_t)$ term vanishes. The cost transformation is based on the assumption that the decision maker is judged no differently whether he obtains inventory from a previous period, or by ordering it. We use the same principle in establishing the dynamic programming formulations in the case of the models presented in this monograph.

2.5 Finite horizon capacitated inventory problem

We now shortly review the capacitated version of the multi-period stochastic inventory problem just described. This review is primarily based on Federgruen and Zipkin (1986a,b) where they assume a fixed capacity level. We denote the capacity level as Q, where $(Q > 0)$. The capacity limit effectively restricts the order quantity. We have shown that for the uncapacitated case, the base stock policy is optimal. We know that the base stock policy instructs to order up to the base stock level in each period. However, if capacity is restricted, this might not be possible in the case the starting inventory position x_t is low. Thus, we expect a somewhat different structure of the optimal policy. It also remains to be shown, if this new policy has the properties of a base stock policy at all.

We rewrite the dynamic programming formulation for the uncapacitated inventory problem given in 2.16, modifying it to account for the capacity limit. The minimal discounted expected cost function $f_t(x_t, Q)$ is now a function of Q also:

$$f_t(x_t, Q) = \min_{x_t \le y_t \le x_t + Q} \{c(y_t - x_t) + C_t(y_t) + \gamma E_{D_t} f_{t+1}(y_t - D_t, Q)\}, \text{ if } 1 \le t \le T, \quad (2.18)$$

where $f_{T+1}(\cdot) = 0$. Observe the difference in y_t range over which minimization is done.

It can be again shown that the cost functions are convex and we are faced with a simple minimization problem with a single optimal solution, therefore the optimal policy is a modified base stock policy. The term *modified* is due to the fact that it is not always possible to attain the target base stock level, which distinguishes it from the base stock policy in the uncapacitated setting. Denoting the optimal base stock level $\hat{y}_t(Q)$, inventory position after ordering is given in the following form:

$$y_t(x_t, Q) = \begin{cases} x_t, & \hat{y}_t(Q) \le x, \\ \hat{y}_t(Q), & \hat{y}_t(Q) - Q \le x < \hat{y}_t(Q), \\ x_t + Q, & x < \hat{y}_t(Q) - Q. \end{cases} \quad (2.19)$$

Now we look at the dependency of $\hat{y}_t(Q)$ on Q. Intuitively, we would expect that, if Q is low, we would compensate by increasing the base stock level. The following Theorem, which is due to Porteus (2002); Lemma 8.5 and Theorem 8.4, shows that this is the case:

Theorem 2.4 *The following holds for all t:*

1. *The optimal ordering policy is a base stock policy with the base stock level $\hat{y}_t(Q)$, which is a decreasing function of Q.*
2. *$f_t(x_t, Q)$ is a convex function on $\{(x_t, Q)|x\}$.*

The proof in Porteus (2002) relies also on showing that f_t is also supermodular, a function property introduced by Topkis (1978)[3]. The optimal policy is expressing a monotone behavior, where the optimal base stock level is a decreasing function of the state - capacity Q.

[3]For more information on supermodularity/submodularity properties see Topkis (1998).

Chapter 3

Inventory management with advance capacity information

The importance of sharing information within modern supply chains has been established by both practitioners and researchers. Accurate and timely information helps firms effectively reduce the uncertainties of a volatile and uncertain business environment. The uncertainty of future supply can be reduced if a company is able to obtain advance capacity information (ACI) about future supply/production capacity availability from its supplier. In this chapter, we address a periodic-review inventory system under stochastic demand and stochastic limited supply, for which ACI is available. We show that the optimal ordering policy is a state-dependent base-stock policy characterized by a base-stock level that is a function of ACI. We establish a link with inventory models that use advance demand information (ADI) by developing a capacitated inventory system with ADI, and we show that equivalence can only be set under a very specific and restrictive assumption, implying that ADI insights will not necessarily hold in the ACI environment. Our numerical results reveal several managerial insights. In particular, we show that ACI is most beneficial when there is sufficient flexibility to react to anticipated demand and supply capacity mismatches. Further, most of the benefits can be achieved with only limited future visibility. We also show that the system parameters affecting the value of ACI interact in a complex way and therefore need to be considered in an integrated manner.

3.1 Introduction

Particularly in the last two decades, companies working in the global business environment have realized the critical importance of effectively managing the flow of materials across the supply chain. Industry experts estimate not only that total supply chain costs represent the majority of operating expenses of most organizations but also that, in some industries, these costs approach 75% of the total operating budget (Monczka et al., 2009). Inventory and hence inventory management play a central role in the operational behavior of a production system or supply chain. The fact is that the average cost of managing and holding inventory in the United States is 30% to 35% of its value, inventory represents about a third the current assets and up to 90% of the working capital of a typical company in the United

States is invested in inventories (Jacobs et al., 2008). Due to the complexities of modern production processes and the extent of global supply chains, inventory appears in different forms at each level of the supply chain. A supply chain member needs to control its inventory levels by applying some sort of inventory control mechanism. The appropriate selection of this mechanism may significantly impact on the customer service level and the members inventory cost, as well as supply chain system-wide costs.

Due to the focus on providing a quality service to the customer, it is no surprise that demand uncertainties attracted the initial attention. However, through time companies have realized that the effective management of supply is equally important. A look at the supply chains production and supply capacities allocated to produce or deliver a certain product reveals that these are generally far from stable over time. On the contrary, supply capacity may be highly variable for several reasons, like frequent changes in the product mix, particularly in a setting where multiple products share the same capacity, changes in the workforce (e.g. holiday leave), a machine breakdown and repair, preventive maintenance etc. To compensate for these uncertainties an extra inventory needs to be kept.

However, there is another, perhaps even more appealing way to tackle uncertainties in supply. Foreknowledge of future supply availability can help managers anticipate possible future supply shortages, while also allowing them to react in a timely manner by either building up stock to prevent future stockouts or reducing stock in the case of favorable supply conditions. Thus, system costs can be reduced by carrying less safety stock while still achieving the same level of performance. These benefits should encourage the parties in a supply chain to formalize their cooperation to enable the requisite information exchange by either implementing necessary information sharing concepts like Electronic Data Interchange (EDI) and Enterprise Resource Planning (ERP) or by using formal supply contracts. We could argue that extra information is always beneficial, but further thought has to be put into investigating in which situations the benefits of information exchange are substantial and when it is only marginally useful. Many companies nowadays have invested millions to improve inventory management through modern planning systems like ERP systems, which allow them to use one software package with a number of integrated modules, rather than multiple, conflicting systems with different operating platforms and data formats. While Bush and Cooper (1988) and Metters (1997) a decade ago attribute problems in determining the appropriate inventory levels to the fact that companies typically do not use any formal analytic approach, the problem persists as companies do not realize that ERP system is only a management tool; a framework that must be effectively applied and integrated to achieve success.

In this chapter, we study the benefits of obtaining advance capacity information (ACI) about future uncertain supply capacity. These benefits are assessed based on a comparison between the case where a manufacturer is able to obtain ACI from its supplier, and a base case without information. Our focus is on a manufacturer which raises its inventory position by placing orders with its supplier. The supplier could be a contract manufacturer to which the manufacturer has outsourced part of its production. As the contract manufacturer has limited capacity, it pre-allocates its capacity to manufacturers and notifies each manufacturer about the allocated capacity slot in advance. This practice is common in the semiconductor industry, where semiconductor foundries routinely share their capacity status with their buyers. Lee and Whang (2000) recognize that capacity information helps companies cope with volatile demand and can contribute substantially to mitigating potential shortage gaming behavior, thereby countering a potential source of the bullwhip effect to which suppliers are particularly prone. In the near future, the supplier can be certain of the capacity share that it can allocate to a particular customer. Similarly, short-term production plans tend to be fixed and uncertainty regarding the size of the available workforce is lower in the near future. Since the supplier has an insight into future capacity availability, it can communicate ACI to its customer (Figure 3.1). As a customer, the manufacturer now faces changing, albeit known, supply capacity availability in near future periods. By anticipating the possible upcoming capacity shortages it can behave proactively by inflating current orders and taking advantage of currently available surplus capacity. Unused capacity that was allocated to a particular manufacturer in a certain period is assumed to be lost for that manufacturer, rather than backlogged, as the supplier wishes to dampen the volatility of the manufacturer's demand. Thus, he chooses to offer its customers advance information rather than capacity flexibility to help them cope with the periods of low supply capacity availability. The supplier can use the remaining capacity to cover demand from other markets, less important customers etc. The focus of this chapter is on establishing the optimal inventory policy that would allow the manufacturer to improve its inventory control by utilizing the available ACI. In addition, we seek to identify the settings in which ACI is most valuable.

We proceed with a brief review of the relevant literature. Although uncapacitated problems form a foundation in the stochastic inventory control research field, we are interested in inventory models that simultaneously tackle the capacity that may limit the order size or the amount of products that can be produced. These models not only recognize that the supply chain's demand side is facing uncertain market conditions, but also look at the risks of limited or even uncertain supply conditions. Researchers are revisiting the early stochastic demand models and extending them to incorporate uncertainty on the supply side. A base-stock policy characterizes the optimal policy for several different capacitated problems. The sense of a base-stock policy differs in a resource constrained case from that in an uncapacitated case. In an uncapacitated case, the base-stock level has a clear interpretation - it is the inventory position to order/produce up to. Yet, in the capacitated case it merely represents a target that may or may not be achieved. If the capacity limit in a certain period is known, there is no use in ordering/producing above that level and we are thus talking about a

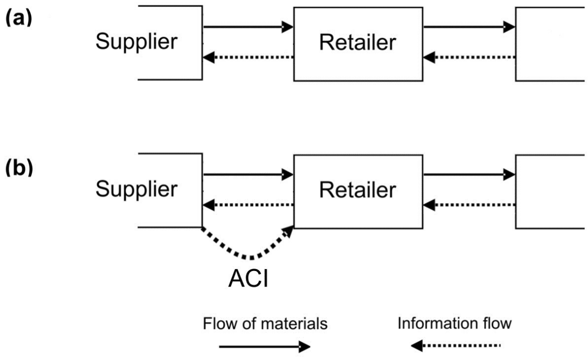


Figure 3.1 Supply chain (a) without and (b) with ACI sharing.

modified base-stock policy. Federgruen and Zipkin (1986a,b) first address the fixed capacity constraint for a stationary inventory problem and prove the optimality of the modified base-stock policy. This result is extended by Kapuscinski and Tayur (1998) for a non-stationary system assuming periodic demand, where it is also shown that a modified base-stock policy is optimal. Later, a line of research extends the focus to capture uncertainty in capacity by analyzing models with limited stochastic production capacity (Ciarallo et al., 1994; Güllü et al., 1997; Khang and Fujiwara, 2000; Iida, 2002). Here we point out the relevance of Ciarallo et al. (1994) to our work. For a finite horizon stationary inventory model they show that the optimal policy remains a base-stock policy, where the optimal base-stock level is increased to account for possible, albeit uncertain, capacity shortfalls in future periods. In the analysis of a single period problem, they show that stochastic capacity does not affect the order policy. The myopic policy of the newsvendor type is optimal, meaning that the decision maker is not better off by asking for a quantity higher than that of the uncapacitated case. Our model builds on these models by assuming both demand and capacity are non-stationary and stochastic, while additionally modeling that some of the future supply capacity is revealed through ACI. We propose that by reducing supply capacity uncertainty due to ACI we can refine the inventory policy and improve its performance. To our knowledge, the relationship between ACI and optimal policy parameters has not yet been explored in the literature[1].

The complexity of a capacitated stochastic non-stationary inventory formulation presents a challenge in terms of obtaining an analytical solution for the parameters of the optimal

[1]We would like to point out the new developments in this field since the work presented in this chapter has been published in Jakšič et al. (2011). We direct the reader to the papers by Altuğ and Muharremoğlu (2011), Çinar and Güllü (2012) and Atasoy et al. (2012)

policy, mainly optimal base-stock levels. Researchers have resorted to developing applicable heuristics (Bollapragada and Morton, 1999; Metters, 1997, 1998). In a capacitated non-stationary stochastic setting, Metters (1997) captures both the effect of deterministic anticipation (anticipating mismatches in demand and capacity, and reacting by building up the inventory), as well as the effect of uncertainty in demand. We note that there is a lack of literature concerning an approximate analysis of inventory systems that also assume uncertain capacity.

We propose that an effective way of circumventing the uncertainty of supply capacity is to use ACI. Advance information can also be obtained on the demand side and is dealt with by the well-established advance demand information (ADI) research stream where classical stochastic demand models are extended to incorporate ADI (Güllü, 1996; Gallego and Özer, 2001; Wijngaard, 2004; Tan et al., 2007). It is usually assumed that future uncertainty can be reduced due to some customers placing their orders in advance of their needs. This forms a stream of early demand that does not have to be satisfied immediately. Since this demand is revealed beforehand through ADI we can use ADI to make better ordering decisions. For a capacitated inventory model with ADI, Özer and Wei (2004) show that the base-stock policy is optimal, where the optimal base-stock level is an increasing function of the future demand levels revealed through ADI. In terms of modeling, our approach resembles the ADI modeling approach so our work also focuses on presenting possible similarities and relevant distinctions between the two.

Our contributions in this study can be summarized as follows: (1) We develop a periodic review inventory model with stochastic demand and limited stochastic supply capacity which enables a decision maker to improve the performance of the inventory control system through the use of ACI. (2) We demonstrate the structural properties of the optimal policy by showing the optimality of a modified base-stock policy with an ACI-dependent base-stock level. Our focus is on showing the dependency of the optimal base-stock level on ACI, which we elaborate on by establishing the monotonicity properties of the base-stock level in relation to ACI. (3) Our computational results provide useful managerial insights into the conditions in which ACI becomes most beneficial. In particular, we show that the biggest savings can be achieved when one is facing a high level of uncertainty in future supply and can effectively reduce this uncertainty by using ACI. However, we also emphasize that the benefits depend greatly on the successfulness of the anticipatory inventory build up, which may be limited by the size of the available capacity. In addition, we demonstrate how the value of ACI changes with respect to the length of the ACI horizon, cost parameters and demand uncertainty. (4) We develop a corresponding capacitated ADI inventory model and comment on its characteristics relative to the proposed ACI model. We show that only with the restrictive assumption of constant supply capacity that is always sufficient to cover the early orders recorded through ADI are the two models equivalent in their optimal base-stock level and optimal cost.

The remainder of the chapter is organized as follows. We present a model incorporating ACI and its dynamic cost formulation in Section 3.2. The optimal policy and its ACI-related properties are discussed in Section 3.3. In Section 3.4 we present the results of a numerical study and point out additional managerial insights. We study the setting with Bernoulli distributed supply capacity in Section 3.5. In Section 3.6 we look at the differences between the ADI modeling and the analysis of ACI model presented in this chapter. Finally, we summarize our findings and suggest directions for future research in Section 3.7.

3.2 Model formulation

In this section, we introduce the notation and our model. The model under consideration assumes periodic-review, non-stationary stochastic demand, limited non-stationary stochastic supply with a zero supply lead time, finite planning horizon inventory control system. The assumption of zero lead time is not a restrictive assumption as the model can be easily generalized to the positive supply lead time case. The supply capacity is assumed to be exogenous to the manufacturer and unused capacity in a certain period is assumed to be lost for this manufacturer, rather than backlogged. The manager is able to obtain ACI on the available supply capacity for orders to be placed in the future and use it to make better ordering decisions. We introduce a parameter n, which represents the length of the ACI horizon, that is, how far in advance the available supply capacity information is revealed. We assume ACI q_{t+n} is revealed at the start of period t for the supply capacity that limits the order z_{t+n}, that will be placed in period $t+n$. The model assumes perfect ACI, meaning that in period t the exact supply capacities for future n periods are known. The supply capacities in more distant periods, from $t+n+1$ towards the end of the planning horizon, remain uncertain (Figure 3.2). This means that, when placing the order z_t in period t, we know the capacity limit q_t and ordering above this limit is irrational.

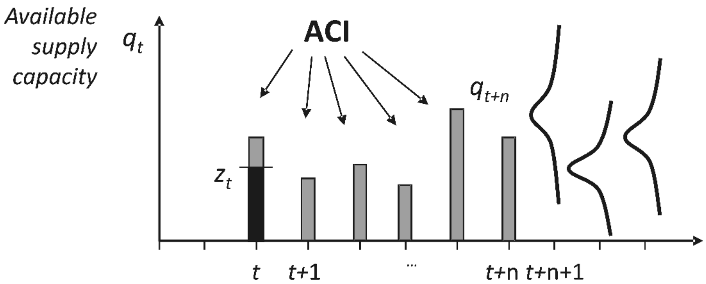


Figure 3.2 Advance capacity information.

Assuming that unmet demand is fully backlogged, the goal is to find an optimal policy that minimizes the relevant costs, that is inventory holding costs and backorder costs. We assume an inventory system with zero fixed costs. The model presented is general because no assumptions are made with regard to the nature of the demand and supply process. Both are assumed to be stochastic and with known independent distributions in each time period. The major notation is summarized in Table 3.1 and some is introduced when needed.

Table 3.1 Summary of the notation

T	:	number of periods in the finite planning horizon
n	:	advance capacity information, $n \geq 0$
h	:	inventory holding cost per unit per period
b	:	backorder cost per unit per period
α	:	discount factor ($0 \leq \alpha \leq 1$)
x_t	:	inventory position in period t before ordering
y_t	:	inventory position in period t after ordering
$\hat{x}_t$	:	starting net inventory in period t
z_t	:	order size in period t
D_t	:	random variable denoting the demand in period t
d_t	:	actual demand in period t
g_t	:	probability density function of demand in period t
G_t	:	cumulative distribution function of demand in period t
Q_t	:	random variable denoting the available supply capacity at time t
q_t	:	actual available supply capacity at time t, for which ACI is revealed at time $t-n$
r_t	:	probability density function of supply capacity in period t
R_t	:	cumulative distribution function of supply capacity in period t

We assume the following sequence of events. (1) At the start of period t, the decision maker reviews the current inventory position x_t and ACI on the supply capacity limit q_{t+n}, for order z_{t+n} that is to be given in period $t+n$, is revealed. (2) Ordering decision z_t is made based on the available supply capacity q_t, where $z_t \leq q_t$ and correspondingly the inventory position is raised to $y_t = x_t + z_t$. Unused supply capacity is lost. (3) The order placed at the start of period t is received. (4) At the end of the period previously backordered demand and demand d_t are observed and satisfied from on-hand inventory; unsatisfied demand is backordered. Inventory holding and backorder costs are incurred based on the end-of-period net inventory.

To determine the optimal cost, we not only need to keep track of x_t but also the supply capacity available for the current order q_t and the supply capacities available for future orders, which constitute ACI. At the start of period t, when the available supply capacity q_{t+n} is already revealed for period $t+n$, the vector of ACI consists of available supply capacities potentially limiting the size of orders in future n periods:

$$\vec{q}_t = \begin{cases} (q_{t+1}, q_{t+2}, \ldots, q_T), & \text{if } T-n \le t \le T, \\ (q_{t+1}, q_{t+2}, \ldots, q_{t+n-1}, q_{t+n}), & \text{if } 1 \le t \le T-n-1. \end{cases} \tag{3.1}$$

Information on the current supply capacity q_t obviously affects the cost, but we chose not to include it in the ACI vector since only ACI for future orders affects the structure and parameters of the optimal policy. All together, the state space is represented by an $n+2$ -dimensional vector $(x_t, q_t, \vec{q}_t)$, where x_t and $\vec{q}_t$ are updated at the start of period $t+1$ in the following manner:

$$x_{t+1} = x_t + z_t - d_t, \tag{3.2}$$

$$\vec{q}_{t+1} = \begin{cases} (q_{t+2}, q_{t+3}, \ldots, q_T), & \text{if } T-n-1 \le t \le T-1, \\ (q_{t+2}, q_{t+3}, \ldots, q_{t+n+1}), & \text{if } 1 \le t \le T-n-2. \end{cases} \tag{3.3}$$

Note also that the probability distributions of both demand and supply capacity affect the optimal cost and optimal policy parameters.

The minimal discounted expected cost function, optimizing the cost over finite planning horizon T from time t onward and starting in the initial state $(x_t, q_t, \vec{q}_t)$ can be written as:

$$\begin{aligned} f_t(x_t, q_t, \vec{q}_t) = \min_{x_t \le y_t \le x_t+q_t} \{C_t(y_t)+ \\ + \begin{cases} \alpha E_{D_t} f_{t+1}(y_t - D_t, q_{t+1}, \vec{q}_{t+1}), & \text{if } T-n \le t \le T, \\ \alpha E_{D_t, Q_{t+n+1}} f_{t+1}(y_t - D_t, q_{t+1}, \vec{q}_{t+1}), & \text{if } 1 \le t \le T-n-1, \end{cases} \end{aligned} \tag{3.4}$$

where $C_t(y_t) = h\int_0^{y_t}(y_t - d_t)g_t(d_t)\mathrm{d}d_t + b\int_{y_t}^{\infty}(d_t - y_t)g_t(d_t)\mathrm{d}d_t$ is the regular loss function, and the ending condition is defined as $f_{T+1}(\cdot) \equiv 0$.

3.3 Analysis of the optimal policy

In this section, we first characterize the optimal policy, as a solution of the dynamic programming formulation given in (3.4). We prove the optimality of a state-dependent modified base-stock policy and provide some properties of the optimal policy. See the Appendix A for proofs of the following theorems.

Let J_t denote the cost-to-go function of period t defined as

$$J_t(y_t, \vec{q}_t) = \begin{cases} C_t(y_t) + \alpha E_{D_t} f_{t+1}(y_t - D_t, q_{t+1}, \vec{q}_{t+1}), & \text{if } T-n \le t \le T, \\ C_t(y_t) + \alpha E_{D_t, Q_{t+n+1}} f_{t+1}(y_t - D_t, q_{t+1}, \vec{q}_{t+1}), & \text{if } 1 \le t \le T-n-1, \end{cases} \tag{3.5}$$

and we rewrite the minimal expected cost function f_t as

$$f_t(x_t, q_t, \vec{q_t}) = \min_{x_t \le y_t \le x_t + q_t} J_t(y_t, \vec{q_t}), \quad \text{for} \;\; 1 \le t \le T.$$

We first show the essential convexity results that allow us to establish the optimal policy. Note that the single-period cost function $C_t(y_t)$ is convex in y_t for any t since it is the usual newsvendor cost function (Porteus, 2002).

Lemma 3.1 *For any arbitrary value of information horizon n and value of the ACI vector $\vec{q}$, the following holds for all t:*

1. *$J_t(y_t, \vec{q_t})$ is convex in y,*
2. *$f_t(x_t, q_t, \vec{q_t})$ is convex in x.*

Based on the convexity results, minimizing J_t is a convex optimization problem for any arbitrary ACI horizon parameter n.

Theorem 3.1 *Let $\hat{y}_t(\vec{q_t})$ be the smallest minimizer of the function $J_t(y_t, \vec{q_t})$. For any $\vec{q_t}$, the following holds for all t:*

1. *The optimal ordering policy under ACI is a state-dependent modified base-stock policy with the optimal base-stock level $\hat{y}_t(\vec{q_t})$.*
2. *Under the optimal policy, the inventory position after ordering $y_t(x_t, q_t, \vec{q_t})$ is given by*

$$y_t(x_t, q_t, \vec{q_t}) = \begin{cases} x_t, & \hat{y}_t(\vec{q_t}) \le x_t, \\ \hat{y}_t(\vec{q_t}), & \hat{y}_t(\vec{q_t}) - q_t \le x_t < \hat{y}_t(\vec{q_t}), \\ x_t + q_t, & x_t < \hat{y}_t(\vec{q_t}) - q_t. \end{cases}$$

This modified base-stock policy is characterized by a state-dependent optimal base-stock level $\hat{y}_t(\vec{q_t})$, which determines the optimal level of the inventory position after ordering. The optimal base-stock level depends on the future supply availability, that is supply capacities $q_{t+1}, q_{t+2}, \ldots, q_{t+n}$, given by ACI vector. Optimal base-stock level $\hat{y}_t(\vec{q_t})$ can be characterized as an unconstrained base-stock level as it might not be reached due to the limited supply capacity. Observe also that $\hat{y}_t(\vec{q_t})$ is independent of the available supply capacity q_t in period t, which follows directly from Theorem 3.1 and the definition of J_t in (3.5).

The optimal policy can thus be interpreted in the following way. In the case that the inventory position at the beginning of the period exceeds the optimal base-stock level, the decision maker should not place an order. However, if the inventory position is lower he should raise the inventory position up to the base-stock level if there is enough supply capacity available; if not, he should take advantage of the full supply capacity available for the current order.

Our model assumes that also in the case of $n = 0$ the realization of supply capacity q_t, available for the current order, is known at the time the order is placed, while only future supply capacity availability remains uncertain. However, when ACI is not available (No-ACI) the decision maker encounters uncertain supply capacity for the order he is currently placing, as has been modeled by Ciarallo et al. (1994). Due to the zero lead time the updating of the inventory position happens before the current period demand needs to be satisfied. Therefore, it is intuitively clear that by knowing only the current period's supply capacity q_t one cannot come up with a better ordering decision, thus the optimal base-stock levels and performances should be the same for both models.

Theorem 3.2 *The ACI model given by* (3.4)*, in the case when* $n = 0$*, and the No-ACI model given by* (A.4)*, are equivalent with respect to the optimal base-stock level and the optimal discounted expected cost.*

In the following theorem we proceed with a characterization of the behavior of the base-stock level in relation to the actual capacity levels revealed through ACI. Intuitively, we expect that when we are facing a possible shortage in supply capacity in future periods we tend to increase the base-stock level. With this we stimulate the inventory build-up to avoid possible backorders which would be the probable consequence of a capacity shortage. Along the same lines of thought, the base-stock level is decreasing when higher supply availability is revealed by ACI. We confirm these intuitive results in Part 3 of Theorem 3.3 and illustrate the optimal ordering policy in Figure 3.3.

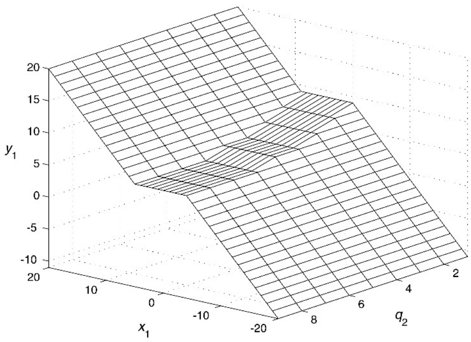


Figure 3.3 Illustration of the optimal ordering policy.

From this point we elect to suppress the subscript t in state variables for clarity reasons. We define the first derivative of functions $f_t(x, \cdot)$ and $J_t(x, \cdot)$ with respect to x as $f'_t(x, \cdot)$

and $J_t'(x,\cdot)$. Observe that for any two ACI vectors $\vec{q}_1$ and $\vec{q}_2$ in period t, $\vec{q}_2 \leq \vec{q}_1$ holds if and only if each element of $\vec{q}_1$ is greater than or equal to the corresponding element of $\vec{q}_2$. In Parts 1 and 2 of Theorem 3.3, we show that the rate of the change in costs is higher at higher capacity levels. Observe that J_t and f_t are also convex nonincreasing functions in $\vec{q}$ and therefore the costs are decreasing with higher future supply capacity availability. The rate of the cost decrease, and thereby the sensitivity of the optimal cost to an increase in capacity, is higher for low capacity and diminishes for higher capacities. Part 3 of Theorem 3.3 suggests that when $\vec{q}_2 \leq \vec{q}_1$ the decision maker has to raise the base-stock level $\hat{y}_t(\vec{q}_2)$ above the one that was optimal in the initial setting, $\hat{y}_t(\vec{q}_1)$.

Theorem 3.3 *For any $q_2 \leq q_1$ and $\vec{q}_2 \leq \vec{q}_1$, the following holds for all t:*

1. $J_t'(y,\vec{q}_2) \leq J_t'(y,\vec{q}_1)$ *for all y,*
2. $f_t'(x,q_2,\vec{q}_2) \leq f_t'(x,q_1,\vec{q}_1)$ *for all x,*
3. $\hat{y}_t(\vec{q}_2) \geq \hat{y}_t(\vec{q}_1)$.

We proceed by offering some additional insights into the monotonicity characteristics of the optimal policy. We continue to focus on how changes in ACI affect the optimal base-stock level. In the first case, we want to assess whether the base-stock level is affected more by a change in supply capacity availability in one of the imminent periods, or whether a change in the available capacity in distant periods is more significant. Let us define unit vector e_i with dimensions equal to the dimensionality of the ACI vector (n-dimensional), where its ith component is 1. With vector e_i we can target a particular component of the ACI vector. We want to establish how the optimal base-stock level is affected by taking away η units of supply capacity in period i from now, in comparison with doing the same thing but one period further in the future.

The results of a numerical study suggest that taking away a unit of supply capacity given by ACI in period i affects the optimal base-stock level more than taking away a unit of supply capacity which is available in later periods, $i+1$ and beyond. We provide the following simple example for the above by setting $n = 2$ and $i = 1$. If we expect the capacity shortage in period $t+1$ (system 1), we will increase $\hat{y}_t(q_{t+1} - \eta, q_{t+2})$ correspondingly to avoid future backorders (assuming $b > h$). However, if the shortage is expected to occur one period later in period $t+2$ (system 2), we will only increase $\hat{y}_t(q_{t+1}, q_{t+2} - \eta)$ if we anticipate that we will not be able to account for the shortage by increasing the optimal base-stock level in period $t+1$. Therefore there is no reason for system 2 to start off with more inventory than system 1. Careful observation also reveals that the same holds for a less interesting setting of $h > b$, even more, both systems will always have the same optimal base- stock level in this case. This suggests that the closer the capacity restriction is to the current period the more we need to take it into account when setting the appropriate base-stock level. We formalize this result in Conjecture 3.1.

Conjecture 3.1 *The following holds for all* t*:*

$\hat{y}_t(\vec{q} - \eta e_i) \geq \hat{y}_t(\vec{q} - \eta e_{i+1})$ *for i=t+1,...,t+n-1.*

Furthermore, we study the sensitivity of the optimal base-stock level to a change in the capacity limit. Assuming the result of Conjecture 3.1 holds, we show in Conjecture 3.2 that the change in the base-stock level should be lower than the change in the capacity limit in absolute terms (see Appendix A for the proof). In other words, each unit decrease in available supply capacity revealed by ACI leads to a lower or at most an equal increase in the optimal base-stock level.

Conjecture 3.2 *The following holds for all* t*:*

$\hat{y}_t(\vec{q} - \eta e_i) - \hat{y}_t(\vec{q}) \leq \eta$ *for i=t+1,...,t+n.*

A dampening effect is present and, together with the result of Conjecture 3.1, this implies that the exact ACI for a distant future period becomes less relevant to a current ordering decision. This is an important result, suggesting there is a diminishing benefit in overextending the ACI horizon n. This is desirable both in terms of having reliable ACI in a practical setting as well as reducing the complexity of determining the optimal parameters by operating with small values of n to avoid the curse of dimensionality.

3.4 Value of ACI

In this section we present the results of a numerical analysis which was carried out to quantify the value of ACI and to gain insights into how the value of ACI changes as some of the system parameters change. Numerical calculations were done by solving the dynamic programming formulation given in (3.4). We first introduce the value of ACI and then proceed by analyzing different settings in three subsections: We construct a set of experiments with different demand and capacity patterns. This enables us to describe the effect that a period-to-period mismatch between demand and capacity has on the inventory cost in Section 3.4.1. Based on a particular demand and capacity pattern we proceed with a more detailed analysis of the influence of the cost structure and the uncertainty of period-to-period demand and capacity in Section 3.4.2. Through an extensive simulation of random demand and capacity patterns we explore the effect of utilization more closely and generalize the results in Section 3.4.3.

To determine the value of ACI, the performance comparison between our ACI model and the case of a capacitated stochastic supply model with no ACI is of interest to us. It follows from the result of Theorem 3.2 that it is sufficient to compare the $n > 0$ setting with the $n = 0$ setting to determine the full value of ACI. We define the relative value of ACI for $n > 0$, $\%V_{ACI}$, as the relative difference between the optimal expected cost of managing the system where $n = 0$, and the system where we have an insight into future supply availability $n > 0$:

$$\%V_{ACI}(n > 0) = \frac{f_t^{(n=0)} - f_t^{(n>0)}}{f_t^{(n=0)}}. \tag{3.6}$$

We also define the marginal change in the value of ACI, $\triangle V_{ACI}$. With this we measure the extra benefit gained by extending the length of the ACI horizon by one time period, from n to $n+1$:

$$\triangle V_{ACI}(n+1) = f_t^{(n)} - f_t^{(n+1)}$$

3.4.1 Effect of demand and capacity mismatch

We proceed with constructing a set of six experiments (experiments number 1-6) with different demand and capacity patterns. The remaining parameters are set at the fixed value of: $T = 8, n = 1, \gamma = 0.99, h = 1, b = 20$, Normal demand and capacity both with a coefficient of variation (CV) of 0.1. We give a graphical illustration of demand and supply capacity patterns, by plotting the expected demand and expected supply capacity for each of the periods in Figures 3.4-3.6. We give the optimal base stock levels $\hat{y}_{(n=0)}$ for $n = 0$ setting. For comparison also the myopic base stock levels $\hat{y}^M$ are given. The myopic base stock level is defined as:

$$\Phi_{t,D^L}(\hat{y}_t^M) = \frac{b}{h+b}. \tag{3.7}$$

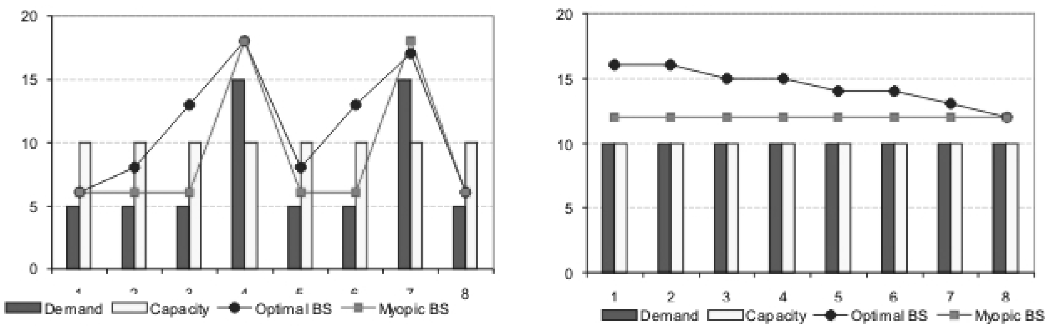
The myopic base stock levels indicate how high should the inventory position be to cover the additional risks due to demand uncertainty, and depending on the backorder and holding cost ratio. However, $\hat{y}^M$ completely neglects the effect of limited supply capacity. The optimal inventory costs are presented in Table 3.3, where also the value of ASI, $\%V_{ASI}$, and the absolute change in the value of ASI, $\triangle V_{ASI}$, is given.

Let us first observe the differences in inventory cost between the proposed settings. In general the inventory cost are higher if the supply capacity is highly utilized (experiment number 2, 3 and 4). However, although the average utilization[2] in all three cases is 100%, there are still considerable cost differences. These are mainly due to period-to-period mismatch between demand and supply capacity pattern. In experiment number 3 presented in Figure 3.5 (a),

[2]Defined as the sum of average demands over the sum of average supply capacities over the whole planning horizon.

which represents a kind of of the ”worst case” scenario, we are first faced with multiple periods of high demand and inadequate capacity. Therefore the extent of the backorders accumulated in these first periods is high, thus inventory cost are high. We see that the optimal policy instructs that we raise the base stock levels in the beginning periods. By doing this, we aim to use as much of the available capacity as possible. However, the probability to achieve these target base stock levels is minor, thus we cannot avoid the backorders. If we reverse this setting in experiment number 4 presented in Figure 3.5 (b), we can use the early excess capacity to build up the necessary inventory to cope with the subsequent capacity shortage. This in turn greatly reduces the cost. We gradually increase the base stock levels as we approach the over-utilized periods. After the peak demand periods, towards the end of the planning horizon, the base stock level drops to the myopic optimal level in the last period[3]. Observe also that the optimal base stock level in the first period is 12, which is above the myopic base stock level of 6. We know that in an uncapacitated setting $\hat{y}$ lies at or below $\hat{y}^M$. We obviously see that this is not the case in the capacitated system. That is why we see the optimal base stock levels increasing as we move towards capacitated periods. In the first four periods the available surplus in capacity (in cumulative terms) is not enough to build up enough inventory to cope with high demand in last periods. A high starting base stock level is a sign that the pre-build phase should commence earlier, and if so, additional savings could be achieved.

Figure 3.4 Expected demand and capacity pattern, and optimal base stock level $\hat{y}_{(n=0)}$ (a) Exp. 1 (b) Exp. 2.



We can further confirm the inventory build up insight by inspecting the experiment number 1 in Figure 3.4 (a), where we are faced with two demand peaks in periods 4 and 7 (Figure 3.4a). To avoid the probable backorders in the two critical and the following periods, the rational thing to do is to pre-build the inventory. Looking at the myopic base stock level and

[3]In period T, at the end of the planning horizon, the myopic and the optimal solution converge. Since this is the last period, we only need to optimize over a single period, which means that the myopic optimization is optimal.

Figure 3.5 Expected demand and capacity pattern, and optimal base stock level $\hat{y}_{(n=0)}$ (a) Exp. 3 (b) Exp. 4.

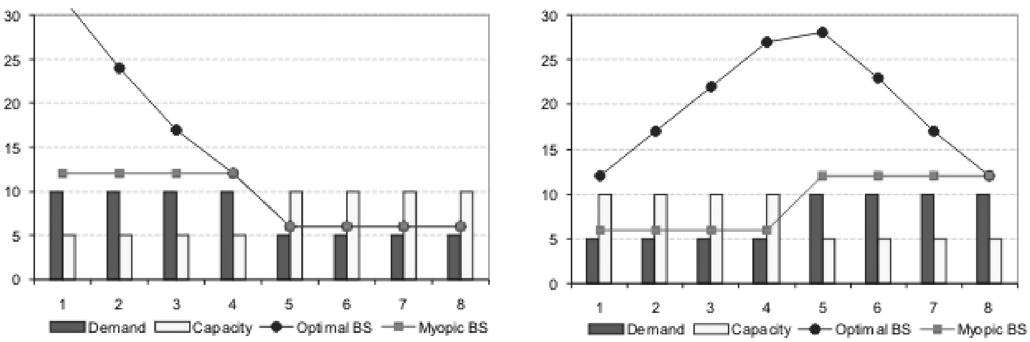
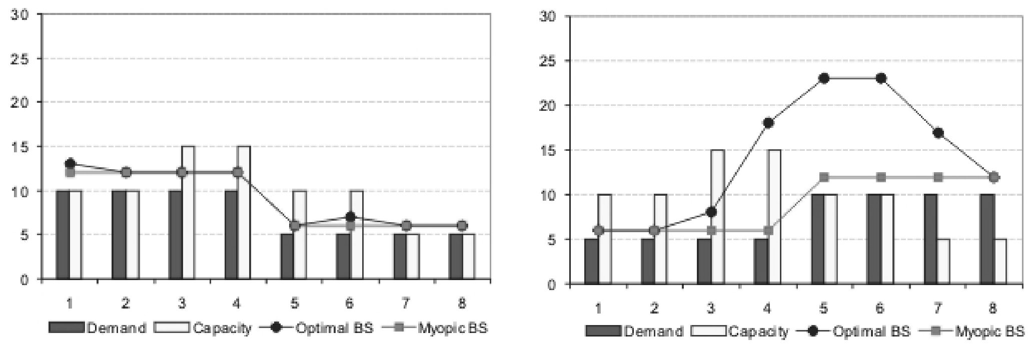


Figure 3.6 Expected demand and capacity pattern, and optimal base stock level $\hat{y}_{(n=0)}$ (a) Exp. 5 (b) Exp. 6.



the optimal base stock level in period 1, we see that they are equal. In period 2 the optimal base stock level increases above the myopic one in the anticipation of the capacity-demand mismatch in period 4. From this we can conclude that two time periods are sufficient, since the first build up phase starts in period 2. The same is true for the second pre-build phase, which immediately follows the peak period, starts in period 5, and build up to period 7.

For comparison with the well researched models of stationary demand and capacity, we include the experiment number 2 presented in Figure 3.4 (b), where the same logic of inventory build up can be noted. Observe that anticipation of future capacity shortages based on the expected values of demand and capacity patterns is not possible here. However, this is not the case if ASI is available. We see in Table 3.3 that the cost reduction can be achieved through the use of ASI, although not substantial in size. In the case of highly utilized capacity (in our case the average utilization is 100%) we are looking to make use of every unit of available capacity, however when we approach to the end of the planning horizon the tendency to do so diminishes. This confirms a well known result for the stationary uncapacitated inventory system that the base stock level is decreasing as we approach the end of the planning horizon (Scarf, 1960). This is often the *end of horizon effect*, where when we approach the planning horizon we care less and less about the future, thus we think more in short terms - myopically. The consequence of this is that the optimal and myopic base stock levels converge.

In Figures 3.6 (a) and (b), we give two additional examples where the effects we have described can also be observed.

Table 3.2 Optimal $\hat{y}_{(n=0)}$ and myopic $\hat{y}^M$ base stock level, optimal system cost, and the value of ASI

Exp.	t	1	2	3	4	5	6	7	8	n	Cost	$\%V_{ASI}$	$\triangle V_{ASI}$
1	$E[D_t]$	5	5	5	15	5	5	15	5	0	31.05		
	$E[Q_t]$	10	10	10	10	10	10	10	10	1	28.84	**7.12**	2.21
	$\hat{y}_{(n=0)}$	6	8	13	18	8	13	17	6	2	27.26	12.22	1.58
	$\hat{y}^M$	6	6	6	18	6	6	18	6	3	27.25	12.24	0.01
2	$E[D_t]$	10	10	10	10	10	10	10	10	0	189.54		
	$E[Q_t]$	10	10	10	10	10	10	10	10	1	189.25	0.15	0.28
	$\hat{y}_{(n=0)}$	16	16	15	15	14	14	13	12	2	189.14	0.21	0.11
	$\hat{y}^M$	12	12	12	12	12	12	12	12	3	189.10	0.23	0.05
3	$E[D_t]$	10	10	10	10	5	5	5	5	0	1579.28		
	$E[Q_t]$	5	5	5	5	10	10	10	10	1	1579.28	**0.00**	0.00
	$\hat{y}_{(n=0)}$	32	24	17	12	6	6	6	6	2	1579.28	0.00	0.00
	$\hat{y}^M$	12	12	12	12	6	6	6	6	3	1579.28	0.00	0.00
4	$E[D_t]$	5	5	5	5	10	10	10	10	0	103.87		
	$E[Q_t]$	10	10	10	10	5	5	5	5	1	103.58	0.28	0.30
	$\hat{y}_{(n=0)}$	12	17	22	27	28	23	17	12	2	103.32	0.53	0.26
	$\hat{y}^M$	6	6	6	6	12	12	12	12	3	103.10	0.74	0.22
5	$E[D_t]$	10	10	10	10	5	5	5	5	0	39.45		
	$E[Q_t]$	10	10	15	15	10	10	5	5	1	38.84	1.55	0.61
	$\hat{y}_{(n=0)}$	13	12	12	12	6	7	6	6	2	38.77	1.73	0.07
	$\hat{y}^M$	12	12	12	12	6	6	6	6	3	38.77	1.73	0.00
6	$E[D_t]$	5	5	5	5	10	10	10	10	0	53.98		
	$E[Q_t]$	10	10	15	15	10	10	5	5	1	52.36	3.01	1.63
	$\hat{y}_{(n=0)}$	6	6	8	18	23	23	17	12	2	51.24	5.07	1.11
	$\hat{y}^M$	6	6	6	6	12	12	12	12	3	50.58	6.30	0.67

We proceed by constructing a set of four experiments (experiments 7-10) with different demand and capacity patterns. The remaining parameters are set at fixed values of: $T = 8, \alpha = 0.99, h = 1$ and $b = 20$. A discrete uniform distribution is used for demand and capacity both with a coefficient of variation (CV) of 0.45. Through careful selection of the type of probability distribution and the parameters used, we were able to limit the number of possible space states, which allowed us to solve the underlying dynamic program. The optimal inventory costs and the optimal base-stock levels $\hat{y}_{(n=0)}$ are presented in Table 3.3, where the value of ACI, $\%V_{ACI}$, and the marginal change in the value of ACI, $\triangle V_{ACI}$, is also given.

The relevance of anticipating future capacity shortages is already well-established in a deterministic analysis of non-stationary capacitated inventory systems. To cope with periods of inadequate capacity, inventory has to be pre-built in advance by inflating orders. Also in the stochastic variant of this problem that we are considering, some anticipation is possible through knowing the probability distributions of demand and supply capacity in future periods. In this chapter we show that we can further improve this anticipative pre-building by using ACI and we can thereby attain additional cost savings.

Table 3.3 Optimal base-stock level $\hat{y}_{(n=0)}$, optimal system cost, and the value of ACI

Exp.	t	1	2	3	4	5	6	7	8	n	Cost	$\%V_{ACI}$	$\triangle V_{ACI}$
7	$E[D_t]$	6	6	6	6	6	6	6	6	0	1108.0		
	$E[Q_t]$	3	3	3	3	9	9	9	9	1	1107.6	0.04	0.43
	$\hat{y}_{(n=0)}$	25	21	17	14	13	13	11	10	2	1107.6	0.04	0.02
										3	1107.6	0.04	0.00
										4	1107.6	0.04	0.00
8	$E[D_t]$	6	6	6	6	6	6	6	6	0	279.1		
	$E[Q_t]$	9	9	9	9	3	3	3	3	1	278.3	0.28	0.78
	$\hat{y}_{(n=0)}$	22	24	25	26	22	18	15	10	2	277.6	0.53	0.70
										3	277.2	0.67	0.40
										4	277.0	0.74	0.20
9	$E[D_t]$	6	6	6	6	6	6	6	6	0	477.4		
	$E[Q_t]$	6	6	6	6	6	6	6	6	1	476.6	0.16	0.78
	$\hat{y}_{(n=0)}$	22	21	20	18	17	15	13	10	2	476.2	0.25	0.41
										3	476.0	0.28	0.15
										4	476.0	0.29	0.04
10	$E[D_t]$	3	3	3	12	3	3	12	3	0	78.8		
	$E[Q_t]$	9	9	9	9	9	9	9	9	1	70.6	10.37	8.16
	$\hat{y}_{(n=0)}$	7	12	17	20	11	16	19	5	2	66.7	15.36	3.93
										3	65.7	16.62	0.99
										4	65.4	17.00	0.30

We first consider three highly utilized settings (experiments 7, 8 and 9) with average utilization[4] of 100% and different expected demand/capacity patterns. Due to high utilization, shortages occur frequently and ACI is therefore needed for anticipation. However, in experiment 7, where in the starting periods we are faced with multiple periods of high demand and inadequate capacity, the over-utilized system prevents inventory pre-building the value of

[4]Defined as the sum of average demand over the sum of average supply capacity over the whole planning horizon.

ACI is almost zero. If we reverse this setting in experiment 8, the costs are greatly reduced and the value of ACI is higher due to having early excess capacity available to build up the necessary inventory to cope with the subsequent capacity shortages.

We see that the value of ACI depends strongly on whether there exists an opportunity to increase orders prior to a capacity shortage, which is affected both by the system's utilization and the nature of the demand/capacity pattern. The results of experiment 10 support this insight, where we study a system with lower average utilization (58%) and expected capacity shortages in periods 4 and 7. By having an insight into the next period's available capacity, we can lower the inventory cost by 10.37%, while an additional period of ACI data gives an additional 4.99% cost reduction. However, prolonging the ACI horizon further does not improve the performance greatly. Additional information on future supply conditions helps in making more effective ordering decisions as $\%V_{ACI}$ increases with the length of information horizon n, however the benefits are diminishing.

3.4.2 Effect of volatility

We proceed by investigating the influence demand uncertainty, supply capacity uncertainty, and cost structure on the value of ACI. The new base scenario is characterized by the following parameters: $T = 8, \alpha = 0.99$ and $h = 1$. A discrete uniform distribution is used to model demand and supply capacity where the expected demand is given as $E[D_{1..8}] = (3, 3, 3, 3, 3, 9, 3, 3)$ and the expected supply capacity as $E[Q_{1..8}] = (6, 6, 6, 6, 6, 6, 6, 6)$. We vary: (1) the cost structure by changing the backorder cost $b = \{5, 20, 100\}$; and (2) the coefficient of variation of demand $CV_D = \{0, 0.25, 0.45, 0.65\}$ and supply capacity $CV_Q = \{0, 0.25, 0.45, 0.65\}$, where the CVs do not change over time[5].

To observe the effect of the demand and the supply capacity uncertainty on the value of ACI, we first analyze the deterministic demand case presented in experiments 12, 13 and 14. Observe that the costs can be substantially decreased, even by up to almost 70% (experiment 12). Since there is no uncertainty in demand, we can attribute the decrease in costs solely to the fact that ACI resolves some of the remaining supply capacity volatility, which greatly increases the relative benefit of ACI. In particular, when CV_Q is low and the ACI horizon is long, it is likely that shortages are fully anticipated and the backorders can be fully avoided. When CV_Q increases, the absolute savings denoted by $\triangle V_{ACI}$ increase for all analyzed experiments. However, this is not the case with the relative savings denoted by $\%V_{ACI}$, which decrease for $b = 20$ and $b = 100$. Although ACI resolves some uncertainty in the system, the remaining uncertainty drives the costs up and the relative value of ACI

[5]Since it is not possible to come up with the exact same CVs for discrete uniform distributions with different means, we give the approximate average CVs for demand and supply capacity distributions with means $E[D_{1..8}]$ and $E[Q_{1..8}]$.

actually decreases.

When we introduce demand uncertainty into the system, we observe that when CV_D increases, both $\%V_{ACI}$ and $\triangle V_{ACI}$ are decreasing, looking at experiments 13, 15 and 16. The latter is intuitively clear since the benefits of a more precise alignment of the base-stock level, made possible by the revealed ACI, are greatly diminished because the volatile demand causes the inventory position to deviate from the planned level. In addition, due to carrying a lot of safety stock to prevent stockouts, the total costs increase, which in turn reduces the relative savings obtained through ACI. However, when both demand and supply uncertainty are increased, we observe the non-monotone behavior of $\%V_{ACI}$ (experiments 16, 18 and 19). We see that $\%V_{ACI}$ is increasing with an increased uncertainty in the system for low b/h ratio ($b = 5$), while the opposite is true for $b = 100$.

To study the effect of the cost structure on the value of ACI, we vary the b/h ratio. For all experiments in Table 3.4 we observe that $\triangle V_{ACI}$ is increasing with an increase in b/h. However, this is not the case with $\%V_{ACI}$, where the behavior is highly non-monotone, depending strongly on other system parameters. For low b/h ratios, we would anticipate low values of ACI, due to the fact that shortage costs are lower and thus there is less need to anticipate possible shortages. However, when there is higher capacity uncertainty in the system, relative savings are higher for low than for high b/h ratio, which can be observed in experiments 13, 14 and 19. We attribute this to the prevalent effect of high CV_Q as described above, where ACI is not able to resolve much of the capacity uncertainty, which in the case of high b/h ratio causes higher increase in total backorder costs and leads to correspondingly low value of ACI. The opposite can be noted for high b/h ratio, looking at experiments 12 and 18, where higher $\%V_{ACI}$ is attained under low capacity uncertainty. In the case of high b/h ratio, holding extra inventory is relatively inexpensive and one would assume that ACI is not needed. Although the probability of a shortage occurring is now very small, ACI allows us to more or less completely avoid costly backorders, thus relatively higher savings are possible particularly when b/h is high.

The observed combined effect of the analyzed system parameters, CV_D, CV_Q and b/h, can be attributed to the complex interaction between them, which clearly suggests that they need to be considered in an integrated manner.

Table 3.4 Optimal system cost and the value of ACI under varying ACI horizon n

			b	5	20	100	5	20	100	5	20	100
Exp.	CV_D	CV_Q	n	Cost			$\%V_{ACI}$			$\triangle V_{ACI}$		
11	0	0	all n	2.88	2.88	2.88	0.00	0.00	0.00	0.00	0.00	0.00
12	0	0.25	0	6.71	9.44	11.51						
			1	4.82	5.77	6.40	28.25	38.95	44.39	1.90	3.68	5.11
			2	3.95	4.15	4.23	41.13	56.08	63.27	0.86	1.62	2.18
			3	3.79	3.82	3.82	43.48	59.56	66.80	0.16	0.33	0.41
			4	3.79	3.79	3.79	43.62	59.90	67.11	0.01	0.03	0.04
13	0	0.45	0	16.10	28.88	59.22						
			1	10.90	20.21	48.08	32.32	30.03	18.81	5.20	8.67	11.14
			2	8.68	15.34	40.33	46.08	46.88	31.90	2.21	4.87	7.75
			3	7.78	12.93	36.39	51.70	55.21	38.56	0.91	2.41	3.94
			4	7.46	11.97	34.87	53.65	58.57	41.11	0.31	0.97	1.51
14	0	0.65	0	35.67	77.84	248.87						
			1	26.71	66.12	237.45	25.10	15.06	4.59	8.95	11.72	11.42
			2	22.71	58.62	227.39	36.34	24.69	8.63	4.01	7.50	10.06
			3	20.84	54.10	220.59	41.58	30.50	11.36	1.87	4.53	6.80
			4	20.05	51.84	217.10	43.80	33.41	12.77	0.79	2.26	3.49
15	0.25	0.45	0	23.23	41.28	86.60						
			1	20.43	35.19	78.13	12.07	14.76	9.78	2.81	6.09	8.47
			2	19.25	32.04	72.53	17.16	22.39	16.24	1.18	3.15	5.60
			3	18.82	30.51	69.77	19.00	26.08	19.43	0.43	1.52	2.76
			4	18.65	29.94	68.69	19.72	27.46	20.68	0.17	0.57	1.08
16	0.45	0.45	0	34.35	61.50	139.65						
			1	32.37	56.89	133.95	5.79	7.50	4.08	1.99	4.61	5.69
			2	31.44	54.33	130.49	8.49	11.66	6.55	0.93	2.55	3.46
			3	31.09	53.02	128.29	9.49	13.79	8.13	0.34	1.31	2.21
			4	30.98	52.50	127.31	9.82	14.63	8.83	0.11	0.52	0.97
17	0.65	0.45	0	47.58	87.57	224.55						
			1	45.85	84.22	221.12	3.63	3.82	1.53	1.73	3.35	3.43
			2	45.04	82.18	219.22	5.34	6.16	2.37	0.82	2.04	1.90
			3	44.73	81.05	217.90	5.98	7.44	2.96	0.30	1.13	1.32
			4	44.64	80.59	217.20	6.19	7.98	3.27	0.10	0.46	0.70
18	0.25	0.25	0	15.81	22.05	28.98						
			1	15.02	20.28	25.55	5.01	8.06	11.87	0.79	1.78	3.44
			2	14.69	19.20	23.38	7.10	12.96	19.34	0.33	1.08	2.17
			3	14.62	18.84	22.28	7.54	14.58	23.12	0.07	0.36	1.10
			4	14.61	18.78	21.96	7.58	14.86	24.22	0.01	0.06	0.32
19	0.65	0.65	0	62.49	135.97	455.85						
			1	58.52	131.15	450.80	6.35	3.55	1.11	3.97	4.83	5.06
			2	56.92	127.94	447.64	8.91	5.91	1.80	1.60	3.21	3.15
			3	56.27	126.15	445.45	9.96	7.22	2.28	0.65	1.79	2.20
			4	56.03	125.37	444.33	10.34	7.80	2.53	0.24	0.78	1.11

3.4.3 Effect of utilization

To generalize some of the above results and to explore the effect of the utilization on the value of ACI, we finally perform an extensive experiment by randomly generating non-stationary demand and capacity patterns with the following distribution parameters, the planning horizon $T = 8$, the set of expected demand values $E[D_t] = \{3, 6, 9, 12\}$ and the expected supply capacity values $E[Q_t] = \{3, 6, 9, 12\}$, for capacity uncertainty $CV_Q = 0.45$ and two demand uncertainty scenarios $CV_D = \{0, 0.45\}$. For each of the two experiments we perform, 10,000 demand/capacity patterns are generated. We keep the cost structure constant with $b = 20$ and $h = 1$, and determine the relative value of ACI, $\%V_{ACI}(n = 3)$, and absolute value of ACI, $f_t^{(n=0)} - f_t^{(n=3)}$, based on a comparison between the $n = 0$ and $n = 3$ settings. The results are presented in Figures 3.7 and 3.8.

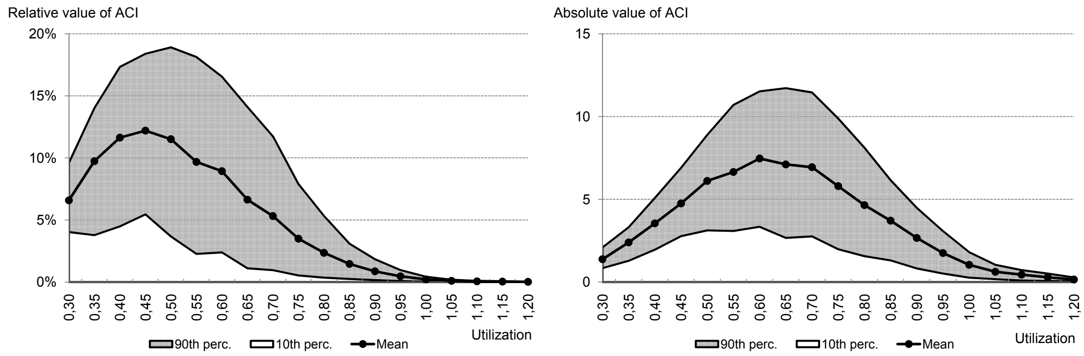


Figure 3.7 The relative and the absolute value of ACI for $CV_D = 0.45$ and $CV_Q = 0.45$

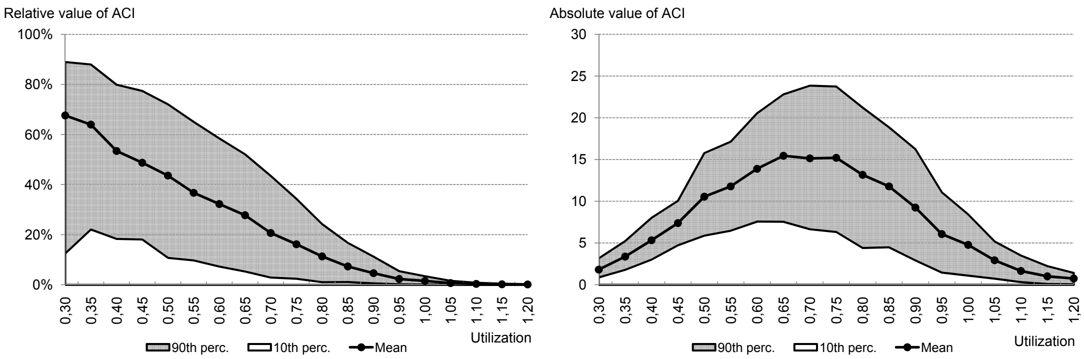


Figure 3.8 The relative and the absolute value of ACI for $CV_D = 0$ and $CV_Q = 0.45$

One of the biggest determinants of the extent of cost savings that can be achieved by using ACI is the average expected capacity utilization of the system through the planning horizon. Observe that both the relative value of ACI and the absolute cost savings vary considerably with a change in utilization. When both demand and capacity are uncertain, we see that the peak relative cost saving of around 12% is achieved when utilization is around 45% (Figure 3.7). While for high utilization there is no surplus capacity available to pre-build the inventory when a capacity shortfall is expected, the value of ACI is also limited for the system with low utilization as there is usually enough capacity available in every period and early anticipation is not required. For absolute cost savings the peak occurs at a higher 60% utilization, which we attribute to the fact that costs rise considerably with greater utilization. In the case of no demand uncertainty, we see in Figure 3.8 that the relative cost savings are substantial, achieving on average well above 60% in the case of low utilization. However, since the costs in such a setting are low, the absolute savings are small. Similar to the above, the peak absolute savings happen at 70% utilization. This is an important observation as the focus in practice is often solely on relative savings, while the perspective of absolute cost savings is unjustifiably neglected.

Finally, we can conclude that the results show considerable cost savings can be achieved through the use of ACI. Further, the uncertain capacitated settings for which we have recognized the greatest benefits are a good representation of the business environment in which many companies actually operate in practice.

3.5 "Zero-full" supply capacity availability

In this section, we model the supply capacity as a Bernoulli processes, meaning that there are randomly interchanging periods of complete capacity unavailability and full availability - denoted as "zero-full" supply capacity availability.

A supply setting described above can also be observed in practice. We give two examples in which a retailer would be facing periods of full or zero supply capacity availability and could take use of ACI if available. When a supplier/manufacturer is facing high setup costs in his production process, which are due to setting up the production process to meet particular customers/retailers needs, he would be reluctant to frequently change production program. A situation like this one can be observed in many capital-intensive industries, where the production resources are usually quite inflexible, resulting in high production switching costs (metal industry, food processing etc.). This results in a highly irregular supply to a particular retailer. As it would be too costly for a manufacturer to fulfill each separate order from a retailer directly from a production line, while at the same time the option of storing high inventories of products is also not possible for longer periods (particularly in food processing industry), a relatively low service level to the retailer is acceptable. However, to optimize his production costs, manufacturer already considers high setup costs in his master production scheduling by determining the production plan for several production periods in advance. Thus, he can communicate the information about the future supply availability to the retailer and while he can not lower the irregularity of supply, he can lower its uncertainty in a number of near-future periods. Advance information about future suppliers capacity availability enables a retailer to prepare for periods of supply unavailability in advance, and by doing so significantly decrease both the inventory holding costs (resulting from high inventory levels needed to cope with supply uncertainty) and the backorder costs (mainly a consequence of high supply variability). A similar practical setting is when the manufacturers transportation costs of supplying the products to the retailer are high and manufacturer resorts to less frequent supplies to the retailer.

We introduce the parameter p_t that describes the probability of full capacity availability in period t, $0 \leq p_t \leq 1$ for every t. To model ACI, we characterize each of the future periods for which ACI is already revealed with the state description a_t. State a_t attains the value of 0 for the zero availability case and 1 in the full availability case. In period t, the manager obtains ACI a_{t+n} on the supply capacity availability in period $t+n$, where n denotes the length of the ACI horizon. Thus, in period t the supply capacity availability for n future periods is known and is record in the ACI state vector $\vec{a}_t = (a_{t+1}, a_{t+2}, \ldots, a_{t+n})$. Note that the capacity availabilities in periods $t+n+1$ and later are still uncertain.

We present the results of the numerical analysis which was carried out to quantify the value of ACI and to gain insights into how the value of ACI changes with a change in the relevant system parameters. In Figure 1 we present an example of an end-customer demand and supply pattern that a retailer is facing. A weekly demand roughly varies between 0 and 100, with an average of 45 and a coefficient of variation of 0.6. The actual supply process is highly irregular with random periods of zero supply and a probability of full supply availability of $p = 0.6$. A gross packaging size from a manufacturer to the retailer contains 20 units.

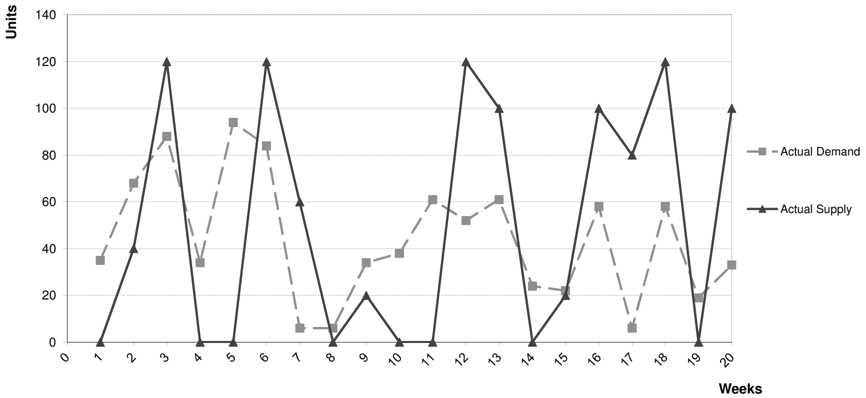


Figure 3.9 Example of end-customer demand and retailers supply pattern.

In the numerical experiment we selected the following set of input parameters: $T = 20$, $n = \{0, 1, 2, 3, 4, 5\}$, $p = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$ is assumed to be constant over the whole planning horizon, the cost structure $h = 1$ and $b = \{5, 20, 100\}$, discount factor $\alpha = 0.99$, uniformly distributed demand with the expected value of $E[D] = 5$ and coefficient of variation $CV_D = \{0, 0.3, 0.6\}$. The results are presented in Figures 3.10-3.11 and Tables 3.5-3.7.

Let us first observe the effect of changing the system parameters on the total cost. Obviously, decreasing the extent of supply capacity availability by decreasing the value of p will increase the costs. Due to the increased probability of multiple consecutive periods of zero capacity, the likelihood of backorders occurring will rise dramatically and the costs will grow particularly in the case of a high b/h cost ratio. In addition, costs rise when one has to deal with the effect of growing demand uncertainty. The demand uncertainty causes deviations in the actual inventory levels from the desired target levels set by the manager. This results in frequent mismatches between the demand and the available on-hand inventory and, consequently, high costs.

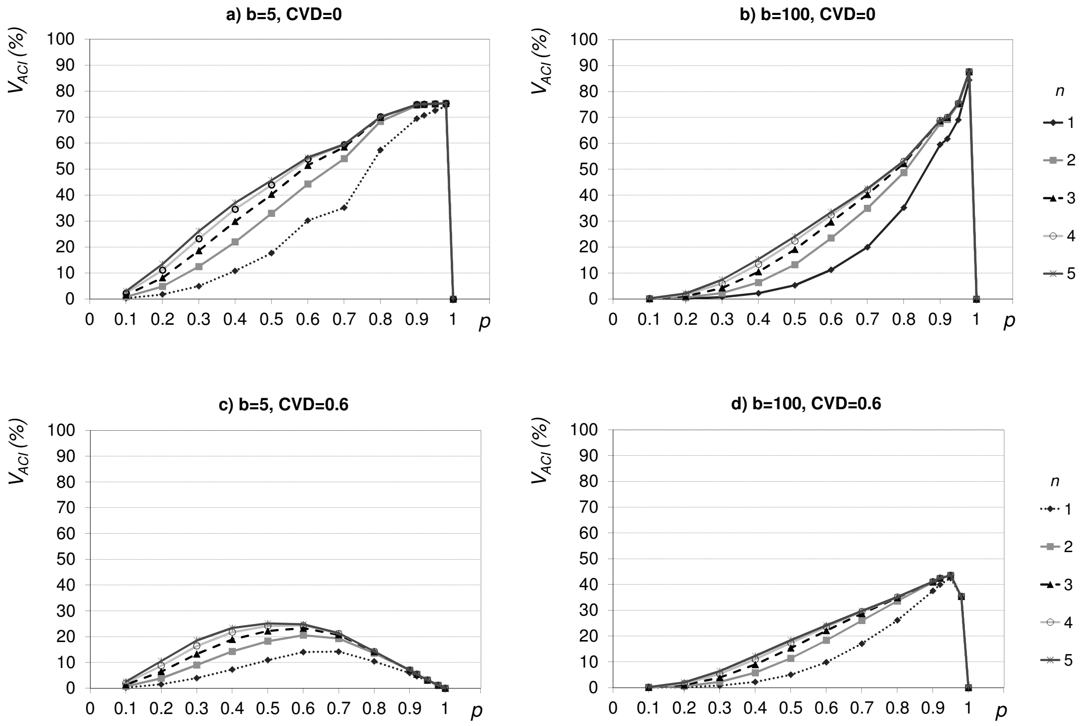


Figure 3.10 Relative value of ACI

These costs can be effectively reduced when ACI is available. We start by looking at the relative value of ACI as defined in (4.10). Extending the ACI horizon obviously also increases the extent of the cost savings. However, the marginal gain of increasing n by 1 period varies substantially depending on the particular setting. When we consider the case of low supply capacity unavailability (p close to 1), we observe a surprisingly high relative decrease in costs measured through $\%V_{ACI}$. This can be attributed to the fact that ACI enables us to anticipate and prepare for the rare periods of complete capacity unavailability. Thus, we can avoid backorders and at the same time lower the inventory levels that we would otherwise need to compensate for the event of multiple successive periods with zero capacity. Especially in the case of low demand uncertainty, and also a high b/h ratio, $\%V_{ACI}$ can reach levels above 80%, even close to 90% (Figure 3.10a and 3.10b, where we denote the settings with the biggest relative value of ACI by denoting the numbers in *Italic*). When the manager wants to gain the most from anticipating future supply capacity unavailability, it would be helpful if no additional uncertainties were present that would prevent him meeting the desired target inventory level. We also observe that these high relative cost savings are already gained with a short ACI horizon. Extending n above 1 only leads to small further cost reductions. This is an important insight regarding the practical use of ACI, when the majority of gains are already possible with limited future visibility it is more likely that the manager will be able to obtain ACI (possibly also more accurate information) from his supplier.

While the short ACI horizon is sufficient in the case of p being close to 1, we see that a longer ACI horizon is needed in a setting with high supply capacity unavailability. Observe that for low p values the relative marginal savings are actually increasing. When multiple periods of zero capacity can occur one after the other it is particularly important to anticipate the extent of future capacity unavailability. In such a setting, it is very important if one can have an additional period of future visibility. Several researchers who have studied a conceptually similar problem of sharing advance demand information suggest that prolonging the information horizon has diminishing returns (Özer and Wei, 2004). Although we consider a special case of zero or full supply availability in this section, this result actually shows that this does not hold in general.

While we have observed a large relative decrease in costs in some settings, it may be more important for a particular company to determine the potential decrease in absolute cost figures. Therefore we continue by studying the absolute marginal change in the value of ACI as defined in (4.5). Intuitively, we would expect that the biggest absolute gains would occur in a setting where the uncertainty of supply is high, and the possible shortage anticipation through ACI would be the most beneficial. We confirm this in Tables 3.5-3.7 (where we denote the settings with the biggest total absolute value of ACI by denoting the numbers in *Italic*; for each setting we also show at which n the majority of the savings are gained by denoting the marginal value of ACI in bold), where we see that the biggest absolute savings are attained for p between 0.2 and 0.4. Here, the cost decrease is bigger for a higher n (Figure 3.11). In fact, the lower the availability the more we gain by prolonging the ACI horizon. In the case of extremely low levels of capacity availability the system becomes too hard to manage due to an extremely long inventory pre-build phase, and the gains from using ACI

Table 3.5 Value of ACI for $b = 5$

							p					
		n	**1**	**0.9**	**0.8**	**0.7**	**0.6**	**0.5**	**0.4**	**0.3**	**0.2**	**0.1**
$CV_D = 0$	Cost	0	0.00	50.27	96.94	125.47	179.06	237.05	330.42	483.98	788.55	1605.02
		1	0.00	15.35	41.34	81.37	124.98	195.20	294.66	459.92	774.45	1600.15
		2	0.00	12.80	30.64	57.70	99.81	158.87	257.85	423.57	750.38	1591.42
		3	0.00	12.63	29.13	52.10	86.90	141.55	231.73	393.87	724.36	1580.07
		4	0.00	12.62	28.94	50.95	82.67	132.92	216.10	371.69	700.84	1568.04
		5	0.00	12.62	28.93	50.75	81.53	128.60	208.02	357.55	682.65	1556.62
	$\%V_{ACI}$	1	0.00	*69.47*	57.35	35.15	30.20	17.65	10.82	4.97	1.79	0.30
		2	0.00	*74.54*	68.39	54.01	44.26	32.98	21.96	12.48	4.84	0.85
		3	0.00	*74.87*	69.95	58.47	51.47	40.29	29.87	18.62	8.14	1.55
		4	0.00	*74.89*	70.14	59.39	53.83	43.93	34.60	23.20	11.12	2.30
		5	0.00	*74.89*	70.16	59.55	54.47	45.75	37.04	26.12	13.43	3.02
	$\triangle V_{ACI}$	1	0.00	**34.93**	**55.59**	**44.10**	**54.08**	**41.85**	35.76	*24.07*	14.10	4.87
		2	0.00	2.55	10.71	23.67	25.17	36.34	**36.81**	***36.35***	24.07	8.73
		3	0.00	0.17	1.51	5.60	12.91	17.31	26.12	*29.70*	**26.01**	11.35
		4	0.00	0.01	0.19	1.15	4.23	8.64	15.63	*22.18*	23.52	**12.03**
		5	0.00	0.00	0.02	0.20	1.14	4.31	8.08	*14.14*	18.19	11.43
$CV_D = 0.3$	Cost	0	36.42	66.32	106.95	143.63	188.38	250.56	341.67	493.68	797.66	1612.92
		1	36.42	51.05	72.54	103.78	148.55	212.64	310.01	470.80	784.39	1608.29
		2	36.42	49.49	66.80	91.44	128.42	185.71	278.23	439.62	762.32	1600.09
		3	36.42	49.34	65.69	87.86	120.28	171.24	257.05	413.51	738.37	1589.57
		4	36.42	49.32	65.47	86.80	117.10	164.06	243.97	394.21	717.10	1578.25
		5	36.42	49.32	65.43	86.50	115.89	160.60	236.26	380.68	699.76	1567.28
	$\%V_{ACI}$	1	0.00	23.02	*32.17*	27.74	21.14	15.13	9.27	4.64	1.66	0.29
		2	0.00	25.38	*37.54*	36.34	31.83	25.88	18.57	10.95	4.43	0.80
		3	0.00	25.61	38.58	*38.83*	36.15	31.66	24.77	16.24	7.43	1.45
		4	0.00	25.64	38.78	*39.56*	37.84	34.52	28.60	20.15	10.10	2.15
		5	0.00	25.64	38.82	*39.77*	38.48	35.90	30.85	22.89	12.27	2.83
	$\triangle V_{ACI}$	1	0.00	**15.27**	**34.41**	**39.85**	**39.83**	**37.92**	31.66	*22.89*	13.27	4.64
		2	0.00	1.56	5.74	12.34	20.13	26.93	**31.78**	***31.18***	22.07	8.20
		3	0.00	0.16	1.11	3.58	8.14	14.47	21.18	*26.11*	**23.95**	10.52
		4	0.00	0.01	0.22	1.05	3.18	7.18	13.08	*19.30*	21.27	**11.32**
		5	0.00	0.00	0.04	0.30	1.21	3.45	7.71	*13.53*	17.34	10.97
$CV_D = 0.6$	Cost	0	82.77	103.57	131.77	170.95	218.26	278.82	369.59	521.15	824.76	1638.21
		1	82.77	97.48	118.05	146.71	187.68	248.42	342.67	500.58	812.40	1633.80
		2	82.77	96.36	113.87	138.02	173.37	227.97	316.89	474.04	792.54	1626.11
		3	82.77	96.27	113.09	135.46	167.47	216.94	299.75	451.86	771.42	1616.46
		4	82.77	96.25	112.93	134.67	165.06	211.39	289.30	435.63	752.82	1606.21
		5	82.77	96.25	112.90	134.45	164.14	208.71	283.18	424.42	737.78	1596.43
	$\%V_{ACI}$	1	0.00	5.89	10.41	*14.18*	14.01	10.90	7.28	3.95	1.50	0.27
		2	0.00	6.96	13.58	19.27	*20.57*	18.24	14.26	9.04	3.91	0.74
		3	0.00	7.05	14.17	20.76	*23.27*	22.19	18.90	13.30	6.47	1.33
		4	0.00	7.07	14.29	21.22	*24.37*	24.18	21.73	16.41	8.72	1.95
		5	0.00	7.07	14.32	21.35	24.80	*25.14*	23.38	18.56	10.55	2.55
	$\triangle V_{ACI}$	1	0.00	**6.10**	**13.72**	**24.24**	**30.59**	**30.40**	**26.92**	*20.57*	12.35	4.41
		2	0.00	1.11	4.17	8.70	14.30	20.45	25.78	***26.54***	19.86	7.69
		3	0.00	0.10	0.78	2.56	5.90	11.04	17.14	*22.18*	**21.12**	9.66
		4	0.00	0.01	0.16	0.79	2.41	5.55	10.45	*16.23*	18.60	**10.25**
		5	0.00	0.00	0.03	0.22	0.92	2.67	6.11	*11.21*	15.05	9.78

Table 3.6 Value of ACI for $b = 20$

		n	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
							p					
$CV_D = 0$	Cost	0	0.00	108.32	179.28	252.26	358.94	508.71	751.09	1202.76	2258.01	5569.65
		1	0.00	36.62	86.10	169.13	277.04	442.65	704.27	1171.85	2241.12	5563.93
		2	0.00	23.29	60.49	119.59	215.45	371.59	630.89	1115.29	2208.71	5553.19
		3	0.00	21.99	54.15	103.61	186.30	324.44	572.82	1055.23	2166.73	5538.60
		4	0.00	21.87	52.65	98.54	172.62	299.17	532.90	1005.40	2123.84	5521.71
		5	0.00	21.86	52.32	96.96	167.01	286.37	508.52	968.87	2085.40	5504.13
	$\%V_{ACI}$	1	0.00	*66.20*	51.97	32.96	22.82	12.98	6.23	2.57	0.75	0.10
		2	0.00	*78.50*	66.26	52.59	39.98	26.95	16.00	7.27	2.18	0.30
		3	0.00	*79.70*	69.80	58.93	48.10	36.22	23.73	12.27	4.04	0.56
		4	0.00	*79.81*	70.63	60.94	51.91	41.19	29.05	16.41	5.94	0.86
		5	0.00	*79.82*	70.82	61.56	53.47	43.71	32.29	19.45	7.64	1.18
	$\triangle V_{ACI}$	1	0.00	**71.70**	**93.18**	**83.13**	**81.90**	66.06	*46.82*	30.91	16.90	5.72
		2	0.00	13.33	25.61	49.54	61.59	**71.07**	***73.38***	56.56	32.40	10.73
		3	0.00	1.30	6.35	15.98	29.15	47.15	*58.07*	**60.06**	41.99	14.59
		4	0.00	0.12	1.50	5.06	13.68	25.27	*39.92*	49.83	**42.89**	16.90
		5	0.00	0.01	0.33	1.58	5.61	12.80	*24.37*	36.53	38.43	**17.57**
$CV_D = 0.3$	Cost	0	36.42	135.85	197.09	275.72	378.34	527.83	778.85	1222.38	2279.71	5593.61
		1	36.42	68.81	120.85	195.27	304.22	466.15	733.12	1192.13	2263.06	5587.95
		2	36.42	63.45	102.20	161.33	254.05	403.82	669.31	1139.93	2232.48	5577.69
		3	36.42	62.96	98.60	150.50	232.04	367.95	621.02	1086.73	2193.87	5564.05
		4	36.42	62.91	97.90	147.27	222.98	349.05	590.23	1044.31	2155.37	5548.48
		5	36.42	62.91	97.77	146.31	219.35	339.39	571.58	1013.44	2121.60	5532.50
	$\%V_{ACI}$	1	0.00	*49.35*	38.68	29.18	19.59	11.69	5.87	2.47	0.73	0.10
		2	0.00	*53.29*	48.15	41.49	32.85	23.49	14.06	6.75	2.07	0.28
		3	0.00	*53.66*	49.97	45.41	38.67	30.29	20.26	11.10	3.77	0.53
		4	0.00	*53.69*	50.33	46.59	41.06	33.87	24.22	14.57	5.45	0.81
		5	0.00	*53.69*	50.39	46.93	42.02	35.70	26.61	17.09	6.94	1.09
	$\triangle V_{ACI}$	1	0.00	**67.04**	**76.24**	**80.45**	**74.12**	61.68	45.73	*30.25*	16.65	5.66
		2	0.00	5.36	18.65	33.93	50.17	**62.33**	**63.81**	*52.20*	30.58	10.25
		3	0.00	0.50	3.60	10.83	22.01	35.87	48.29	***53.20***	**38.61**	13.64
		4	0.00	0.04	0.70	3.23	9.06	18.90	30.79	*42.42*	38.50	15.57
		5	0.00	0.00	0.13	0.96	3.63	9.65	18.66	*30.87*	33.77	**15.98**
$CV_D = 0.6$	Cost	0	91.05	172.61	244.32	319.62	423.43	574.03	817.42	1272.77	2335.62	5652.68
		1	91.05	125.66	176.03	249.67	356.90	516.59	773.42	1243.09	2319.01	5646.91
		2	91.05	122.83	164.82	225.10	317.03	464.83	717.15	1195.14	2289.68	5636.73
		3	91.05	122.53	162.49	217.56	300.44	435.73	675.66	1148.12	2253.91	5623.40
		4	91.05	122.50	162.04	215.35	293.76	420.89	649.29	1111.34	2219.02	5608.53
		5	91.05	122.50	161.96	214.72	291.19	413.61	633.51	1085.03	2189.11	5593.55
	$\%V_{ACI}$	1	0.00	27.20	*27.95*	21.88	15.71	10.01	5.38	2.33	0.71	0.10
		2	0.00	28.84	*32.54*	29.57	25.13	19.02	12.27	6.10	1.97	0.28
		3	0.00	29.02	*33.49*	31.93	29.04	24.09	17.34	9.79	3.50	0.52
		4	0.00	29.03	*33.68*	32.62	30.62	26.68	20.57	12.68	4.99	0.78
		5	0.00	29.04	*33.71*	32.82	31.23	27.95	22.50	14.75	6.27	1.05
	$\triangle V_{ACI}$	1	0.00	**46.95**	**68.29**	**69.95**	**66.53**	**57.44**	44.01	*29.68*	16.61	5.77
		2	0.00	2.83	11.21	24.57	39.87	51.77	**56.27**	***47.95***	29.33	10.19
		3	0.00	0.31	2.33	7.53	16.58	29.10	41.49	*47.02*	**35.77**	13.33
		4	0.00	0.03	0.45	2.21	6.69	14.84	26.37	*36.78*	34.88	14.87
		5	0.00	0.00	0.09	0.63	2.56	7.27	15.78	*26.31*	29.91	**14.99**

Table 3.7 Value of ACI for $b = 100$

			p									
		n	**1**	**0.9**	**0.8**	**0.7**	**0.6**	**0.5**	**0.4**	**0.3**	**0.2**	**0.1**
$CV_D = 0$	Cost	0	0.00	228.71	378.08	593.46	917.66	1428.07	2357.22	4301.25	9380.07	26309.55
		1	0.00	92.58	244.86	474.87	814.32	1352.24	2305.11	4268.18	9362.27	26303.63
		2	0.00	73.78	193.67	386.03	701.78	1239.56	2207.25	4201.86	9327.24	26292.43
		3	0.00	71.44	180.58	354.33	645.25	1155.18	2109.67	4118.46	9278.93	26277.00
		4	0.00	71.17	177.43	343.94	620.31	1108.20	2040.57	4041.37	9225.11	26258.77
		5	0.00	71.14	176.79	340.62	609.62	1083.50	1995.91	3980.49	9173.66	26239.36
	$\%V_{ACI}$	1	0.00	*59.52*	35.24	19.98	11.26	5.31	2.21	0.77	0.19	0.02
		2	0.00	*67.74*	48.77	34.95	23.52	13.20	6.36	2.31	0.56	0.07
		3	0.00	*68.76*	52.24	40.29	29.69	19.11	10.50	4.25	1.08	0.12
		4	0.00	*68.88*	53.07	42.04	32.40	22.40	13.43	6.04	1.65	0.19
		5	0.00	*68.90*	53.24	42.61	33.57	24.13	15.33	7.46	2.20	0.27
	$\triangle V_{ACI}$	1	0.00	**136.14**	**133.22**	**118.59**	103.34	75.83	*52.11*	33.07	17.79	5.93
		2	0.00	18.80	51.19	88.84	**112.54**	**112.68**	***97.86***	66.33	35.04	11.20
		3	0.00	2.33	13.09	31.70	56.53	84.38	*97.58*	**83.40**	48.31	15.43
		4	0.00	0.27	3.15	10.39	24.93	46.98	*69.10*	77.08	**53.81**	18.23
		5	0.00	0.03	0.64	3.33	10.70	24.71	*44.66*	60.88	51.46	**19.40**
$CV_D = 0.3$	Cost	0	36.42	242.76	405.88	623.04	943.58	1461.26	2392.68	4343.79	9435.50	26375.73
		1	36.42	131.39	279.88	503.45	843.04	1385.07	2339.59	4309.98	9417.12	26369.44
		2	36.42	116.14	238.30	431.75	746.22	1278.49	2245.67	4244.88	9382.14	26357.98
		3	36.42	114.73	229.71	408.36	700.99	1208.93	2159.98	4167.18	9335.43	26342.61
		4	36.42	114.59	227.95	400.95	681.75	1171.12	2100.90	4098.11	9285.20	26324.81
		5	36.42	114.58	227.61	398.74	673.90	1151.68	2063.92	4045.61	9238.70	26306.24
	$\%V_{ACI}$	1	0.00	*45.88*	31.04	19.20	10.66	5.21	2.22	0.78	0.19	0.02
		2	0.00	*52.16*	41.29	30.70	20.92	12.51	6.14	2.28	0.57	0.07
		3	0.00	*52.74*	43.40	34.46	25.71	17.27	9.73	4.07	1.06	0.13
		4	0.00	*52.80*	43.84	35.65	27.75	19.86	12.20	5.66	1.59	0.19
		5	0.00	*52.80*	43.92	36.00	28.58	21.19	13.74	6.86	2.09	0.26
	$\triangle V_{ACI}$	1	0.00	**111.37**	**126.00**	**119.60**	**100.54**	76.19	*53.09*	33.82	18.38	6.28
		2	0.00	15.25	41.58	71.70	96.82	**106.58**	***93.92***	65.10	34.98	11.47
		3	0.00	1.41	8.58	23.39	45.24	69.55	*85.70*	**77.70**	46.71	15.36
		4	0.00	0.13	1.77	7.40	19.24	37.81	*59.08*	69.07	**50.23**	17.80
		5	0.00	0.01	0.34	2.22	7.85	19.44	*36.98*	52.51	46.49	**18.57**
$CV_D = 0.6$	Cost	0	91.05	305.27	466.78	687.14	1010.66	1532.47	2470.47	4432.75	9543.15	26482.59
		1	91.05	190.60	344.85	569.93	911.29	1455.87	2416.16	4397.50	9523.54	26475.69
		2	91.05	180.61	310.01	508.33	825.00	1357.91	2326.60	4332.98	9487.67	26463.49
		3	91.05	179.79	303.93	489.69	787.16	1297.26	2249.08	4259.50	9441.23	26447.36
		4	91.05	179.71	302.77	484.30	771.89	1265.60	2197.44	4196.44	9392.55	26428.97
		5	91.05	179.70	302.56	482.77	765.97	1249.83	2165.92	4149.50	9348.54	26410.08
	$\%V_{ACI}$	1	0.00	*37.56*	26.12	17.06	9.83	5.00	2.20	0.80	0.21	0.03
		2	0.00	*40.84*	33.59	26.02	18.37	11.39	5.82	2.25	0.58	0.07
		3	0.00	*41.10*	34.89	28.73	22.11	15.35	8.96	3.91	1.07	0.13
		4	0.00	*41.13*	35.14	29.52	23.62	17.41	11.05	5.33	1.58	0.20
		5	0.00	*41.13*	35.18	29.74	24.21	18.44	12.33	6.39	2.04	0.27
	$\triangle V_{ACI}$	1	0.00	**114.66**	**121.94**	**117.21**	**99.37**	76.60	*54.31*	35.26	19.61	6.90
		2	0.00	10.00	34.84	61.60	86.29	**97.96**	***89.55***	64.52	35.87	12.19
		3	0.00	0.81	6.08	18.64	37.84	60.65	*77.52*	**73.48**	46.44	16.13
		4	0.00	0.08	1.15	5.39	15.26	31.65	*51.64*	63.06	**48.69**	18.39
		5	0.00	0.01	0.21	1.53	5.93	15.77	*31.52*	46.93	44.00	**18.88**

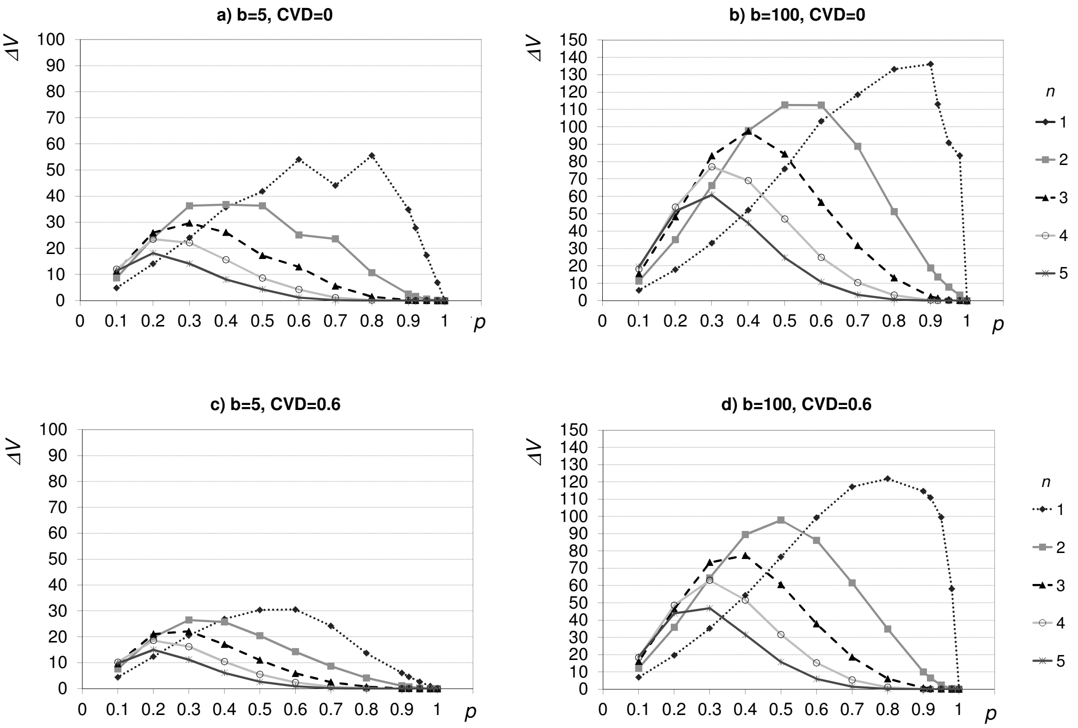


Figure 3.11 Absolute change in the value of ACI

are limited.

The effect of demand uncertainty on an absolute decrease in costs is not as obvious as it was in the relative case. This can be attributed to the interaction of two factors. While stronger demand uncertainty intensifies the difficulties of managing inventories as described above, it also contributes to higher costs and thus provides more potential for savings through ACI.

To summarize, while the relative cost savings are highest for the case of close to full availability due to the fact that one can completely avoid backorders with only a small extra inventory, the cost reduction in absolute terms is greater for cases with medium to low supply capacity availability. Further, we show that in most cases having only a little future visibility already offers considerable savings, although when one faces the possibility of consecutive periods of supply unavailability it can be very beneficial to extend the ACI horizon. In general, managers should recognize that the extent of savings shown clearly indicates that sharing ACI should be encouraged in supply chains with unstable supply conditions. In our experience, the current dynamic programming cost formulation is manageable in terms of the complexity of the calculations and can also be used for larger practical problems.

3.6 Relation between ACI and ADI

The use of advance demand information (ADI) has been widely studied in the literature. We refer the reader to the review in the introductory section. In this section, we construct a capacitated ADI model with structural similarities to our ACI model in order to explore the key differences and possible similarities between ACI and ADI. In particular, we show that the models cannot be equivalent because, for equivalence to hold, the available capacity should always be able to cover at least that part of demand that is revealed by ADI and hence should never be binding. We therefore conclude that the model we propose in this chapter clearly poses a new problem and can serve as the basis for a new line of research.

We first construct a capacitated ADI model. In this model, we assume that demand is generated by two independent customer groups: customers that place their orders N periods ahead, and those that place their orders for the current period in a traditional way without ADI. At the end of period t, we record the demand from the "traditional" customers that had not been observed prior to its materialization, which we refer to as the unobserved part of demand d_t and we also record the observed part of demand o_{t+N} for the future period $t+N$. Parameter N represents the length of the information horizon over which ADI is available, assuming perfect ADI. Also, we assume that there is a fixed supply capacity Q^c limiting the size of an order in each period. The idea behind construction of the capacitated ADI model is as follows: The capacity Q^c available in each period is used both to cover the observed part of demand and the unobserved part of demand. If the observed part of demand is modeled as realization of a random variable and it is met first due to the first-come-first-served principle, the remaining capacity to cover the unobserved part is also a random variable. This suggests that we have a random capacity available to satisfy the unobserved part of demand d_t, which is conceptually similar to our stochastic demand ACI inventory model.

To derive a formal ADI model description we start by writing x_t in a modified form $\tilde{x}_t$, as

$$\tilde{x}_t = \hat{x}_t - o_t. \tag{3.8}$$

Observe that $\tilde{x}_t$ already accounts for the observed part of the current period's demand. By ordering, we raise $\tilde{x}_t$ to the corresponding modified inventory position after ordering $\tilde{y}_t$, given as $\tilde{y}_t = \tilde{x}_t + \tilde{z}_t$, where $\tilde{z}_t \leq Q^c$. Due to the update of the inventory position with observed demand, $\tilde{y}_t$ only needs to cover the remaining, unobserved part of demand. From (3.8), we can show that at the start of period $t+1$, $\tilde{x}_t$ is updated in the following manner:

$$\tilde{x}_{t+1} = \tilde{x}_t + \tilde{z}_t - d_t - o_{t+1}.$$

We also need to track the demand observations, given through ADI, that will affect future orders. These constitute the ADI vector $\vec{o}_t = (o_{t+1}, \ldots, o_{t+N-1})$ that gets updated at the start of period $t+1$

$$\vec{o}_{t+1} = (o_{t+2}, \ldots, o_{t+N}),$$

by including the new information o_{t+N} collected through period t and purging the oldest data point o_{t+1}, with which we update $\tilde{x}_{t+1}$. The state space is described by an N -dimensional vector $(\tilde{x}_t, \vec{o}_t)$.

The minimal discounted expected cost function of the proposed ADI model, optimizing the cost over a finite planning horizon T from time t onward and starting in the initial state $(\tilde{x}_t, \vec{o}_t)$, can be written as:

$$\begin{aligned} f_t(\tilde{x}_t, \vec{o}_t) = & \min_{\tilde{x}_t \leq \tilde{y}_t \leq \tilde{x}_t + Q^c} \{C_t(\tilde{y}_t) + \\ + & \begin{cases} \alpha E_{D_t} f_{t+1}(\tilde{y}_t - D_t, \vec{o}_{t+1}), & \text{if } T-N-1 \leq t \leq T, \\ \alpha E_{D_t, O_{t+N}} f_{t+1}(\tilde{y}_t - D_t, \vec{o}_{t+1}), & \text{if } 1 \leq t \leq T-N, \end{cases} \end{aligned} \tag{3.9}$$

where $f_{T+1}(\cdot) \equiv 0$. Note that the ADI model given by (3.9) is a special case of the model introduced by Özer and Wei (2004). Consequently, it can be shown that the optimal ordering policy solving (3.9) is a state-dependent modified base-stock policy, where the optimal base-stock level is a function of the ADI vector $\vec{o}_t$.

Now that we have constructed the ADI model (3.9), we can write the formulations which establish the relationships between the ACI and ADI model dynamics. For the models to be equivalent in their base-stock levels we need to assume that $\tilde{y}_t = y_t$ holds. This is intuitively clear since both need to cover the same demand. Note that we assume the unobserved part of demand in the ADI model is modeled in the same way as the ACI model's demand given Section 3.2. By substituting the relevant ACI dynamics formulations into the corresponding ADI formulations, we show that the ACI model's x_t and z_t are related to their ADI model counterparts $\tilde{x}_t$ and $\tilde{z}_t$ in the following way:

$$\begin{aligned} x_t &= \tilde{x}_t + o_t, \\ z_t &= \tilde{z}_t - o_t. \end{aligned}$$

By updating $\tilde{x}_t$ at the start of period t, we already use some of total capacity Q^c to cover the observed part of demand o_t. Now we only have remaining capacity $Q^c - o_t$ to sufficiently raise y_t to cover the unobserved part of demand. This directly relates to having finite capacity q_t, which limits the extent to which we can raise y_t in the ACI model. We can write the following relationship:

$$q_t = Q^c - o_t. \tag{3.10}$$

By comparing the above ADI model with our ACI model, we first derive the relationship between the two information horizon parameters. Observe that at the start of period t the ADI vector $\vec{o}_t$ gives observed demand for $N-1$ future periods, which will affect the size of the following $N-1$ orders. This corresponds to having ACI for future n periods given by the ACI vector $\vec{q}_t$. The ACI and ADI horizon parameters n and N are therefore related in the following way:

$$n = N - 1.$$

We make our most important observation by substituting the ADI dynamics formulations into (3.4). Comparing the constructed new ADI dynamic programming formulation with the one proposed by (3.9) shows an inconsistency at the lower bound over which the minimization is made. Instead of looking for the optimal $\tilde{y}_t$ by minimizing the cost function over $\tilde{x}_t \leq \tilde{y}_t \leq \tilde{x}_t + Q^c$, minimization is made over $\tilde{x}_t + o_t \leq \tilde{y}_t \leq \tilde{x}_t + Q^c$. For direct equivalence of the ACI model and the ADI model to hold, we always have to be able to raise $\tilde{y}_t$ above $\tilde{x}_t + o_t$ for all t. This means that the available capacity Q^c should be sufficient to cover at least the observed part of demand. This "negativity issue" also stems directly from (3.10) where, for $q_t \geq 0$ to hold, $Q^c - o_t \geq 0$ has to hold. The fixed capacity therefore has to exceed all possible realizations of the observed part of demand, $Q^c \geq o_t$. This assumption only holds when $Q^c \geq \max o_t$, which happens either when an uncapacitated system, $Q^c \to \infty$, is considered or in a more realistic scenario where the observed part of demand only represents a small portion of the total demand. However, this implies that the remaining capacity that covers the unobserved part of demand cannot vary considerably. Thus, the equivalence of the two models would only hold if we consider our ACI model with a near-stationary capacity distribution with low uncertainty. As we have learned from the numerical analysis this in fact describes the "less interesting" setting where the value of ACI is small.

Based on these findings we conclude that restrictive assumptions are needed to guarantee direct equivalence between the capacitated ADI model and the proposed ACI model in terms of optimal base-stock levels and optimal costs. Hence, we conclude that the model we propose in this chapter is essentially different from capacitated ADI models.

3.7 Summary

In this chapter, we develop a model that incorporates advance capacity information (ACI) into inventory decision making and explore its effect on making effective ordering decisions within a periodic review inventory planning system facing limited stochastic supply. Based on the convexity of the relevant cost functions, we are able to show that the form of the optimal policy is a modified base-stock policy with a single state-dependent base-stock level. Essentially the base-stock level depends on realizations of future supply capacities revealed by ACI, and is a decreasing function of the ACI extent. We complement this result by showing additional monotonicity properties of the optimal policy. Another contribution of this work lies in establishing a link with advance demand information modeling by derivation of a capacitated ADI model. We show that the ACI model is only equivalent to the ADI model under a very specific and restrictive assumption. This implies that, although there are structural similarities, research on ACI will require new analyses of extensions of the model proposed in this chapter since ADI insights will not necessarily hold for the ACI model. The ACI is structurally different from ADI due to the capacity restriction.

By means of a numerical analysis we develop some additional managerial insights. In particular, we establish the following conditions in which inventory costs can be decreased by using ACI: (1) when there is a mismatch between demand and supply capacity, which can be anticipated through ACI, and there is an opportunity to pre-build inventory in an adequate manner; (2) when uncertainty in future supply capacity is high and ACI is used to reduce it effectively; and (3) in the case of high backorder costs, which further emphasizes the importance of avoiding stock outs. In these, managers should seek to establish long-term contractual agreements which encourage the sharing of ACI. Such relationships may bring considerable operational cost savings.

There are multiple ways to extend the work presented in this chapter. While the proposed model assumes perfect ACI, the ACI model can be extended to describe the situation where the communicated supply limit might not be completely accurate. This information can be denoted as imperfect ACI. The consequence of ACI not being exact is that there is still some uncertainty in the actual supply capacity availability. This leads to a situation where the inventory position does not reflect the actual realization of orders in the pipeline and anticipating future supply conditions is made harder due to the remaining share of the uncertainty. The present ACI model assumes that ACI reveals the supply capacity availability for current and n future orders, meaning that when placing an order exact supply capacity realization is known. An interesting idea for future research would be that supply information is received only after an order has been placed, in case there is a positive supply lead time. In this instance, the order has to be placed not knowing the available supply capacity; however, we observe *advance supply information* for the order that is already given and is currently still in the pipeline. Advance supply information indicates whether the order will be filled fully or just partially before the actual delivery, and thus enables the decision maker to react if necessary.

Chapter 4

Inventory management with advance supply information

It has been shown in numerous situations that sharing information between the companies leads to improved performance of the supply chain. In this chapter, we study a positive lead time periodic-review inventory system of a retailer facing stochastic demand from his customer and stochastic limited supply capacity of the manufacturer supplying the products to him. The manufacturer is willing to share the advance supply information (ASI) about the actual future replenishments of the pipeline orders placed by the retailer. ASI is provided at a certain time after the order has been placed and the retailer can now use this information to decrease the uncertainty of the supply, and thus improve its inventory policy. For this model we develop a dynamic programming formulation, and characterize the optimal ordering policy as a state-dependent base-stock policy. In addition, we show some properties of the base-stock level. While the optimal policy is highly complex, we obtain some additional insights by comparing it to the state-dependent myopic inventory policy. Our numerical results show the benefits of using ASI and based on them we provide useful managerial insights.

4.1 Introduction

Nowadays companies are facing difficulties in effectively managing their inventories mainly due to the highly volatile and uncertain business environment. While they are trying their best to fulfill the demand of their customers by using more or less sophisticated inventory control policies, their efforts can be severely hindered by the unreliable and limited deliveries from their suppliers. Due to the widespread trend of establishing plants overseas or outsourcing to specialists it has become increasingly more difficult for the companies to retain control over their procurement process. As the complexity of supply networks grows, so do the challenges and inefficiencies the companies are facing; orders get lost or are not delivered in full, shipment are late or don't arrive at all.

It has been well acknowledged both in the research community as well as by practitioners that these uncertainties can be reduced and better supply chain coordination can be achieved through the improved provision of information (Lee and Padmanabhan, 1997; Chen, 2003).

While the performance of a supply chain depends critically on how its members coordinate their decisions, sharing information can be considered as the most basic form of coordination in supply chains. In light of this, the concept of achieving so-called Supply Chain Visibility is gaining on importance, as it provides accurate and timely information throughout the supply chain processes and networks. This enables companies to share the information through often already established B2B communication channels and ERP solutions. EDI formatted electronic notifications on the status of the order fulfillment process, such as order acknowledgements, inventory status, Advance Shipment Notices (ASN), and Shipment Status Messages (SSM) are shared, enabling companies to track and verify the status of their order and consequently foresee supply shortages before they happen (Choi, 2010). There are also multiple examples of companies like UPS, FedEx and others in shipping industry, and Internet retailers like eBay and Amazon, that are offering real time order fulfillment information also on the B2C level.

Real visibility in the supply chain can be regarded as a prerequisite for the companies to reach new levels of operating efficiency, service capabilities, and profitability for suppliers, logistics partners, as well as their customers. However, while the technological barriers to information sharing are being dismantled, the abundance of the available information by itself is not a guarantee for improved performance. Therefore the focus now is on developing new tools and technologies that will use this information to improve on current state of the inventory management practices.

In this chapter, we investigate the benefits of advance information sharing (ASI). We consider a retailer facing stochastic demand from the end customer and procuring the products from a single manufacturer with stochastic limited supply capacity. We assume that the order is replenished after a given fixed lead time, which constitutes of order processing, production and shipping delay. However it can happen that the quantity received by the retailer is less than what he ordered originally. This supply uncertainty can be due to, for instance, the allocation policy of the manufacturer, which results in variable capacity allocations to her customers or to an overall capacity shortage at certain times. This stochastic nature of capacity itself may be due to multiple causes, such as variations in the workforce (e.g. holiday leaves), unavailability of machinery or multiple products sharing the total capacity.

We assume that the manufacturer tracks the retailer's order evolution and at certain point, when she can assess the extent to which the order will be fulfilled, she shares ASI with the retailer, giving him feedback on the actual replenishment quantity ahead of the time of the physical delivery of products. ASI enables the retailer to respond to the possible shortage by adjusting his future order decisions, and by doing this possibly offset the negative impact of the shortage. Based on this rationale we pose the following two research questions: (1) How can we integrate ASI into inventory decision model, or, more specifically can we characterize the optimal policy that would account for the availability of ASI, and describe its properties? (2) Can we quantify the value of ASI and establish the system settings where utilizing ASI is of the most importance?

The practical setting in which the above modeling assumptions could be observed is food processing industry, where the food processing facilities/manufacturers are being supplied with the agricultural products. The products are harvested periodically and the product availability is changing through time depending on a variety of factors: weather, harvesting capacity etc. Also it is reasonable to assume that supply capacity cannot be backordered as in a lot of cases harvested products cannot be stored for longer periods. The supply process taking a number of time periods is done in two phases. After an order is made by the retailer, the harvesting part of the production process commences, where the production outcome is uncertain. Then the products are delivered to the food processing facility. At this point the product availability is revealed and is no longer uncertain, and ASI is communicated to the customer in the form of ASN. The actual replenishment follows after the product is fully processed.

It is also worthwhile to shed some light on the differences between the ASI model proposed in this chapter and the ACI model from Chapter 3. We first describe the practical setting and the relevant differences in the way these two proposed types of upstream information are shared. Later within this chapter we give more insights into the modeling and structural differences of the two inventory models.

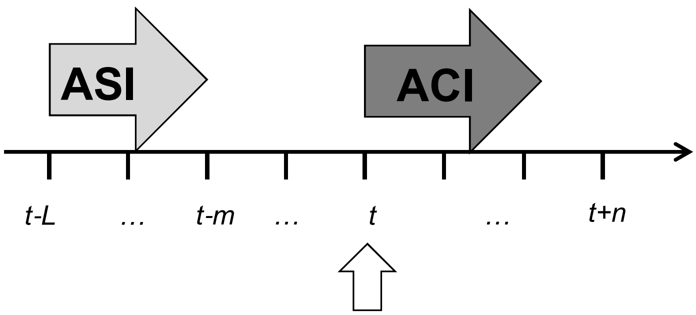


Figure 4.1 The time perspective of sharing ASI and ACI.

The main difference in the way information is shared in the case of ASI and ACI model is in the time delay between the placement of the order and the time the information on the available supply capacity is revealed. In the ASI case, the supply capacity information is revealed after the order has been placed and the lack of supply capacity availability results in the replenishment below the initial order. In the case of ACI, the information is available for future supply capacity availability, thus the order placed is aligned with this availability. ASI thus only allows the decision maker to respond to the actual realized shortages in a more timely manner. While based on ACI, the decision maker can anticipate the potential future shortages and accordingly adopt his ordering strategy.

Below we give some additional differences between the two inventory models in terms of the level of reliability and availability of information, and the extent of the potential savings:

- **Reliability**: ASI can be considered as more reliable/perfect information as it is communicated after the order has been placed with the manufacturer. Therefore it is reasonable to assume that the manufacturer has already put the order on the list of orders for execution within the Manufacturing Execution System, which also means the capacity availability has been thoroughly checked. In the case of ACI, the order has not been placed yet. The supply capacity availability will depend on the manufacturer's plans within the Master Production Scheduling. It is reasonable to assume that the MPS will be less reliable than MES.
- **Availability**: Due to the above argument, it is also reasonable to assume that the manufacturer is more willing to share ASI than ACI. In addition to this, manufacturer may not be willing to share the future capacity availability due to strategic reasons. While sharing ASI particularly in the case when the manufacturer cannot deliver the full order can be considered as a required business practice.
- **Savings potential**: Through sharing ACI the supply capacity availability can be anticipated and consequently the supply shortages can be avoided. Thus it is reasonable to assume that the potential savings will be higher than in the case of sharing ASI.

We proceed with a short literature review. Our work builds on the broad research stream of papers assuming uncertainty in the supply processes. In the literature the supply uncertainty is commonly attributed to one of the two sources: yield randomness and randomness of the available capacity. Our focus lies within the second group of problems, where Federgruen and Zipkin (1986a,b) were the first to address the capacitated stationary inventory problem with a fixed capacity constraint and have proven the optimality of the modified base-stock policy. Kapuscinski and Tayur (1998) extend this result by studying the non-stationary version of the model, where they assume periodic demand. Later, a line of research extends the focus to capture the uncertainty in capacity, by analyzing models with limited stochastic production capacity (Ciarallo et al., 1994; Güllü et al., 1997; Khang and Fujiwara, 2000; Iida, 2002). Ciarallo et al. (1994) explore different cases of the stochastic capacity constraint in a single and multiple periods setting. In the analysis of a single period problem, they show that stochastic capacity does not affect the order policy. The myopic policy of newsvendor type is optimal, meaning that the decision maker is not better off by asking for a quantity higher than that of the uncapacitated case. For a finite horizon stationary inventory model they show that the optimal policy remains to be a base-stock policy, where the optimal base-stock level is increased to account for the possible, however uncertain, capacity shortfalls in the future periods. Iida (2002) extends this result for the non-stationary environment.

Although a lot of attention in recent decades has been put in assessing the benefits of sharing information in the supply chains, the majority of the research is focused on studying the effect of sharing downstream information, in particular demand information (Gallego and Özer, 2001; Karaesmen et al., 2003; Wijngaard, 2004; Tan et al., 2007; Özer and Wei, 2004). Review papers by Chen (2003), Lau (2007) and Choi (2010) show that sharing upstream information has been considered in the literature in the form of sharing lead time information, production cost information, production yield, and sharing capacity information.

Capacity information sharing is of particular interest to us, where several papers have been discussing sharing information on future capacity availability (Jakšič et al., 2011; Altuğ and Muharremoğlu, 2011; Çinar and Güllü, 2012; Atasoy et al., 2012). Jakšič et al. (2011) study the benefits of sharing perfect information on future supply capacity available for orders to be placed in future periods. They show that the optimal ordering policy is a state-dependent base-stock policy characterized by the base-stock level that is a function of advance capacity information. Altuğ and Muharremoğlu (2011) work on a similar model; however they assume that the evolution of the capacity availability forecasts is done via the Martingale Method of Forecast Evolution (MMFE). Zhang et al. (2006) discuss the benefits of sharing advance shipment information. A setting in which a company receives the exact shipment quantity information is closely related to the one proposed in this chapter, however they assume that inventory is controlled through a simple non-optimal base-stock policy and as such it fails to capture the uncertainty of supply. Our model can be considered as a generalization of the model by Zhang et al. (2006), as we allow for both, demand and supply capacity, to be stochastic, and more importantly we model the optimal system behavior by considering considering the optimal inventory policy that is able to account for the supply uncertainty by setting appropriate safety stock levels. We propose that by having timely feedback on actual replenishment quantities through ASI, we can refine the inventory policy and improve its performance. To our knowledge the exploration of the relationship between the proposed way of modeling ASI and the optimal policy parameters has not yet received any attention in the literature.

The remainder of the chapter is organized as follows. We present a model incorporating ASI and its dynamic cost formulation in Section 4.2. The optimal policy and its properties are discussed in Section 4.3. We proceed by the study of the approximate inventory policy based on the state-dependent myopic policy in Section 4.4. In Section 4.5 we present the results of a numerical study and point out additional managerial insights. Finally, we summarize our findings and suggest directions for future research in Section 4.6.

4.2 Model formulation

In this section, we introduce the notation and the model of advance supply information for orders that were already placed, but are currently still in the pipeline. The model under consideration assumes periodic-review, stationary stochastic demand, limited stationary stochastic supply with fixed supply lead time, finite planning horizon inventory control system. Unmet demand is fully backlogged. However, the retailer is able to obtain ASI on supply shortages affecting the future replenishment of the orders in the pipeline from the manufacturer. We introduce the ASI parameter m that represents the time delay in which ASI is communicated with the retailer. The parameter m effectively denotes the number of periods between the time the order has been placed with the manufacturer and the time ASI is revealed. More specifically, ASI on the order z_{t-m} placed m periods ago is revealed in period t after the order z_t is placed in the current period (Figure 4.2). Depending on the available supply capacity q_{t-m}, ASI reveals the actual replenishment quantity, determined as the minimum of the two, $\min(z_{t-m}, q_{t-m})$. We assume perfect ASI. Observe that the longer the m, the larger is the share of the pipeline orders for which the exact replenishment is still uncertain. Furthermore, we assume that the unfilled part of the retailer's order is not backlogged at the manufacturer, but it is lost. We give the summary of the notation in Table 4.1 and we introduce some later upon need.

Table 4.1 Summary of the notation

T	:	number of periods in the finite planning horizon
L	:	constant nonnegative supply lead time, a multiple of review periods ($L \geq 0$);
m	:	advance supply information parameter, $0 \leq m \leq L$
h	:	inventory holding cost per unit per period
b	:	backorder cost per unit per period
α	:	discount factor ($0 \leq \alpha \leq 1$)
x_t	:	inventory position in period t before ordering
y_t	:	inventory position in period t after ordering
$\tilde{x}_t$	:	starting on-hand inventory in period t
z_t	:	order size in period t
D_t	:	random variable denoting the demand in period t
d_t	:	actual demand in period t
Q_t	:	random variable denoting the available supply capacity at time t
q_t	:	actual available supply capacity limiting order z_t given at time t, for which ASI is revealed m periods later

We assume the following sequence of events. (1) At the start of period t, the decision maker reviews the current inventory position x_t. (2) The ordering decision z_t is made up to uncertain supply capacity and correspondingly the inventory position is raised to $y_t = x_t + z_t$. (3) Order placed in period $t - L$ is replenished in the extent of $\min(z_{t-L}, q_{t-L})$, depending

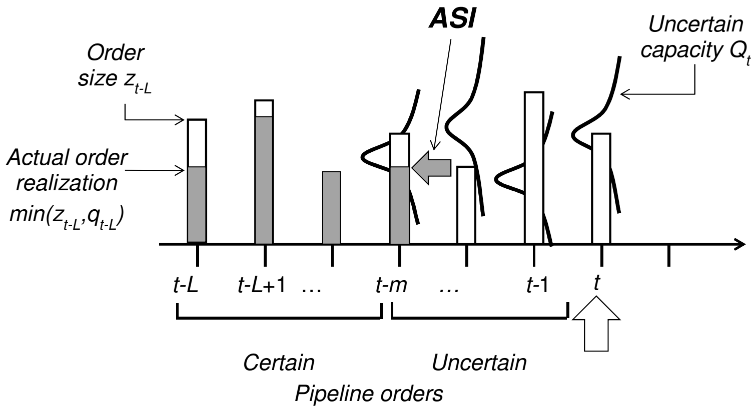


Figure 4.2 Advance supply information.

on the available supply capacity. ASI on the order placed in period $t-m$ is revealed, which enables the decision maker to update the inventory position by correcting it downward in the case of insufficient supply capacity, $y_t - (z_{t-m} - q_{t-m})^+$ (where $(x)^+ = max(x, 0)$). (4) At the end of the period previously backordered demand and demand d_t are observed and satisfied from on-hand inventory; unsatisfied demand is backordered. Inventory holding and backorder costs are incurred based on the end-of-period on-hand inventory.

Due to the positive supply lead time, each of the orders remains in the pipeline stock for L periods. For orders placed m periods ago or earlier we have already obtained ASI, while for more recent orders the supply information is not available yet. Therefore we can express the inventory position before ordering x_t as the sum of net inventory and the certain and uncertain pipeline orders:

$$x_t = \tilde{x}_t + \sum_{s=t-L}^{t-m-1} \min(z_s, q_s) + \sum_{s=t-m}^{t-1} z_s. \tag{4.1}$$

Note, that due to perfect ASI the inventory position x_t reflects the actual quantities that will be replenished for the orders for which ASI is already revealed, while there is still uncertainty in the actual replenishment sizes for recent orders for which ASI is not known yet.

Observe also that m denotes the number of uncertain pipeline orders. Therefore, m lies within $0 \leq m \leq L$, and the two extreme cases can be characterized as:

- $m = L$, or so-called *"No information case"*, which corresponds to the most uncertain setting as the actual replenishment quantity is revealed no sooner than at the moment of actual arrival. This setting is a positive lead time generalization of of the Ciarallo et al. (1994) model.

- $m = 0$, or so-called *"Full information case"*, which corresponds to the full information case, where before placing the new order, we know the exact delivery quantities for all pipeline orders. This is the case with the least uncertainty within the context of our model. Observe however that the current order is still placed up to uncertain supply capacity.

When moving from period t to $t+1$, we obtain ASI for the order z_{t-m} placed in period $t-m$. Correspondingly, the inventory gets corrected downwards if the order exceeds the available supply capacity, thus inventory position x_t is updated in the following manner:

$$x_{t+1} = x_t + z_t - (z_{t-m} - q_{t-m})^+ - d_t. \tag{4.2}$$

Note, that there is dependency between the order quantity and the size of the correction to x_t. If z_t is high, it is more probable that the available supply capacity will restrict the replenishment of the order, thus the correction will be bigger, and vice versa for low z_t. To fully describe the system behavior, we do not only need to keep track of x_t, but also have to track the pipeline orders for which we do not have ASI yet. We denote the stream of uncertain pipeline orders with the vector $\vec{z}_t = (z_{t-m}, z_{t-m+1}, \ldots, z_{t-2}, z_{t-1})$. In period $t+1$, $\vec{z}_{t+1}$ gets updated by the inclusion of the new order z_t, and the order z_{t-m} is dropped out as its uncertainty is resolved through the received ASI.

A single period expected cost function is a function of x_t and all uncertain orders, including the most recent order z_t, given in period t. Cost charged in period $t+L$, $\tilde{C}_{t+L}(\tilde{x}_{t+L+1})$, reassigned to period t when ordering decision is made, can be expressed as:

$$C_t(y_t, \vec{z}_t, z_t) = \alpha^L E_{\vec{Q}_t, Q_t, D_t^L} \tilde{C}_{t+L}(y_t - \sum_{s=t-m}^{t} (z_s - Q_s)^+ - D_t^L), \tag{4.3}$$

where the inventory position after ordering accounted for the possible future supply shortages, $y_t - \sum_{s=t-m}^{t} (z_s - Q_s)^+$, is used to cover the lead time demand, $D_t^L = \sum_{s=t}^{t+L} D_s$

The minimal discounted expected cost function, optimizing the cost over a finite planning horizon T, from time t onward, and starting in the initial state $(x_t, \vec{z}_t)$, can be written as:

$$f_t(x_t, \vec{z}_t) = \min_{x_t \le y_t} \{C_t(y_t, \vec{z}_t, z_t) + \alpha E_{D_t, Q_{t-m}} f_{t+1}(y_t - (z_{t-m} - Q_{t-m})^+ - D_t, \vec{z}_{t+1})\}, \text{ for } t \le T, \tag{4.4}$$

where $f_{T+1}(\cdot) \equiv 0$. The cost function f_t is a function of inventory position before ordering and orders given in last m periods, for which ASI has not yet been revealed.

4.3 Analysis of the optimal policy

In this section, we show the necessary convexity results of the relevant cost functions. This allows us to establish the structure of the optimal policy and show some of its properties.

Lets define J_t as the cost-to-go function of period t:

$$J_t(y_t, \vec{z}_t, z_t) = C_t(y_t, \vec{z}_t, z_t) + \alpha E_{D_t, Q_{t-m}} f_{t+1}(y_t - (z_{t-m} - Q_{t-m})^+ - D_t, \vec{z}_{t+1})\}, \text{ for } t \leq T. \quad (4.5)$$

The minimum cost function f defined in (4.4) can now be expressed as:

$$f_t(x_t, \vec{z}_t) = \min_{x_t \leq y_t} J_t(y_t, \vec{z}_t, z_t), \text{ for } t \leq T, \quad (4.6)$$

We proceed by establishing the necessary convexity results that allow us to establish the structure of the optimal policy. Observe that the single period cost function $C_t(y_t, \vec{z}_t, z_t)$ is not convex already for the zero lead time case as was originally shown by Ciarallo et al. (1994), where we elaborate on this in detail in Lemma C.1 in the Appendix B. $C_t(y_t, \vec{z}_t, z_t)$ is shown to be convex in y_t and quasiconvex in z_t (Figure 4.3), which however still suffice for the optimal policy to exhibit the structure of the base-stock policy.

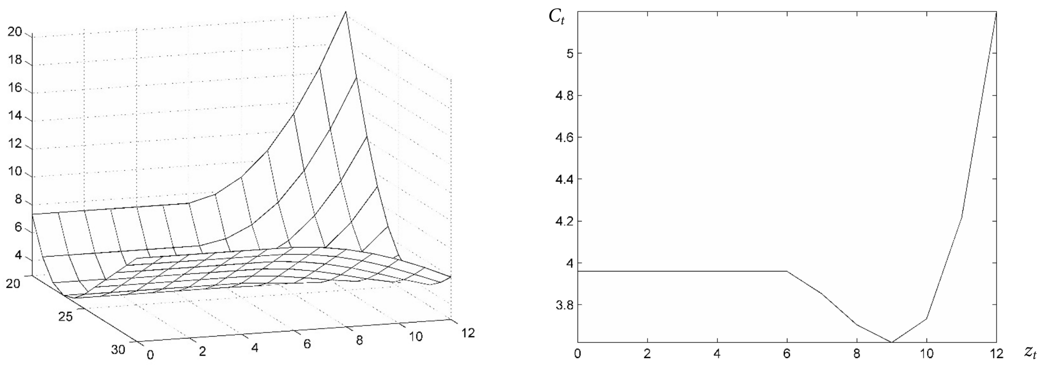


Figure 4.3 (a) $C_t(y_t, \vec{z}_t, z_t)$ as a function of y_t and z_t, and (b) $C_t(y_t, \vec{z}_t, z_t)$ as a function of z_t for a particular y_t.

We show that the results of the zero lead time case can be generalized to the positive lead time case, where the convexity of the costs functions in the inventory position is established given a more comprehensive system's state description $(x_t, \vec{z}_t)$. In Lemma B.1 in the Appendix B, we show that the single period cost function $C_t(y_t, \vec{z}_t, z_t)$ is not a convex function in general, but it exhibits a unique although state-dependent minimum. Based on this result one can show that the related multi-period cost functions $J_t(y_t, \vec{z}_t, z_t)$ and $f_t(x_t, \vec{z}_t)$ are convex in the inventory position y_t and x_t respectively (we show in Appendix B that the convexity holds also for other characterizations of the inventory position), as shown in the next Lemma:

Lemma 4.1 *For any arbitrary value of information horizon m, value of the ASI vector $\vec{z}_t$ and the order z_t, the following holds for all t:*

1. $J_t(y_t, \vec{z}_t, z_t)$ *is convex in* y_t,
2. $f_t(x_t, \vec{z}_t)$ *is convex in* x_t.

Based on the results following Lemma 4.1, we establish a structure of the optimal policy in the following Theorem:

Theorem 4.1 *Assuming that the system is in the state $(x_t, \vec{z}_t)$, let $\hat{y}_t(\vec{z}_t)$ be the smallest minimizer of the function $J_t(y_t, \vec{z}_t, z_t)$. For any $\vec{z}_t$, the following holds for all t:*

1. *The optimal ordering policy under ASI is the state-dependent base-stock policy with the optimal base-stock level $\hat{y}_t(\vec{z}_t)$.*
2. *Under the optimal policy, the inventory position after ordering $y_t(x_t, \vec{z}_t)$ is given by*

$$y_t(x_t, \vec{z}_t) = \begin{cases} x_t, & \hat{y}_t(\vec{z}_t) \leq x_t, \\ \hat{y}_t(\vec{z}_t), & x_t < \hat{y}_t(\vec{z}_t). \end{cases} \tag{4.7}$$

The proof is by induction, where we provide the details in the Appendix B. The optimal inventory policy is characterized by a single optimal base-stock level $\hat{y}_t(\vec{z}_t)$ that determines the optimal level of the inventory after ordering. The optimal base-stock level however is state-dependent as it depends on uncertain pipeline orders $\vec{z}_t$, for which ASI has not yet been revealed. Observe that due to not knowing the current period's capacity, we are not limited in how high we set the inventory position after ordering. The logic of the optimal policy is such that y_t should be raised to the optimal base-stock level $\hat{y}_t$, although in fact y_t does not reflect the actual inventory position as it is possible that the order will not be delivered in its full size.

We proceed by studying the properties of the optimal base-stock level in relation to the outstanding uncertain pipeline order status. In the following theorem we show that due to convexity results of Lemma 4.1 the optimal base-stock level is higher in the case of larger uncertain pipeline orders (Part 3 of Theorem 4.2). In the following formulations we suppress the subscript t in state variables for clarity reasons. We define the first derivative of functions $f_t(x, \cdot)$ and $J_t(x, \cdot)$ with respect to x as $f'_t(x, \cdot)$ and $J'_t(x, \cdot)$. Observe that for any two ASI vectors $\vec{z}_1$ and $\vec{z}_2$ in period t, $\vec{z}_1 \leq \vec{z}_2$ holds if and only if each element of $\vec{z}_1$ is smaller than or equal to the corresponding element of $\vec{z}_2$. In Parts 1 and 2 of Theorem 4.2, we show that the rate of the change in costs is higher in the case where there are lower uncertain pipeline orders in the system.

Theorem 4.2 *For any $\vec{z}_1 \leq \vec{z}_2$ and $z_1 \leq z_2$, the following holds for all t:*

1. $J'_t(y, \vec{z}_1, z_1) \geq J'_t(y, \vec{z}_2, z_2)$ *for all y,*
2. $f'_t(x, \vec{z}_1) \geq f'_t(x, \vec{z}_2)$ *for all x,*
3. $\hat{y}_t(\vec{z}_1) \leq \hat{y}_t(\vec{z}_2)$.

The dependency of the optimal base-stock level on $\vec{z}_t$ can be intuitively attributed to the following; if we were placing high orders (with regards to expected supply capacity available) in past periods, it is likely that a lot of the orders will not be realized in its entirety. This leads to probable replenishment shortages and demand backordering due to insufficient inventory availability. Therefore it is rational to set the optimal base-stock level higher with a goal of taking advantage of every bit of available supply capacity in the current period. By setting high targets, we aim to get the most out of the capacity, that is, we want to exploit the opportunity of the possibility of a large supply availability, although the chances that it will be actually realized can be small. If currently, we are not facing supply shortages the tendency to use the above logic diminishes. The result is also confirmed in Table 4.2. This observation is equivalent to what we observed in ACI model in Chapter 3, where the optimal base-stock level is aligned with the future supply capacity availability revealed through sharing ACI.

Table 4.2 Optimal base-stock levels $\hat{y}_t(z_{t-2}, z_{t-1})$ ($L = 3$, $m = 2$, $E[D] = 5$, $CV_D = 0$, $E[Q] = 6$, $CV_Q = 0.33$)

z_{t-2} \ z_{t-1}	0	1	2	3	4	5	6	7	8	9
0	20	20	20	20	20	20	21	22	23	24
1	20	20	20	20	20	20	21	22	23	24
2	20	20	20	20	20	20	21	22	23	24
3	20	20	20	20	20	20	21	22	23	24
4	20	20	20	20	20	21	21	22	23	24
5	20	20	20	20	21	21	22	23	**23**	24
6	21	21	21	21	21	22	22	23	24	25
7	22	22	22	22	22	23	23	24	25	25
8	23	23	23	23	23	**24**	24	25	25	26
9	24	24	24	24	24	24	25	25	26	27

In addition to the monotonicity properties of the optimal base-stock level, we study the sensitivity of the base-stock level to a change in the vector of outstanding uncertain pipeline orders $\vec{z}$. In the case of ACI, we point out in Conjecture 3.1 in Chapter 3, that taking away a unit of future supply capacity in period $t+i$ affects the optimal base-stock level more than taking away a unit of supply capacity which is available in later periods, $t+i+1$ and beyond. One could suggest that in the case of ASI increasing the size of the older pipeline order will affect the optimal base-stock level more than doing the same with one of the more recent pipeline orders. It can be quickly checked that this intuition is wrong. In Table 4.3, we calculate the differences in the base-stock levels for the setting presented in Table 4.2 above. Observe that increasing the pipeline order by 1 unit for any of the two orders, results in all possible scenarios, where none, both or either of the two base stock levels, $\hat{y}_t(z_{t-2}+1, z_{t-1})$ and $\hat{y}_t(z_{t-2}, z_{t-1}+1)$, are increased. The above suggestion does not hold due to the fact that y_t includes all the pipeline orders. If any of the uncertain pipeline orders is increased, the exposure to a potential shortage increases. Due to the fact that the shortage cannot be avoided, the best one can do is to immediately increase the optimal base-stock level to try to assure sufficient inventory at least L periods from now. In the ACI case such an immediate reaction is not necessary if future capacity availability allows the postponement of the decision to increase the inventories. Thus, it is not a surprise that the Table 4.2 is close to being symmetric (there are exceptions to this, where we point one below), which can be helpful in reducing the computational efforts in determining the optimal base-stock levels (again, this is not the case with the ACI model).

Table 4.3 The difference in the base-stock levels, $\hat{y}_t(z_{t-2}+1, z_{t-1}) - \hat{y}_t(z_{t-2}, z_{t-1}+1)$, and vice versa.

	$z_{t-1}+1$								
$z_{t-2}+1$	1	2	3	4	5	6	7	8	9
1	0	0	0	0	0	-1	-1	-1	-1
2	0	0	0	0	0	-1	-1	-1	-1
3	0	0	0	0	0	-1	-1	-1	-1
4	0	0	0	0	0	0	-1	-1	-1
5	0	0	0	0	0	0	0	0	-1
6	1	1	1	0	0	0	-1	0	0
7	1	1	1	1	0	1	0	0	0
8	1	1	1	1	0	1	0	0	0
9	1	1	1	1	0	0	0	0	0

While the above sensitivity property does not hold in general for the ASI case, we point out that it might hold for the special case where all the initial uncertain pipeline orders are equal. For the case denoted in bold in Table 4.2, it holds that $\hat{y}_t(8, 5) \geq \hat{y}_t(5, 8)$, where the initial state is $(5, 5)$ and the orders were increased by 3 units.

Observe. also that the change in the base-stock level is always at most equal to the change in the size of the uncertain pipeline order. We expect that if the pipeline order is increased by η, the corresponding increase in the base-stock level is at most equal to η or lower. Based on the above observations, we form the following two conjectures:

Conjecture 4.1 *The following holds for all* t*:*

1. *If* $z_{t-m} = z_{t-m+1} = \cdots = z_{t-1}$*, then* $\hat{y}_t(\vec{z}+\eta e_i) \geq \hat{y}_t(\vec{z}+\eta e_{i+1})$ *for i=t-m,...,t-1.*
2. $\hat{y}_t(\vec{z}+\eta e_i) - \hat{y}_t(\vec{z}) \leq \eta$ *for i=t-m,...,t-1.*

4.4 Insights from the myopic policy

We proceed by establishing the approximate inventory policy that would capture the relationship between the uncertain pipeline orders and the target inventory position. Approximate policy is an extension of a solution to a single-period problem (myopic solution). The relevant inventory costs that are influenced by the ordering decision z_t in period t are reassigned costs charged in period $t+L$ based on the on-hand inventory $\hat{x}_{t+L+1}$:

$$\tilde{x}_{t+L+1}(:, z_t) = y_t - D_t^L - \sum_{s=t-m}^{t} (z_{t-m} - q_{t-m})^+. \tag{4.8}$$

The myopic solution satisfies the following relationship for the non-stockout probability:

$$P\{\tilde{x}_{t+L+1}(:, z_t) \geq 0\} = b/b+h. \tag{4.9}$$

The steps of the approximate policy algorithm are as follows: (1) Determine the distributions $V_t(\vec{z}_t, z_t) = D_t^L + \sum_{s=t-m}^{t} (z_{t-m} - q_{t-m})^+$. (2) Determine the approximate state-dependent base-stock levels $\tilde{\hat{y}}_t(\vec{z}_t)$. (3) Assuming that the approximate policy instructs the decision maker to place the order with an accordance to (2), simulate the ordering policy and evaluate the relevant costs.

While it would be great if myopic policy would provide a reliable estimate of the optimal costs (which we show for the dual sourcing model in Chapter 5), one can easily see that this is not the case for ASI model (and similarly for the ACI model). The myopic policy cannot account for future supply shortages (that is for the orders that are still to be placed in the future). Particularly in the case of a highly utilized system and in the case of high demand/supply capacity uncertainty that leads to probable demand and supply mismatches. Thus, the following study is primarily concerned with capturing the state-dependency of the optimal base-stock levels by exploring the relationship between the vector of uncertain pipeline orders and the corresponding approximate base-stock levels.

In Table 4.4 we present the base-stock levels for the optimal policy, myopic policy and the differences between the two. Note that the myopic policy is optimal when there is no supply capacity uncertainty (which corresponds to a base-stock level of 23). Looking at the differences between the base-stock levels we see that the myopic base-stock levels are always lower and thus can be regarded as a lower bound for the optimal base-stock levels. The differences are decreasing with increasing uncertain pipeline orders. This can be attributed to the fact that myopic policy accounts for the potential shortages in the replenishment of the pipeline orders, but fails to account for future supply unavailability. For high uncertain pipeline orders, the additional inventory to cover the supply shortage is also sufficient to cover the risk of future shortages. In fact, we observe a risk pooling effect, where the base-stock level of 29 is sufficient to cover both risks.

Table 4.4 The optimal and myopic base-stock levels, ($L = 3$, $m = 2$, $E[D] = 5$, $CV_D = 0.5$, $E[Q] = 6$, $CV_Q = 0.33$)

Myopic					z_{t-1}					
z_{t-2}	0	1	2	3	4	5	6	7	8	9
0	23	23	23	23	23	23	24	24	25	26
1	23	23	23	23	23	23	24	24	25	26
2	23	23	23	23	23	23	24	24	25	26
3	23	23	23	23	23	23	24	24	25	26
4	23	23	23	23	23	23	24	25	25	26
5	23	23	23	23	23	24	24	25	26	27
6	24	24	24	24	24	24	25	25	26	27
7	24	24	24	24	25	25	25	26	27	28
8	25	25	25	25	25	26	26	27	28	29
9	26	26	26	26	26	27	27	28	29	29
Optimal					z_{t-1}					
z_{t-2}	0	1	2	3	4	5	6	7	8	9
0	27	27	27	27	27	27	27	28	29	29
1	27	27	27	27	27	27	27	28	29	29
2	27	27	27	27	27	27	27	28	29	29
3	27	27	27	27	27	27	27	28	29	29
4	27	27	27	27	27	27	28	28	29	29
5	27	27	27	27	27	27	28	29	29	29
6	27	27	27	27	28	28	28	29	29	29
7	28	28	28	28	28	29	29	29	29	29
8	29	29	29	29	29	29	29	29	29	29
9	29	29	29	29	29	29	29	29	29	29
Difference					z_{t-1}					
z_{t-2}	0	1	2	3	4	5	6	7	8	9
0	4	4	4	4	4	4	3	4	4	3
1	4	4	4	4	4	4	3	4	4	3
2	4	4	4	4	4	4	3	4	4	3
3	4	4	4	4	4	4	3	4	4	3
4	4	4	4	4	4	4	4	3	4	3
5	4	4	4	4	4	3	4	4	3	2
6	3	3	3	3	4	4	3	4	3	2
7	4	4	4	4	3	4	4	3	2	1
8	4	4	4	4	4	3	3	2	1	0
9	3	3	3	3	3	2	2	1	0	0

4.5 Value of ASI

In this section we estimate the extent of the savings gained through incorporating ASI into the inventory system. We perform a numerical numerical analysis to quantify the value of ASI and assess the influence of the relevant system parameters. Numerical calculations were done by solving the dynamic programming formulation given in (4.4).

To determine the influence of ASI parameter m, demand uncertainty, supply capacity uncertainty, and system utilization on the value of ASI, we set up the base scenario that is characterized by the following parameters: $T = 10, L = 3, \alpha = 0.99$ and $h = 1$ and $b = 20$. A discrete uniform distribution is used to model demand and supply capacity where the expected demand is given as $E[D] = 4$ and the expected supply capacity varies $E[Q] = \{4, 6, 8\}$, which means that the utilization of the system is $Util = \{1, 0.75, 0.5\}$. In addition we vary the coefficient of variation of demand $CV_D = \{0, 0.65\}$ and supply capacity $CV_Q = \{0, 0.33, 0.65\}$, and the ASI parameter $m = \{3, 2, 1, 0\}$, covering both the *No information* and *Full information* case.

We define the relative value of ASI for $m \leq L$, $\%V_{ASI}$, as the relative difference between the optimal expected cost of managing the system in the *No information* case ($m = L$), and the system where we have obtained ASI on a number of pipeline orders ($m \leq L$):

$$\%V_{ASI}(m \leq L) = \frac{f_t^{(m=L)} - f_t^{(m \leq L)}}{f_t^{(m=L)}}. \tag{4.10}$$

We also define the marginal change in the value of ASI, $\triangle V_{ASI}$. With this we measure the extra benefit gained by decreasing the number of uncertain pipeline orders by obtaining ASI sooner, from m to $m - 1$:

$$\triangle V_{ASI}(m-1) = f_t^{(m)} - f_t^{(m-1)}$$

We present the results in Figures 4.4 and 4.5. Not surprisingly, the insights about the value of ASI very much correspond to the ones for ACI model. The interplay of system parameters is relatively complex, which is exhibited in the fact that the value of ASI changes in a non-monotone manner. This is particularly the case in the effect of the system's utilization, where the majority of the gains are made at (in our case) moderate utilizations. Increasing capacity uncertainty is where we would anticipate that the value of ASI will be increasing and majority of the gains would be made. This can be observed in $\triangle V_{ASI}$ for both low and high demand uncertainty scenario, while this only partially holds for $\%V_{ASI}$. The increasing demand uncertainty decreases both the relative and the absolute value of ASI. While the relative value of ASI extends over 30%, and for most of the scenarios above 10%, it drops below 4% for all scenarios under high demand uncertainty.

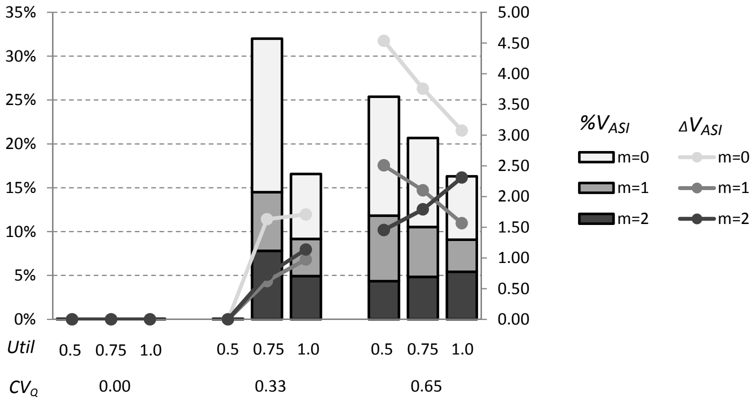


Figure 4.4 The relative and the absolute value of ACI for $CV_D = 0$.

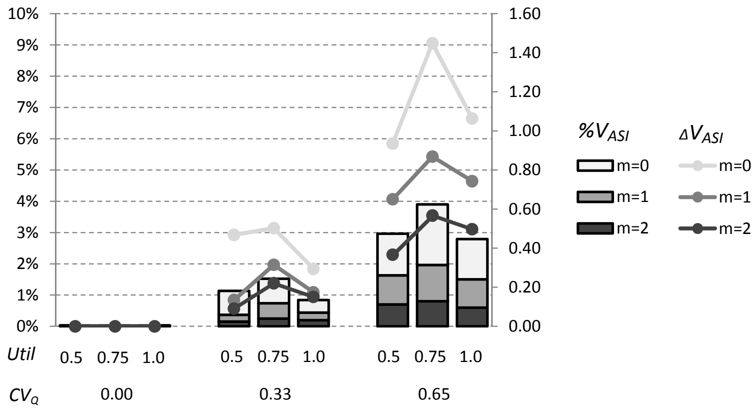


Figure 4.5 The relative and the absolute value of ACI for $CV_D = 0.65$.

It is expected that the benefits of ASI fall short of the ones observed when studying the ACI model. The interesting observation is made when studying the influence of the ASI parameter m. While we have observed that with a limited insight into future supply capacity availability the majority of the benefits can be gained in most of the scenarios under ACI model, this is not the case with ASI model. When m is decreased from $m = L$ in the *No information* case, we observe the increasing returns with m approaching 0. As ASI only allows for a response to shortages in replenishment, it is the scenario in which ASI is revealed soon after the order is placed that allows the decision maker to promptly respond to the shortages.

4.6 Summary

In this chapter, we analyze a periodic review inventory system with positive lead time and stationary stochastic demand and supply capacity. As the *No information* case of our model can be considered as a positive lead time generalization of the paper by Ciarallo et al. (1994), we also extend the scope of the model by incorporating the possibility to obtain information about the available supply capacity for the pipeline orders. ASI is revealed after the order has been placed, but before it is replenished.

We show that the optimal policy is highly complex due to the extensive system's state description, where apart from the inventory position, the stream of uncertain pipeline orders has to be monitored and adapted constantly. However, despite this complexity, we show that the optimal policy is a state-dependent base-stock policy. We show that the base-stock levels should be increased to compensate for the increased replenishment uncertainty. Despite the fact that the myopic policy does not provide a good approximation for the optimal base-stock levels (and optimal costs), we show that by inclusion of the safety factor that would compensate for the future supply capacity uncertainty, the myopic policy adequately captures the risk of shortages in pipeline orders.

Numerical calculations show that the benefits obtained through ASI can be relatively big (although also highly dependent on other system parameters), however in this case ASI should be revealed for the most of the uncertain pipeline orders as we observe the increasing returns with the increasing ASI availability.

The analysis in the future could explore different alternatives to the presented ASI model. Additional insights could be gained by studying simplified settings (for instance constant demand, Bernoulli distributed supply capacity etc.), where explicit expressions could be obtained that would better capture the supply uncertainty structure in the system.

Chapter 5

Dual sourcing: trading off stochastic capacity limitations and long lead times

In this chapter, we model a periodic review inventory system with non-stationary stochastic demand, in which a manufacturer is procuring a component from two available supply sources. The faster supply source is modeled as stochastic capacitated with immediate delivery, while the slower supply source is modeled as uncapacitated with a longer fixed lead time. The manufacturer's objective is to choose how the order should be split between the two supply sources in each period, where the slower supply source is used to compensate for the supply capacity unavailability of the faster supply source. Note that this is different from the conventional dual-sourcing problem, and motivated by the new reality of near-shoring options. We derive the optimal dynamic programming formulation that minimizes the total expected inventory holding and backorder costs over a finite planning horizon and show that the optimal policy is relatively complex. We extend our study to show that the myopic two-level base-stock policy provides a nearly perfect estimate of the optimal costs. Our numerical results reveal the benefits of the dual sourcing and explain the trade-off between the replenishment responsiveness and its reliability. We show that in most cases the manufacturer should develop a hybrid procurement strategy, taking advantage of both supply sources to minimize its expected total cost.

5.1 Introduction

With the rise of reshoring options and growth of B2B supply platforms, the complexity of sourcing management in the supply chain is increasing. Recent studies by BCG (2014b) and others suggest that the cost of sourcing in the U.S.A. may be approaching the cost of producing in China due to matters such as the rising wages in coastal China, the increase in transportation costs from more remote regions in China, the exchange rate developments of the RMB vs the USD, and the growth in productivity in the U.S.A. due to extensive automation. The realization that the economics of manufacturing are swinging in favor of the U.S.A. has seen companies becoming interested in shifting manufacturing back to the U.S.A., for both goods to be sold at home and those intended for major export markets. The BCG's survey found that the number of respondents saying that their companies are already bringing production back from China to the United States had risen by 20 %, from roughly 13% to 16% in the past year (BCG, 2014a). Similar developments are taking place in Europe.

Although the comparative attractiveness of the U.S.A. and other developed economies has increased, the current nearshoring trend is facing several challenges. The traditionally presumed unlimited capacity availability of nearshore manufacturing options may be decreasing due to the fact that so much capacity has been taken out of the system over the past two decades and that widespread automation is reducing the ability to respond to short-term changes in demand. Companies would have to rebuild their supply chains and identify people with the right skills to handle the increasingly sophisticated automated operations. In an executive survey by Alix Partners (2014), the respondents mention that the challenges center on the availability and capability of the local workforce, and the capability and flexibility of suppliers. In addition, with one-third of U.S. 3PLs reporting increased volumes and revenues as a result of nearshoring, their CEOs are also reporting capacity shortages across transport modes, including truckload, LTL, intermodal and rail, with the outcome that *"The capacity crunch has led to higher rates and longer transit times with 3PLs struggling to meet on-time service goals and cost targets"* (Penske Logistics, 2014).

The fact that manufacturing jobs are coming back to developed economies due to reshoring, does not mean that they are not continuing to develop in Asia and other developing countries. In fact, the number of global sourcing options is still growing, with the global supply chains effectively pooling the global manufacturing capacities and making them available to companies worldwide. For instance, the rise of B2B platforms such as Alibaba, enables suppliers from further away to flexibly respond to market needs, albeit with longer (transportation) lead times.

As a consequence, the trade-offs in dual sourcing are changing. Most research in dual sourcing studies the trade-offs between a nearshoring option that is more expensive and faster against an offshoring option that is cheaper but requires a longer lead time. Yet the developments described above suggest that at least for part of the product portfolio the trade-off may be less about product cost, and nearby suppliers may be faster but not necessarily more flexible. They may suffer from limited capacity availability.

In this chapter, we focus on such a newly developing trade-off. We study the problem of a manufacturer procuring a component for the production of a finished product, where two sourcing options are considered: a faster, yet partially available source, and a slower, yet fully available source. Both sources supply at the same landed cost. In this case, it is unlikely that the faster source is always favorable. In any case, supplying from the faster nearshore source will involve uncertainties related to the current capacity availability, which could result in uncertain replenishment to the manufacturer. This uncertainty would need to be hedged using the slower, presumably uncapacitated, offshore source. When a manufacturer experiences or anticipates a supply shortage of the nearshore source, a decision has to be made concerning the extent to which the slower offshore source should also be utilized. Due to the sourcing lead time, the ordering decisions are made before demand for the finished product is realized. Thus, the decision maker is facing both, the uncertain replenishment from the fast supply source, and the uncertainty of demand for the finished product, and needs to allocate the replenishment between the two supply sources.

To study this, we model a zero-lead-time supply source that is stochastic capacitated, where the supply capacity is exogenous to the manufacturer and the actual capacity realization is only revealed upon replenishment. The reliable positive lead time supply source is modeled as uncapacitated with a fixed one-period lead time. Both the supply capacity of the nearshore source and the demand are assumed to be stochastic and non-stationary with known distributions in each time period. Unmet demand is backordered. In each period the manufacturer places the order with the fast source, the slow source, or with both sources. Our goal is to find an optimal policy that minimizes the inventory holding costs and backorder costs over a finite planning horizon. See Figure 5.1 for a sketch of the supply chain under study.

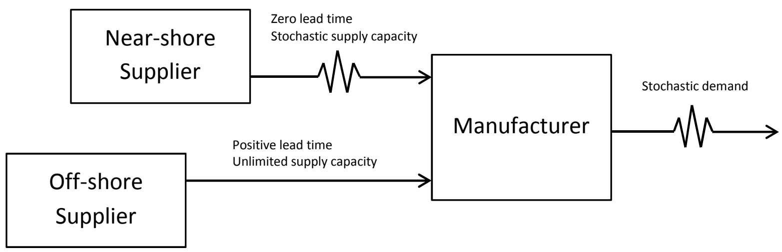


Figure 5.1 Sketch of the supply chain under study.

We proceed with a review of the relevant literature on supply uncertainty models in a single-stage setting, where our interest lies in two research tracks: single sourcing inventory models with random capacity and dual-sourcing models with suppliers that differ in their delivery times and/or supply capacity availability. The way we model the supply capacity of the faster supply source is in line with the work of Ciarallo et al. (1994); Khang and Fujiwara (2000); Iida (2002); Jakšič et al. (2011) and Jakšič and Fransoo (2015), where the random supply/production capacity determines a random upper bound on the supply availability in each period. For a finite horizon stationary inventory model they show that the optimal policy is a base-stock policy (or a modified base-stock policy if capacity is known prior to placing an order), where the optimal base-stock level is increased to account for possible, albeit uncertain, capacity shortfalls in future periods. An important observation for our work is the insight provided by Ciarallo et al. (1994) in their analysis of a single-period problem, where they show that stochastic capacity does not affect the order policy. The myopic policy of the newsvendor type is optimal to cover the demand uncertainty, meaning that the decision maker is not better off by asking for a quantity higher than that of an uncapacitated case.

For a general review of multiple supplier inventory models we refer the interested reader to Minner (2003). The review is based on the important criteria for the supplier choice, mainly the price and the supplier's service. A more focused review of multiple sourcing inventory models when supply components are uncertain by Tajbakhsh et al. (2007) reveals that most of these models consider uncertainty either in supply lead time, supply yield, or supplier availability. More specifically, the review of dual-sourcing literature shows that a series of papers shares some basic modeling assumptions with our model: dual-sourcing periodic review with deterministic lead times that are different for the two supply sources. These papers can be divided into two streams depending on the assumptions on supply capacity availability, where for the first stream the assumption of unconstrained suppliers holds, while for the second some sort of capacity constraint is introduced at the faster supplier or both suppliers. Most of these papers rely on the price difference between the two suppliers that stimulates the manufacturer to partially source from the cheaper, slower supplier. However, we argue that in the case where the supply capacity availability at the faster supply source is limited, variable, and potentially stochastic, the above-mentioned price incentive is not needed. Thus, in our case the incentive to find the optimal dual-sourcing strategy lies in finding the right balance between the responsiveness of the faster supply source and the reliability of the slower supply source.

In the first of the two above-mentioned streams that assumes unconstrained suppliers, the search for the best dual-sourcing strategy revolves around the dilemma of when to use a faster and necessarily more expensive supplier to compensate for a slow response by a cheaper supplier. Several papers discuss the setting in which the lead times of the two suppliers differ by a fixed number of periods (Daniel, 1963; Fukuda, 1964; Bulinskaya, 1964; Whittemore and Saunders, 1977). More specifically, Fukuda (1964) shows the optimality of the two-level base-stock policy for the case where the two suppliers' lead times differ by one period (so-called consecutive lead times). The policy instructs that first the order with the fast supply source is placed so that the inventory position is raised to the first base-stock level, and then the slow supply source is used to raise the inventory position to the second base-stock level. For general, nonconsecutive, lead times, Whittemore and Saunders (1977) found the optimal policy to be quite complex and lacking structure.

Within the second stream of papers, the first to assume a capacitated faster supplier is Daniel (1963). He showed that a modified two-level base-stock policy is also optimal in this case, where ordering with the faster supplier is up to the first base-stock level if the fixed capacity limit allows it. Yazlali and Erhun (2009) impose minimum and maximum fixed capacity limits on the availability of the two supply sources, and again show that the two-level base-stock policy is optimal when leadtimes are consecutive. Similarly, Xu (2011) assumes fixed capacity bounds on the cumulative order quantity to the two suppliers. He proves the optimality of a myopic ordering policy for a setting where the faster supply mode provides instant delivery, while the slower delivers an order one period later. While the modeling perspective of the papers referred to above is similar to ours, they all still primarily focus on exploring the trade-off between the price difference and responsiveness. An exception to this is the work by Yang et al. (2005) who explore a trade-off between variable capacity restrictions and responsiveness. They consider a Markovian capacity constraint on the fast supply mode, and show that the base-stock type policy is optimal in the situation where lead times differ by one period and there are no fixed ordering costs. While their paper comes closest to our focus on exploring the effect of reliability of a faster supply mode on the optimal dual sourcing strategy, there are still relevant differences in our work. They assume that the capacity is known at the time orders are placed with the two supply channels, and also that the current period's demand is realized before an order with a longer lead time supply channel is placed.

The remainder of the chapter is organized as follows. We present the model formulation in Section 5.2. In Section 5.3, the general structure of the optimal policy is characterized and the myopic policy is developed. In Section 5.4, we present the results of a numerical study to determine the optimal costs of dual sourcing, to study the order allocation between the supply modes and quantify the benefits of dual sourcing over single sourcing. Additionally, we explore the option of obtaining the advance capacity information on future supply capacity availability of the faster supplier in Section 5.5. Finally, we summarize our findings and suggest possible extensions in Section 5.6.

5.2 Model formulation

In this section, we give the notation and the model description. The faster, zero-lead-time, supply source is stochastic capacitated where the supply capacity is exogenous to the manufacturer and the actual capacity realization is only revealed upon replenishment. The slower supply source is modeled as uncapacitated with a fixed one period lead time. The demand and supply capacity of the faster supply source are assumed to be stochastic non-stationary with known distributions in each time period, although independent from period to period. In each period, the customer places an order with either an unreliable, or a reliable supply mode, or both.

Presuming that unmet demand is fully backordered, the goal is to find the optimal policy that would minimize the inventory holding costs and backorder costs over finite planning horizon T. We intentionally do not consider any product unit price difference and fixed ordering costs as we are primarily interested in studying the trade-off between the capacity uncertainty associated with ordering from a faster supply source and the delay in the replenishment from a slower source. Any fixed costs would make the dual-sourcing strategy less favorable, and the difference in the fixed costs related to any of the two ordering channels would result in a relative preference of one channel over the other. The notation used throughout the chapter is summarized in Table 5.1 and some is introduced when needed.

Table 5.1 Summary of the notation.

T	:	number of periods in the finite planning horizon
c_h	:	inventory holding cost per unit per period
c_b	:	backorder cost per unit per period
α	:	discount factor ($0 \leq \alpha \leq 1$)
$\tilde{x}_t$	:	on-hand inventory before ordering in period t
x_t	:	inventory position before ordering in period t
y_t	:	inventory position after ordering from a faster capacitated supply source in period t
w_t	:	inventory position after ordering from a slower uncapacitated supply source in period t
z_t	:	order placed with the faster supply source in period t
v_t	:	order placed with the slower supply source in period t
d_t, D_t	:	actual realization and random variable denoting demand in period t
$g_t(D_t)$	:	probability density function of demand in period t
$G_t(D_t)$	:	cumulative distribution function of demand in period t
q_t, Q_t	:	actual realization and random variable denoting the available supply capacity of the faster capacitated supply source in period t
$r_t(Q_t)$	:	probability density function of the supply capacity of the faster supply source in period t
$R_t(Q_t)$	:	cumulative distribution function of the supply capacity of the faster supply in period t

We assume the following sequence of events. (1) At the start of the period, the manager reviews the inventory position before ordering x_t, where $x_t = \tilde{x}_t + v_{t-1}$ is the sum of the on-hand stock $\tilde{x}_t$ and the order v_{t-1} to a slower supply source made in the previous period. (2) Order z_t to a faster supply source and order v_t to a slower supply source are placed. For the purpose of the subsequent analysis, we define two inventory positions after the order placement, y_t and w_t. First, after placing order z_t the inventory position is raised to y_t, $y_t = x_t + z_t$, and order v_t is raised to w_t, $w_t = x_t + z_t + v_t$. Observe that it makes no difference in which sequence the orders are actually placed as long as both are placed before the current period's capacity of the fast supply source q_t and demand d_t are revealed. (3) The order with the slower supply source from the previous period v_{t-1} and the current period's order z_t are replenished. The inventory position can now be corrected according to the actual supply capacity realization $w_t - (z_t - q_t)^+ = x_t + min(z_t, q_t) + v_t$, where $(z_t - q_t)^+ = \max(z_t - q_t, 0)$. (4) At the end of the period, demand d_t is observed and satisfied through on-hand inventory;

otherwise, it is backordered. Inventory holding and backorder costs are incurred based on the end-of-period on-hand inventory, $\tilde{x}_{t+1} = y_t - (z_t - q_t)^+ - d_t$. Correspondingly, the expected single-period cost function is defined as $C_t(y_t, z_t) = \alpha E_{Q_t,D_t} \tilde{C}_t(\tilde{x}_{t+1}) = \alpha E_{Q_t,D_t} \tilde{C}_t(y_t - (z_t - Q_t)^+ - D_t)$, where $\tilde{C}_t(\tilde{x}_{t+1}) = c_h(\tilde{x}_{t+1})^+ + c_b(-\tilde{x}_{t+1})^+$. The minimal discounted expected cost function that optimizes the cost over a finite planning horizon T from period t onward, starting in the initial state x_t, can be written as:

$$f_t(x_t) = \min_{x_t \leq y_t \leq w_t} \left\{ C_t(y_t, z_t) + \alpha E_{Q_t,D_t} f_{t+1}(w_t - (z_t - Q_t)^+ - D_t) \right\}, \text{ for } 1 \leq t \leq T \quad (5.1)$$

and the ending condition is defined as $f_{T+1}(\cdot) \equiv 0$.

5.3 Characterization of the near-optimal myopic policy

In this section, we first characterize the optimal policy for the non-stationary demand and supply capacity setting. We show that the structure of the optimal policy is relatively complex, where the order with the faster supply source depends on the inventory position before ordering, while the order with the slower supply source is raised to a state-dependent base-stock level. We continue by studying the stationary setting by introducing the myopic policy where the myopic orders are the solutions to the extended single period problem, and show the properties of the two myopic base-stock levels[1]. We conduct the numerical analysis to show that the costs of the myopic policy provide a very accurate estimate of the optimal costs. See the Appendix D for proofs of the following propositions.

In the literature review, we refer to a series of papers studying the dual sourcing inventory problem with consecutive lead times. The two-level base-stock policy characterizes the structure of the optimal policy in all of them, both in the case of uncapacitated supply sources and in the case where one or both supply sources exhibit the fixed capacity limit. However, by studying the convexity properties of the cost functions given in (5.2)-(5.4), we show that these are not convex in general. In addition, we show that the optimal inventory position after ordering with the faster supplier $\hat{y}_t$ is not independent of the inventory position before ordering x_t, and therefore cannot be characterized as the optimal base-stock level.

As single-period costs C_t in period t are not influenced by order v_t, we can rewrite (5.1) in the following way:

$$f_t(x_t) = \min_{x_t \leq y_t} \left\{ C_t(y_t, z_t) + \min_{y_t \leq w_t} \alpha E_{Q_t,D_t} f_{t+1}(w_t - (z_t - Q_t)^+ - D_t) \right\}, \text{ for } 1 \leq t \leq T, \quad (5.2)$$

[1]With the phrase "myopic policy" we denote the policy that optimizes the single period problem. As we later demonstrate that the myopic policy is not optimal in a multiperiod setting, we avoid using the phrase "optimal myopic policy" to avoid confusion.

which now enables us to introduce auxiliary cost functions $J_t(y_t, z_t)$ and $H_t(w_t, z_t)$:

$$J_t(y_t, z_t) = C_t(y_t, z_t) + \min_{y_t \le w_t} \alpha E_{Q_t,D_t} f_{t+1}(w_t - (z_t - Q_t)^+ - D_t), \text{ for } 1 \le t \le T \tag{5.3}$$

$$H_t(w_t, z_t) = \alpha E_{Q_t,D_t} f_{t+1}(w_t - (z_t - Q_t)^+ - D_t), \text{ for } 1 \le t \le T \tag{5.4}$$

Based on the above observations, we give the structure of the optimal policy under stochastic non-stationary demand and supply capacity in the following proposition:

Proposition 5.1 *Let $\hat{y}_t(x_t)$ be the smallest minimizer of the function $J_t(y_t, z_t)$ and $\hat{w}_t(\hat{z}_t)$ be the smallest minimizer of the function $H_t(w_t, \hat{z}_t)$. The following holds for all t:*

1. *The optimal inventory position after placing the order with the faster supplier $\hat{y}_t(x_t)$ is a function of the inventory position before ordering x_t.*
2. *The optimal inventory position after placing the order with the slower supplier $\hat{w}_t(\hat{z}_t)$ is a state-dependent base-stock level.*
3. *The inventory position $w_t(x_t)$ after placing the order with the slower supply source is:*

$$w_t(x_t) = \begin{cases} x_t, & \hat{w}_t(\hat{z}_t = 0) \le x_t, & \hat{z}_t = 0, \hat{v}_t = 0, \\ \hat{w}_t(\hat{z}_t = 0), & \hat{y}_t(x_t) \le x_t < \hat{w}_t(\hat{z}_t = 0), & \hat{z}_t = 0, \hat{v}_t = \hat{w}_t(\hat{z}_t = 0) - x_t, \\ \hat{w}_t(\hat{z}_t > 0), & x_t < \hat{y}_t(x_t), & \hat{z}_t = \hat{y}_t(x_t) - x_t, \hat{v}_t = \hat{w}_t(\hat{z}_t > 0) - \hat{y}_t(x_t). \end{cases} \tag{5.5}$$

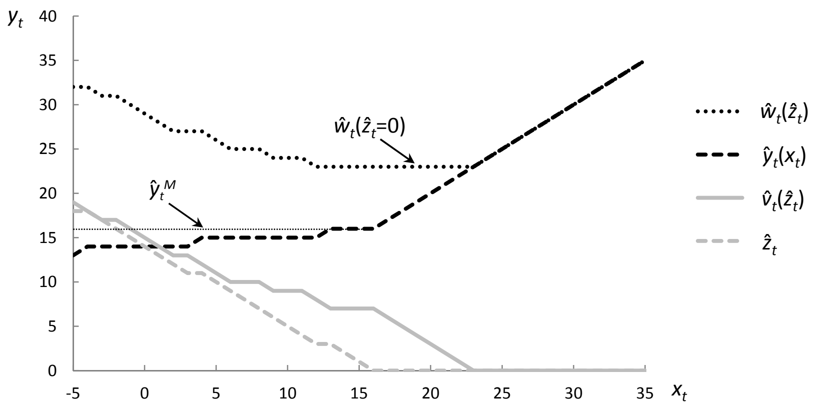


Figure 5.2 The optimal inventory positions after ordering and the optimal orders.

For a clearer representation of the optimal policy as given in (5.5), we depict the optimal order sizes depending on the initial inventory position x_t and the corresponding inventory positions after ordering in Figure 5.2. For comparison, we also plot the myopic base-stock level $\hat{y}_t^M$ that optimizes the single period cost function C_t (Part 3 of Lemma C.1 in Appendix D). For $x_t < \hat{y}_t^M$ we order up to $\hat{y}_t^M$ for small optimal orders with a faster supplier $\hat{z}_t$, while for higher $\hat{z}_t$, $\hat{y}_t(x_t)$ lies below $\hat{y}_t^M$. For $x_t \geq \hat{y}_t^M$ no $\hat{z}_t$ is placed. Similarly, looking at the optimal order with the slower supply source $\hat{v}_t$, it is only placed if $x_t \leq \hat{w}_t(\hat{z}_t = 0)$, and at $\hat{y}_t(x_t) \leq x_t < \hat{w}_t(\hat{z}_t = 0)$ the inventory position x_t is increased to a constant base-stock level $\hat{w}_t(z_t = 0)$. For $x_t < \hat{y}_t(x_t)$ the size of the order with the slower supply source depends on the anticipated supply shortage of the faster supply source, thus the state-dependent base-stock level $\hat{w}_t(\hat{z}_t)$ depends on the size of the order $\hat{z}_t$ placed with the faster supply source.

Based on the above results, the optimal policy instructs that the order with the slower supply source should compensate for the probable supply shortage of the faster supply source. The goal here is to bring the next period's starting inventory position to the optimal level (where the demand uncertainty also needs to be taken into account). In the next period, this enables the decision-maker to place the order with the faster supply source in such a way that the period's inventory costs are minimized given the supply capacity uncertainty and demand uncertainty.

Next, we focus on a stationary demand and supply capacity setting. Based on the insights obtained studying the optimal inventory policy, one might suggest that the myopic base-stock level $\hat{y}^M$ is a good approximation for $\hat{y}_t(x_t)$. To study this, we introduce the notion of the myopic policy, which optimizes the so-called extended single period problem in every period. As the model under consideration assumes different lead times for the faster and the slower supplier, the extended period covers the time interval in which both ordering decisions are made that directly influence the single period costs C_t in period t. It effectively starts by placing an order v_{t-1} with the slower supplier in period $t-1$, followed by placing an order z_t with the faster supplier in period t. The ability to cover the demand D_t at the end of period t depends on the replenishment of the two orders and the realization of the demand (Figure 5.3). Observe also that with the order v_t we cannot influence the costs in period t.

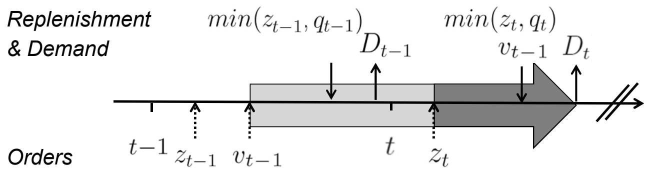


Figure 5.3 The scheme for the myopic inventory policy.

The myopic equivalent to the optimal cost function as given in (5.2) can therefore be written in the following way:

$$f_t^M(x_t) = \min_{x_t \leq y_t} C_t(y_t, z_t) + \min_{y_t \leq w_t} \alpha E_{Q_t,D_t} f_{t+1}^M(w_t - (z_t - Q_t)^+ - D_t), \text{ for } 1 \leq t \leq T, \quad (5.6)$$

where the search for the optimal y_t resorts to minimizing the single period cost function C_t.

In the following proposition, we show that the myopic policy can be characterized as the two-level base-stock policy, where the optimal order $\hat{z}_t$ is placed up to the first base-stock level $\hat{y}^M$, and the optimal order $\hat{v}_t$ is placed up to the second state-dependent base-stock level $\hat{w}(\hat{z})$. For clarity reasons we omit the subscript t in parts of the following text.

Proposition 5.2 *Under stationary stochastic demand and supply capacity, the following results hold for all t:*

1. *$f_t^M(x_t)$ is convex in x_t.*
2. *The optimal myopic policy is the two-level base-stock policy with the constant base-stock levels $\hat{y}^M$ and $\hat{w}^M(\hat{z})$, where:*
   $\hat{y}^M = \hat{y_t}^M$ is the solution to $\min_{x_t \leq y_t} C_t(y_t, z_t)$: $\hat{y}^M = G^{-1}\left(\frac{c_b}{c_b + c_h}\right)$, and
   $\hat{w}^M(\hat{z}) = \hat{w_{t-1}}^M(\hat{z}_{t-1})$ is the solution to $\min_{y_{t-1} \leq w_{t-1}} \alpha E_{Q_{t-1}, D_{t-1}} \min_{x_t \leq y_t} C_t(y_t, z_t)$.

In Proposition 5.3 we derive the properties of the state-dependent myopic base-stock level $\hat{w}^M(\hat{z})$. Part 1 suggests that for the pair of optimal orders $\hat{z} = \{\hat{z}_1, \hat{z}_2\}$ placed with the faster supply source in any period, the decision-maker has to raise the base-stock level $\hat{w}^M(\hat{z}_1)$ above $\hat{w}^M(\hat{z}_2)$, when $\hat{z}_1 \geq \hat{z}_2$. From this $\hat{v}^M(\hat{z}_1) \geq \hat{v}^M(\hat{z}_2)$ follows directly given the fact that $\hat{z}_1$ and $\hat{z}_2$ were placed up to the constant $\hat{y}^M$. With an increase in order $\hat{z}$, the probability of a supply shortage at the faster supply source increases. To compensate for this, it is optimal that a higher order $\hat{v}$ is placed with the slower supply source. In Part 2, we show that the level of compensation to account for the additional supply uncertainty (due to the higher $\hat{z}$ placed) should at most be equal to the difference between the optimal order sizes η.

Proposition 5.3 *The following holds for all t:*

1. *For any pair $\hat{z} = \{\hat{z}_1, \hat{z}_2\}$, where $\hat{z}_1 \geq \hat{z}_2$: $\hat{w}(\hat{z}_1) \geq \hat{w}(\hat{z}_2)$.*
2. *For $\eta \geq 0$: $\hat{w}(\widehat{z + \eta}) - \hat{w}(\hat{z}_t) \leq \eta$.*

Observe that the constant base-stock level $\hat{y}^M$ is the solution to the multiperiod uncapacitated single source inventory model. For the capacitated single supplier model, Ciarallo et al. (1994) show that while $\hat{y}^M$ optimizes a single period problem, it is far from optimal in a multiperiod setting. However, we show that in the dual sourcing model under consideration the appropriate combination of the two myopic base-stock levels provides a very good substitute for the optimal base-stock levels. In Table 5.2, we present the optimal and the myopic inventory positions. Numbers in bold are used to denote the cases, where the myopic inventory positions and orders differ from the optimal ones. The results for $\hat{y}(x)$ confirm that $\hat{y}$ is a function of x. We see that with increasing x, $\hat{y}(x)$ is increasing, approaching the myopic $\hat{y}^M$. For lower x, the optimal policy suggests that it is not optimal anymore to place $\hat{z}$ up to $\hat{y}^M$. This can be attributed to the increased uncertainty about the replenishment of $\hat{z}$. To compensate for the relatively smaller $\hat{z}$ placed, the optimal decision is to rely more heavily on the slower supplier, by increasing $\hat{v}$ above $\hat{v}^M$.

Table 5.2 The optimal and the myopic inventory positions and orders.

x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
$\hat{y}(x)$	14	14	14	14	15	15	15	15	15	15	15	15	15	16	16	16	16							
$\hat{y}^M$	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	16	16	16	16							
$\hat{z}$	14	13	12	11	11	10	9	8	7	6	5	4	3	3	2	1	0							
$\hat{z}^M$	**16**	**15**	**14**	**13**	**12**	**11**	**10**	**9**	**8**	**7**	**6**	**5**	**4**	3	2	1	0							
$\hat{w}(\hat{z})$	29	28	27	27	27	26	25	25	25	24	24	24	23	23	23	23	23	23	23	23	23	23	23	23
$\hat{w}^M(\hat{z}^M)$	**31**	**30**	**29**	**28**	27	**27**	**26**	25	25	**25**	24	24	**24**	23	23	23	23	23	23	23	23	23	23	23
$\hat{v}(\hat{z})$	15	14	13	13	12	11	10	10	10	9	9	9	8	7	7	7	7	6	5	4	3	2	1	0
$\hat{v}^M(\hat{z}^M)$	15	14	13	**12**	**11**	11	10	**9**	**9**	9	**8**	**8**	8	7	7	7	7	6	5	4	3	2	1	0

Despite the fact that the myopic base-stock levels generally differ from the optimal inventory positions, we now proceed to show that the costs of the myopic policy provide a nearly perfect estimate of the optimal costs. In Figure 5.4, we provide the histogram of the relative differences in costs of the optimal and myopic policy. The accuracy of the myopic policy is tested on 300 scenarios, where in almost 60% of the scenarios the costs of the myopic policy are equal to the optimal costs. In only 4 cases the myopic costs differed for around 0.3%, which is also the highest cost difference observed. A careful study of suboptimal scenarios has not revealed a clear pattern that would point out the characteristics of these scenarios. In general, the accuracy is lower when both the demand and supply capacity uncertainty are high, which is not unexpected.

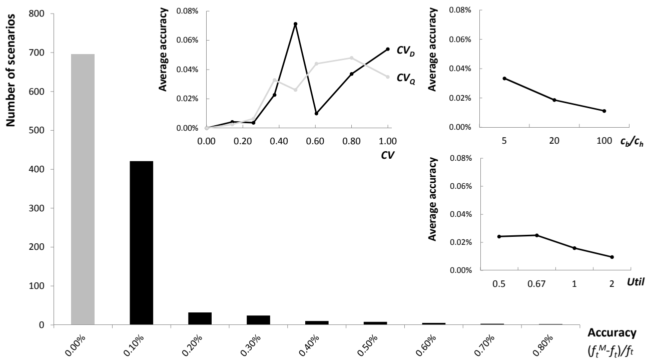


Figure 5.4 The relative difference in costs between the optimal and the myopic policy.

We conclude this section with the discussion on the optimality of the myopic policy. The high accuracy of the myopic policy might suggest that myopic policy could be optimal under certain assumptions. However, the policy does not exhibit the necessary properties of the myopic optimality. Heyman and Sobel (1984) point out the following two conditions (among others) for the myopic policy to be optimal: single period cost function needs to be additively separable on the state and action, and that the myopic policy guarantees that the set of consistent states (from which the optimal action can be taken) are visited in the next period. It is clear that the single period cost function C_t does not satisfy the first condition, as it does not depend additively on x_t and y_t. While for $\hat{w}^M$ it does hold that the optimal action will again be feasible in the next period, this is not the case for $\hat{y}^M$ as the system can easily end up in $x_t > \hat{y}^M$ from which the optimal action is infeasible. Despite the fact that the myopic base-stock levels differ from the optimal inventory positions after ordering, the myopic policy provides a nearly perfect estimate of the optimal costs. The accuracy can be attributed to the possibility of balancing the orders placed with the two supply sources, which in most situations leads to the optimal costs.

5.4 Value of dual sourcing

In this section, we present the results of the numerical analysis, where we investigate the effect of different system parameters on: (1) the optimal costs of dual sourcing; (2) the relative utilization of the two supply sources; and (3) the benefits of dual sourcing compared to single sourcing from either supply source.

Numerical calculations were carried out by solving the dynamic programming formulation given in (5.1). We used the following set of input parameters: $T = 12$, $c_h = 1$, $\alpha = 0.99$, and a discrete uniform distribution to model stochastic demand and supply capacity. Throughout the experiments we varied the utilization[2] of the faster supply source, $Util = (0, 0.5, 0.67, 1, 2, \infty)$, the per unit backorder cost $c_b = \{5, 20, 100\}$, and the coefficient of variation of demand, $CV_D = \{0, 0.14, 0.26, 0.37, 0.49, 0.61\}$, and the supply capacity of the faster supply source, $CV_Q = \{0, 0.14, 0.26, 0.37, 0.49, 0.61\}$, where the CVs do not change over time[3].

5.4.1 Optimal costs of dual sourcing

We start by studying the optimal costs under different system parameters and by comparing the performance of the dual sourcing model with the two base cases: the worst case in which

[2] Defined as the expected demand over the available capacity of the faster supply source.

[3] We give the approximate average CVs for demand and supply capacity distributions since it is impossible to come up with the exact same CVs for discrete uniform distributions with different means.

the supply is only available through the slower supply source (the faster supply source is unavailable, $Util = \infty$), and the best case in which the faster supply source is assumed to be uncapacitated, and thus becomes fully available ($Util = 0$). The results are presented in Figure 5.5 and Table 5.3.

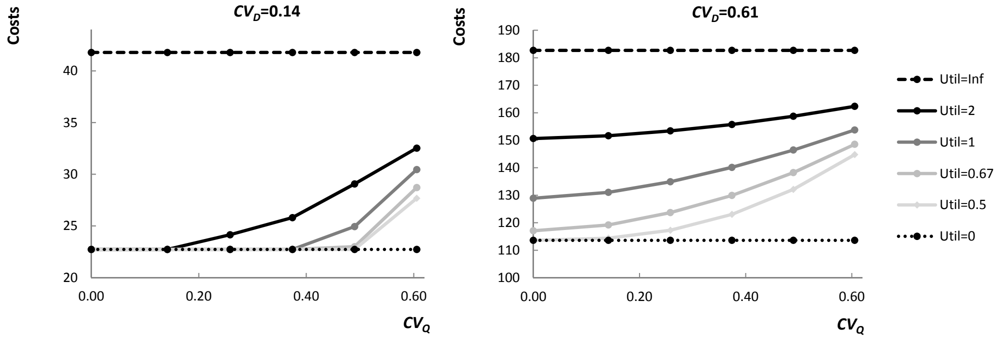


Figure 5.5 Optimal system costs.

As expected, the system costs rise with increasing demand and capacity uncertainty, as well as with increasing utilization of the faster supply source. The sensitivity of costs is highest in the case of increasing CV_D. Moreover, it is at a high CV_D where the costs are the most sensitive to an increase in CV_Q and $Util$. For relatively low CV_D and up to moderate CV_Q, the system works with minimal costs even for the cases of high utilization. In this setting, the capacity and demand realizations are easy to anticipate, and sourcing from the slower supply source to compensate for the lack of supply capacity availability with the faster supply source does not increase the costs compared to the best case. However, for a high CV_D, due to the longer lead time, ordering with the slower supply source becomes riskier as the exposure to demand uncertainty increases. This leads to a relatively high increase in costs already for moderate utilizations.

5.4.2 Relative utilization of the two supply sources

To investigate to what extent either of the two supply sources is utilized under different system parameters, we present the calculations for the share of inventories replenished from a faster capacitated supply source in Figure 5.6. The higher the CV_Q and $Util$, the lower the share of inventories replenished from the faster supply source. A high CV_Q leads to more probable shortages in replenishment from the faster source and there is thus a greater need to compensate for these shortages by placing a bigger share of orders with the slower source. However, as an order with the slower supply source needs to be placed one period earlier, one cannot take advantage of learning about the demand realization in the current period. The accuracy of this compensation strategy suffers when CV_D is increasing, which is reflected in a high increase in costs.

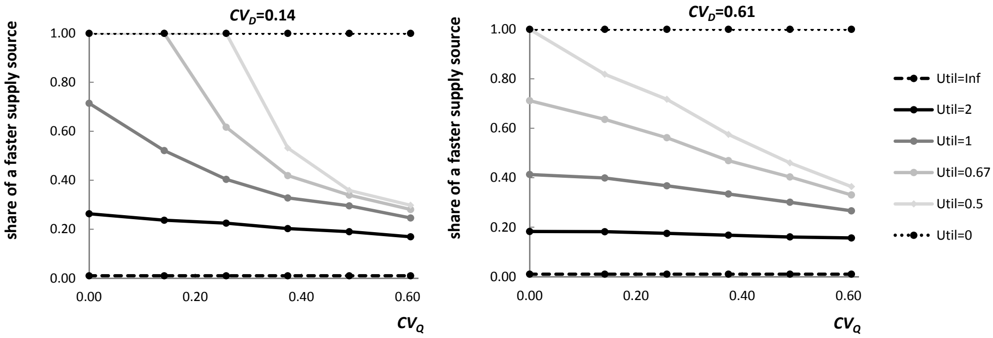


Figure 5.6 The share of inventories replenished from a faster capacitated supply source.

Although utilization of the faster supply source represents a hard limit on the size of possible replenishment, the actual utilization falls well below its maximum (theoretical) availability. For instance, for $Util = 1$, the share of inventories sourced from the faster source is at 71%, and decreases further to 41% as demand uncertainty increases (even when $CV_Q = 0$). Still, observe that up to moderate $Util$ and low CV_Q the sourcing is done almost exclusively through the faster source (100% for $CV_D = 0.14$), which suggests that the faster supply source is actually sufficiently available in this setting. While we would expect increasing demand uncertainty to make sourcing from the slower supply source less favorable, somewhat counter-intuitively the opposite holds when CV_Q is relatively low. In a setting with high CV_D more safety stock is required to avoid backorders. To keep inventories at this relatively higher level, occasionally a large order with the faster supply source needs to be placed as a response to the high demand realization (which is more probable due to the high CV_D). However, this might lead to a supply shortage even for relatively low system utilizations, and consequently to costly backorders. It therefore makes sense to replenish some inventories from the slower supply source, so that we start the next period with a higher starting inventory position. While this can lead to higher inventory holding costs, it is still less costly than incurring backorder costs.

Next, we study the effect of the cost structure on the actual utilization of the two supply sources. When backorder costs increase relative to holding costs, increasing the share of sourcing from the slower supply source is optimal (Figure 5.7). This is in line with the above reasoning where, through sourcing from the slower supplier, one can avoid backorders while incurring higher inventory holding costs instead. The share of inventories replenished from a slower supplier also increases relatively more for higher CV_Q, particularly when CV_D is also high, as the need for a more reliable supply source increases. This effect is higher for lower utilizations, which can be attributed to the fact that at high $Util$ ordering through the slower source is already extensively used (around 80% of inventories is replenished from

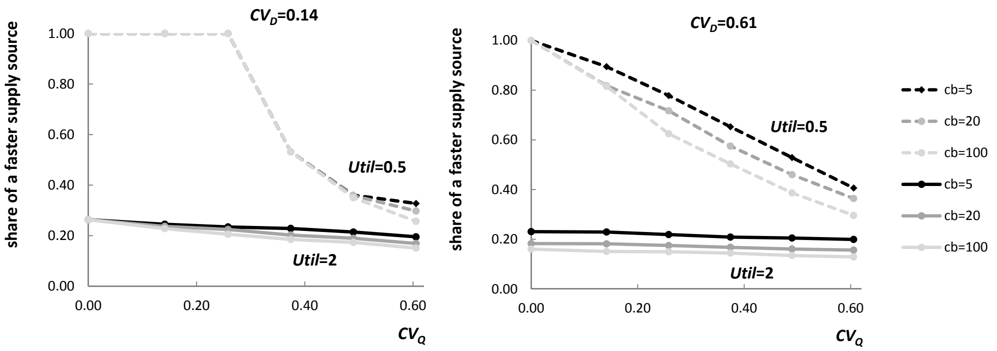


Figure 5.7 The share of inventories replenished from the faster capacitated supply source.

the slower source) and despite the increased backordering costs, increasing the exposure to the slower supply source further only marginally increases the benefits.

5.4.3 Benefits of dual sourcing

Lastly, the benefits of dual sourcing are assessed relative to the performance of the two single sourcing cases. To quantify the benefits of dual sourcing, we define the *relative value* of dual sourcing, $\%V_{DS}$, as the relative cost savings over a single-source setting in which either a faster (FS) or slower (SS) supply source is used:

$$\%V_{DS/\{FS,SS\}} = \frac{f_t^{\{FS,SS\}} - f_t^{DS}}{f_t^{\{FS,SS\}}}, \tag{5.7}$$

where f_t^{FS}, f_t^{SS} and f_t^{DS} represent the corresponding cost functions from (5.1) that apply to a chosen decision policy.

We present the results on the relative value of dual sourcing compared to single sourcing in Table 5.3. In the table, numbers in bold are used to denote the single supply source with the lower costs in each of the settings.

Looking at the benefits of dual sourcing over sourcing from a faster capacitated supply source, it is expected that its utilization largely influences the extent of the savings that can be achieved through the dual sourcing. For an overutilized system, $\%V_{DS/FS}$ approaches 100%, while for moderate utilizations the relative savings still range from 20% to over 50%. The lack of availability and reliability of supply from a faster source inherently exposes the system to supply shortages, and leads to substantial backorder costs. For up to moderate $Util$ with high CV_Q, the average supply availability is sufficient, although the system is exposed to supply shortages due to the highly variable supply capacity of the faster supply source. This again results in considerable benefits from utilizing the slower uncapacitated

Table 5.3 The optimal costs and the relative value of dual sourcing.

			Costs (f_t^{DS})				$\%V_{DS/FS}$				$\%V_{DS/SS}$			
Util	CV_Q	CV_D	0.00	0.14	0.37	0.61	0.00	0.14	0.37	0.61	0.00	0.14	0.37	0.61
∞			0.0	41.8	112.1	182.7	100.0	93.8	85.3	79.1	-	**0.0**	**0.0**	**0.0**
2	0.00		0.0	22.7	82.9	150.6	100.0	93.6	81.6	73.0	-	**45.6**	**26.1**	**17.6**
2	0.14		0.0	22.7	84.1	151.6	100.0	93.8	81.5	72.9	-	**45.6**	**25.0**	**17.0**
2	0.37		0.0	25.8	89.5	155.7	100.0	93.4	80.9	72.7	-	**38.2**	**20.1**	**14.8**
2	0.61		0.0	32.5	96.8	162.3	100.0	92.4	80.4	72.5	-	**22.2**	**13.6**	**11.1**
1	0.00		0.0	22.7	69.5	128.9	-	66.8	62.0	56.5	-	**45.6**	**38.1**	**29.4**
1	0.14		0.0	22.7	71.3	131.1	100.0	75.7	63.3	56.8	-	**45.6**	**36.5**	**28.3**
1	0.37		0.0	22.7	78.3	140.1	100.0	87.2	68.5	59.1	-	**45.6**	**30.2**	**23.3**
1	0.61		0.0	30.5	91.1	153.7	100.0	88.8	71.9	61.9	-	**27.1**	**18.8**	**15.8**
0.67	0.00		0.0	22.7	68.2	117.1	-	**0.0**	**1.0**	**15.3**	-	45.6	39.2	35.9
0.67	0.14		0.0	22.7	68.2	119.2	-	**0.0**	**8.5**	**19.6**	-	45.6	39.2	34.8
0.67	0.37		0.0	22.7	72.2	129.9	100.0	68.7	44.8	36.5	-	**45.6**	**35.6**	**28.9**
0.67	0.61		0.0	28.7	87.6	148.5	100.0	84.7	62.1	49.8	-	**31.3**	**21.9**	**18.7**
0.5	0.00		0.0	22.7	68.2	113.6	-	**0.0**	**0.0**	**0.0**	-	45.6	39.2	37.8
0.5	0.14		0.0	22.7	68.2	114.4	-	**0.0**	**0.0**	**0.2**	-	45.6	39.2	37.4
0.5	0.37		0.0	22.7	69.4	123.1	100.0	**37.1**	**20.5**	**16.5**	-	45.6	38.1	32.6
0.5	0.61		0.0	27.7	85.2	144.8	100.0	81.4	54.7	40.7	-	**33.8**	**24.0**	**20.7**
0			0.0	22.7	68.2	113.6	0.0	0.0	0.0	0.0	-	45.6	39.2	37.8

supply source as an alternative. It is the settings with up to moderate *Util* and low CV_Q where supply solely through the faster supply source is adequate to run the system at close to minimum costs.

Sourcing exclusively through the slower supply source is optimal in the setting where there is no demand uncertainty (for $CV_D = 0$, the cases denoted with "-" represent the settings in which the costs of using a single supply mode are zero), and obviously when the faster supply source is completely unavailable ($Util = \infty$). For the remaining settings, we observe that $\%V_{DS/SS}$ ranges from 15% to 45%, and is less sensitive to the changes in *Util* and CV_Q than in the case of sourcing solely through the faster supply source, as expected. While utilizing the slower supplier successfully resolves the CV_Q and *Util* related problems in the settings where the faster supplier's replenishment is inadequate (when $\%V_{DS/FS}$ is high), we see that considerable savings are still to be gained by partially sourcing through the faster supply source.

As already noted, demand uncertainty negatively affects the accuracy of using the slower yet reliable supply source to compensate for potential supply shortages of the faster source. It is therefore somewhat surprising that $\%V_{DS/SS}$ are decreasing with increasing CV_D. However, the opposite holds when looking at the absolute cost savings. These are increasing considerably with an increase of CV_D, particularly for up to moderate $Util$ and low CV_Q. This is in-line with our previous observations where such a set of parameter values characterizes the settings in which sourcing is performed almost exclusively through the faster supply source. $\%V_{DS/FS}$ is in general also decreasing with increasing CV_D, which is in agreement with our initial observation that a slower supply source is inherently more exposed to the risk of demand uncertainty. Here the absolute cost savings exhibit non-monotonic behavior with increasing CV_D. For settings with high CV_Q the slower reliable source is already extensively used and the absolute savings are decreasing with CV_D (more so for lower $Util$). However, this is not the case for low CV_Q (particularly at moderate $Util$), where the absolute savings are increasing with CV_D. This is the setting we already addressed in which limited sourcing from the slower supply source is beneficial for tackling instances of supply unavailability due to the placement of big orders in response to large demand variations. Obviously, the absolute cost reduction is significant in the case where we need to compensate for a general lack of supply availability (high $Util$ and high CV_Q), and smaller when the system only faces occasional supply shortages due to large demand uncertainty.

Finally, we may conclude with the observation that, besides the setting in which the faster supply source is close to being fully available and essentially reliable, the relative cost savings are considerable, ranging from just above 10% to close to 50%. Based on these results, we can conclude that dual sourcing can successfully resolve the issues related to an overutilized and uncertain availability of a faster supply source. In addition, a smart decision on how to split the orders between the two supply sources depending on the system setting, also substantially reduces the costs of demand uncertainty.

5.5 Value of advance capacity information

In this section, we study a situation in which the faster supplier provides an upfront information on exact future supply capacity availability. This information is modeled in the same way as in Jakšič et al. (2011) in a single supplier setting (Chapter 3). When ACI is available, the supply capacity availability is known prior to placing the order with the two suppliers, either as ACI on current period's capacity or on capacity of future periods.

Observe that due to the near-optimality of the myopic policy, it is sufficient to obtain the information on current period's capacity and next period's capacity, while further future insight into supply capacity availability of the faster supplier will not bring additional benefits. Due to the myopic nature of the policy it is sufficient to resolve the supply capacity uncertainty for the time span in which first the order with the slower supply source, followed by the order with the faster supply source in the next period.

We present the results of the numerical analysis, which was carried out to confirm the above insight and determine the value of sharing ACI of the faster supply channel. We show that ACI can effectively help the company to reduce the negative effects of the uncertain replenishment from the faster supply channel.

In Figure 5.8 we present the cost comparison between the base setting without ACI and the settings in which ACI is available for the current and for the future period, for different utilizations and levels of supply capacity uncertainty of the faster supply source. The cost curve depicted as $Util = 0$ represents the scenario with the lowest costs, where ordering is done solely with the faster supplier with infinite capacity. The worst case scenario is the case where the faster supplier has no capacity and all orders are placed to with the slower supplier, and is depicted with a $Util = Inf$ cost curve. Our interest lies in studying the intermediate scenarios, where we are predominantly interested in the extent of visibility needed to obtain the majority of the benefits of ACI, and the effect the supply capacity uncertainty CV_Q has on the costs under both, the situation without and with ACI.

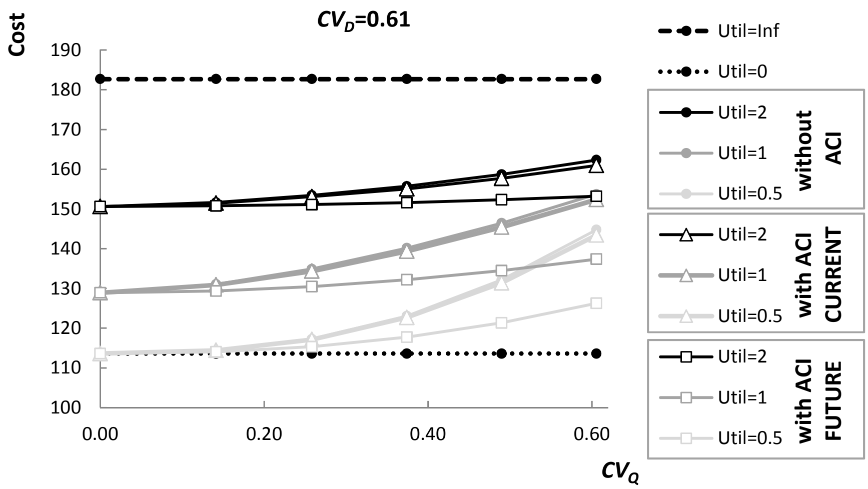


Figure 5.8 System costs under ACI for different unreliable supply channel utilizations.

In Chapter 3, we have shown that generally the additional benefit of extending the supply capacity visibility diminishes with revealing ACI for future periods. However, we quickly see that this is not the case in a dual sourcing model under consideration. The value of ACI, defined as the relative difference in costs between a situation without and with ACI, is up to 1.5% in the case of ACI on current period's capacity. For the case of future ACI availability the cost savings become considerable, approaching 20%. Thus, we only get the majority of the benefits if we extend the information horizon to the following period. Observe first that ACI will not affect the order placed with the faster supplier as it is always placed up to the myopic base-stock level. It is the possibility that the order with the slower supplier can be modified in accordance to ACI, which results in reduced costs. Relatively small benefits obtained through learning only about the current supply capacity availability, can be explained by the fact that with the order with the slower supplier one can only influence the inventory levels in the following period. While unfavourable ACI may result in the capacity shortage already in the current period, the order with the slower supplier can only be used to influence the costs in the following period. However, in the case of future ACI the capacity shortage in the following period can be anticipated and the order with the slower supplier can be increased accordingly.

Looking at the effect of supply capacity uncertainty of the faster supplier, the costs of the situation without and with ACI are equal for $CV_Q = 0$. It is expected that the reduction in costs through ACI is increasing when CV_Q is increasing, as ACI effectively reduces the uncertainty of the supply capacity availability. The largest savings are attained at settings with high CV_Q, no matter what the utilization of the faster supplier is.

While the supply capacity uncertainty of the faster supplier has the predominant effect on the benefits of ACI, the demand uncertainty also influences the possible savings attained through ACI. In the case of a highly utilized faster supplier, the slower supplier is used more heavily. While ACI helps the decision maker to come up with a better ordering decision with the slower supplier, the delayed replenishment together with demand uncertainty results in high costs due to demand and supply mismatches, thus the value of ACI diminishes. This effect is lower for the case of low utilization of the faster supplier. Here ACI is helpful in covering the mismatches mainly through the faster supplier, therefore the exposure to the demand uncertainty risk faced by a supply through the slower supplier is lower.

5.6 Summary

In this chapter, we analyze a sourcing situation in which both sourcing from a nearshore and offshore supply source may be used. As the product purchase costs of sourcing from the two sources are more or less similar in various industry sectors nowadays, we argue that the trade-off is leaning towards finding the right balance between the limited supply availability of the nearshore supply options and the longer lead times of offshore sourcing.

Accordingly, we model a dual-sourcing inventory model with a stochastic capacitated, zero-lead-time nearshore supplier and an uncapacitated offshore supplier with a longer lead time. We derive the dynamic programming formulation for the optimal inventory holding and backorder costs, where we show that the structure of the optimal inventory policy is not a two-level base-stock policy as it is the case in both unlimited and fixed capacity dual sourcing models studied in the literature. The optimal inventory position to which we order by placing an order with the stochastic capacitated faster supplier depends on the inventory position before ordering. To reduce the complexity of the policy we study the myopic policy, which proves to be a two-level base-stock policy, where the first myopic base-stock level corresponds to the solution of the single period newsvendor problem. We show that the myopic policy is not optimal, however it still provides a very accurate approximation of the optimal costs.

The numerical analysis reveals several managerial insights. The results show that the costs increase substantially when the capacity of the faster supply source is stochastic (already for moderate utilization of the faster supply source). The extent to which we rely on the faster supply source will therefore depend on the trade-off between its current availability and the benefits we obtain by delaying the order decision, and thus taking advantage of the latest demand information. However, somewhat counter-intuitively, we observe that in some settings the increasing demand uncertainty leads to an increase in sourcing from the slower supplier. We attribute this to the high costs related to incurring backorders, which results in a strategy primarily oriented to avoiding supply shortages, namely a strategy that instructs the decision-maker to prebuild inventories by utilizing the slower supply source.

In addition, we show that the relative cost savings of dual sourcing over single sourcing range from 10% to close to 50%. Exceptions to this are cases where the faster supply source is only moderately utilized and the demand uncertainty is low. This shows that, despite the lack of a price incentive for placing orders with the slower offshore supplier (which is a common assumption in the literature), the offshore supply option is used to obtain considerable operational cost savings. Given the limitations and uncertainties that the move towards nearshoring brings in terms of supply availability, we show that a mixed strategy which allocates the orders between the two sourcing options is optimal.

Our results provide a potential justification for continuing to use offshore sources following the commencement of nearshoring operations, despite the fact that the costs are similar and the lead times are longer. Managers will need to take the capacity flexibility of the nearshore option into account when moving an operation nearshore.

Future research could relax the assumptions about the two delivery lead times. However, assuming a non-zero-lead time of the faster supply source greatly increases the complexity of the model as the inventory position becomes a function of multiple uncertain pipeline orders. Accordingly, also the myopic policy could be very complex (state-dependent first base-stock level) and probably less accurate, even in the case where the lead times of the two supply sources differ by one period.

Chapter 6

Conclusions

We now present the conclusions of the research presented in this monograph. We provide answers to the relevant research questions formulated in Section 1.4. Note, that this is a general overview of the insights we have gained throughout our study, as the individual model's specifics have already been discussed extensively in conclusions of each of the chapters.

The work presented in this monograph is aimed at giving insights into managing inventories in a stochastic demand and limited stochastic supply capacity setting. We have developed four inventory models with different information sharing and supply options aimed at improving the inventory system's performance.

The theoretical contribution of the monograph revolves around the first two relevant research questions:

Q1. *How can we incorporate supply capacity information and alternative supply options into the underlying stochastic capacitated inventory model?*

The road towards developing the proposed inventory models started through derivation of a discrete model formulation, identification of the system control variables, description of the state space, and representation of the evolution of the system by giving the relevant functional equations. In the case of ACI and ASI models in Chapters 3 and 4, the state space needs to be expanded to incorporate supply capacity information. In the case of ACI model, the state of the system is described by the starting inventory position and the information on supply capacity availability in future periods. In the case of ASI model, advance replenishment information is used to correct the inventory position in the case of supply shortage. For the remaining pipeline orders the replenishment remains uncertain, which needs to be accounted for by keeping track of these orders in the state space description. In the dual sourcing model in Chapter 5 the orders to the two supply sources are interdependent. Apart from ACI model, the inventory position before replenishment is uncertain in the remaining two models, which presents a difficulty in terms of estimating the resulting on-hand stock availability to cover demand. This is subsequently reflected in increased computational complexity associated with the quantification of the relevant inventory costs. Finally, we derive a dynamic programming formulation for the finite horizon optimal discounted expected inventory costs.

Q2. *What is the structure of the optimal ordering policy and can we derive its properties?*

By studying the convexity properties of the relevant cost functions, we are able to characterize the structure of the optimal policy for all the models under study. We first show that in the case of ACI model in Chapter 3 the auxiliary cost function is convex, thus there exists a single minimizer that minimizes the system costs. We show that the optimal inventory position after ordering does not depend on the inventory position before ordering, which characterizes the base-stock policy. We show that the optimal policy is the modified base-stock policy with a single base-stock level, which is a function of ACI, where the order in each period is placed up to the base-stock level if sufficient supply capacity is available.

The ASI model in Chapter 4 is an extension of the underlying stochastic inventory model studied in Ciarallo et al. (1994). Despite the fact that the single period cost function (and consequently the multiperiod cost functions) exhibits the quasiconvex shape, we show that the minimum costs are attained at the single state-dependent inventory position. In line with this, we show that the optimal policy is the base-stock policy, where the optimal base-stock level depends on the remaining uncertain pipeline orders. For both advance information models we derive additional structural properties that describe the state dependency of the optimal policy parameters. Favourable information about future supply capacity availability decreases the optimal base-stock level in the ACI case. In the case of ASI, the exposure to the potential supply shortages depends on the size of the remaining uncertain pipeline orders. Thus, the optimal base-stock level needs to be increased in the case when higher orders were placed in the recent past.

The properties needed for the optimality of the base-stock policy cannot be proven for the dual sourcing model in Chapter 5. One would expect that the optimal policy would be a two-level base-stock policy observed in the uncapacitated case and the case with fixed capacity. We show that the inventory position after placing the order with the faster supplier in fact depends on the inventory position before ordering, and thus does not behave as a base-stock policy. The inventory position after placing the order with the slower supplier can be characterized as the secondary base stock level, however it is state-dependent on the order placed with the faster supplier. Study of the myopic policy showed that it accurately approximates the cost of the optimal policy despite the simplified structure of the policy. Myopic policy is a two-level base-stock policy, where the first base-stock level corresponds to the newsvendor base-stock level of an uncapacitated single period inventory problem. Even in the cases where the target inventory positions of the optimal policy and the myopic policy differ a lot, we observe nearly perfect accuracy of the myopic policy.

In general, it holds that for an effective inventory control it is not practical to derive the optimal action for each of the possible system states, but rather an effort should be put in finding an optimal policy, a structured rule that describes the optimal behavior. However, while we have shown that the optimal policies have a relatively simple structure, the calculation of the optimal base-stock levels remains a difficult task. The reason for this is twofold. Firstly, due to the state-dependency, the base-stock level generally needs to be determined for each system state. The second reason is that due to the complex state-description, and the stochastic nature of both the demand and supply process, the problem complexity becomes large and eventually too large to enable the evaluation of the optimal costs, the problem commonly referred to as *"Curse of dimensionality"* (Puterman, 1994). This greatly reduces the likelihood that a realistic inventory problem can be solved. It turns out that the numerical analysis of the optimal policies is not computationally efficient and even not feasible for larger problems. At the same time, we show that oversimplification and disregarding the stochastic nature of the model leads to poor results. While, these observations may limit the applicability of the optimal policy in a practical setting, we now proceed to review the managerial insights gained through the numerical analysis of the proposed inventory policies.

By providing answers to the second group of research questions we aim to reveal the extent of the possible gains obtained through integrating information or alternative supply options into the inventory system with uncertain capacity. As the underlying model is inherently difficult to manage, both due to the limited as well as uncertain supply availability, we show that the potential for the cost reduction is considerable. This should motivate the companies to explore the insights gained through the study of the proposed four inventory models, and correspondingly adapt their existing inventory control policies.

Q3. *What is the value of supply capacity information and alternative supply options in terms of inventory cost reduction?*

Foreknowledge of future uncertain events is always useful in managing an inventory system. We argued that through advance information about supply availability one can better anticipate the potential mismatches between demand and supply capacity in the future. This anticipation of future supply shortages is beneficial to make timely and correct ordering decisions, which result in either building up stock to prevent future stockouts, or reducing the stock in the case the supply conditions in the future might be favorable. Thus, system costs can be reduced by carrying less safety stock while still achieving the same level of performance. In the case of ACI model in Chapter 3, we observe the reduction of inventory costs of up to 30% and more, particularly for certain settings in the special case of "zero-full" supply capacity availability in Section 3.5. The cost savings in the case of ASI model in Chapter 4 are lower, going up to around 10%. This is expected as the supply capacity visibility is limited to the orders already placed, and therefore only allows the decision maker to react to the supply shortage revealed through ASI.

While advance information will not increase the supply availability, it improves either the anticipation of the future supply shortages or enables a more timely reaction to inadequate replenishment of pipeline orders. The idea behind the dual sourcing model in Chapter 5 is that the exposure to the supply shortages of the primary supply channel can be reduced by relying to alternative supply option. The option to place a part of the order with the slower reliable supplier reduces the costs substantially compared to sourcing solely from the stochastic capacitated supplier, as well as relying fully to the slower supplier.

To further elaborate the setting in which the proposed inventory policies provide the majority of the benefits, we review the last research question:

Q4. *Which are the determining factors of the magnitude of the expected cost benefits, or more specifically, what is the influence of the relevant system parameters?*

The numerical study of the value of information sharing and alternative supply options revolves around analyzing the effect of the relevant system parameters on the inventory costs: utilization of the primary stochastic capacitated supply channel, the level of demand and supply capacity uncertainty, and the ratio between the inventory holding and the backorder costs. We show that the utilization has the predominant effect on the inventory cost reduction. A highly utilized or even overutilized system is in most of the settings the primary candidate in which one should try to take advantage of the proposed supply alternatives.

In general, we observe that the remaining parameters effect the inventory cost reduction in a complex way. Due to the interaction between the multiple parameters, we show that each cannot be treated independently and therefore need to be considered in an integrated manner. In the case of ACI in ASI models, interesting conclusions were drawn based on the analysis of the effect of changes in uncertainty of demand and supply capacity. Increasing the uncertainty of demand caused a decrease in the value of advance information. This can be attributed to the fact that advance information is of limited use when the system gets more and more chaotic due to increased demand variability. However the opposite is true when supply capacity becomes more stochastic. We observe that the value of advance information significantly increases. We attributed this to the fact that when one has to deal with highly variable and uncertain supply capacity availability every bit of information on the capacity realization helps in reducing the uncertainty. In the case of the alternative supply option, the relationship between the demand and supply capacity uncertainty and the inventory cost reduction is nontrivial. We show that a hybrid strategy of employing both the primary uncertain supplier and the alternative slower reliable supplier needs to be implemented. The share of the order placed with each of the suppliers will heavily depend on the particular system characteristics.

By showing the potential gains that can be made by incorporating advance information and alternative supply options, we provide an incentive to companies to more effectively incorporate already available information into their inventory policies, establish the necessary mechanisms that would facilitate the additional information exchange, and proactively look for the possible alternative supply channels to improve the supply availability.

Appendix A

Chapter 3 Proofs

Preliminaries for Theorem 3.1

Lemma A.1 *Let $b \in B$, where B is a convex set, and $y \in Y(b)$, where $Y(b)$ is a nonempty set for every b, assume that $g(y,e)$ is convex in y and e. If $f(b,e) := \min_{Ay \leq b} g(y,e)$ and $f(b,e) \geq -\infty$ for every b and e, then f is also convex in b and e.*

Proof: Let $0 \leq \theta \leq 1$. Then

$$\begin{aligned}
\theta f(b,e) + (1-\theta) f(\bar{b}, \bar{e}) &= \theta \min_{Ay \leq b} g(y,e) + (1-\theta) \min_{Ay \leq \bar{b}} g(y, \bar{e}) \\
&= \theta g(y_0^*, e) + (1-\theta) g(y_1^*, \bar{e}) \\
&\geq g(\theta(y_0^*, e) + (1-\theta)(y_1^*, \bar{e})) && \text{(A.1)} \\
&\geq \min_{Ay \leq \theta b + (1-\theta)\bar{b}} g(y, \theta e + (1-\theta)\bar{e}) && \text{(A.2)} \\
&= f(\theta(b,e) + (1-\theta)(\bar{b}, \bar{e}))
\end{aligned}$$

(A.1) is due to convexity of g and (A.2) is because $Ay_0^* \leq b$ and $Ay_1^* \leq \bar{b}$, which implies $A(\theta y_0^* + (1-\theta) y_1^*) \leq \theta b + (1-\theta)\bar{b}$. □

Lemma A.2 *If $J(y,e)$ is convex in y and e, then $f(x,q,e) = \min_{x \leq y \leq x+q} J(y,e)$ is convex in x.*

Proof: Let $h(b,e) := \min_{Ay \leq b} J(y,e)$ where $A = [-1, 1]$ and $b = [-x, x+q]$. By Lemma A.1, we conclude that $h(b,e)$ is convex in b and e. Let $b = x + z$, where $0 \leq z \leq q$ for any q, then we have $h(b,e) = f(x,q,e)$ and since this is affine mapping, f is also convex in x. □

Proof of Theorem 3.1: The theorem can be proven by regular inductive arguments and the results of Lemmas A.1 and A.2. □

Proof of Theorem 3.1: Convexity results of Theorem 3.1 directly imply the proposed structure of the optimal policy. □

Proof of Theorem 3.2: In the case with no ACI, the decision maker has to decide for the order size without knowing what the available supply capacity for the current period will be. Inventory position after ordering y_t only gets updated after the order is actually received, where it can happen that the actual delivery size is less than the order size due to the limited supply capacity. The single-period cost $c(x,z)$ for $q \geq 0$ is:

$$\begin{aligned} c(x,z) &= (1-R(z))\left[b\int_{x+z}^{\infty}(d-x-z)g(d)\mathrm{d}d + h\int_{0}^{x+z}(x+z-d)g(d)\mathrm{d}d\right] \\ &+ b\int_{0}^{z}\int_{x+q}^{\infty}(d-x-q)g(d)\mathrm{d}d\, r(q)\mathrm{d}q \\ &+ h\int_{0}^{z}\int_{0}^{x+q}(x+q-d)g(d)\mathrm{d}d\, r(q)\mathrm{d}q. \end{aligned} \tag{A.3}$$

The first line in (A.3) represents the case where the capacity q is higher than the order size z, which means that the capacity is not limiting the order placed. While the rest accounts for the case of the capacity constrained system, where the order size is reduced due to limited supply capacity. We proceed by writing the optimal discounted expected cost function from period t to T:

$$\begin{aligned} H_t(x_t,z_t) &= c_t(x_t,z_t) + \alpha(1-R_t(z_t))\int_{0}^{\infty}H_{t+1}(x_t+z_t-d_t)g(d_t)\mathrm{d}d_t \\ &+ \alpha\int_{0}^{z_t}\int_{0}^{\infty}H_{t+1}(x_t+q_t-d_t)g(d_t)\mathrm{d}d_t\, r_t(q)\mathrm{d}q, \text{ for } 1 \leq t \leq T \end{aligned} \tag{A.4}$$

where $H_{T+1}(\cdot) \equiv 0$. The first-order condition for optimality is obtained by taking first partial derivative of (A.4), where z_t^* is the optimal order size in period t:

$$(1-R_t(z_t^*))\left((h+b)\left(G(x_t+z_t^*) - \frac{b}{h+b}\right) + \alpha\int_{0}^{\infty}{H_{t+1}^{*}}'(x_t+z_t^*-d_t)g(d_t)\mathrm{d}d_t\right) \equiv 0. \tag{A.5}$$

The optimal ordering policy is a base-stock policy with the optimal base-stock level y_t^*.

We are now ready to show the equivalence of the two models. By using (3.5) and (A.7) we write the optimal discounted expected cost function f_t for $n=0$ as:

$$f_t(x_t,q_t) = \begin{cases} C_t(x_t) + \alpha E_{D_t,Q_{t+1}}f_{t+1}(x_t-D_t), & \hat{y}_t \leq x_t, \\ C_t(\hat{y}_t) + \alpha E_{D_t,Q_{t+1}}f_{t+1}(\hat{y}_t-D_t), & \hat{y}_t - q_t \leq x_t < \hat{y}_t, \\ C_t(x_t+q_t) + \alpha E_{D_t,Q_{t+1}}f_{t+1}(x_t+q_t-D_t), & x_t < \hat{y}_t - q_t, \end{cases} \tag{A.6}$$

Assuming that the single-period cost function $C_t(y_t)$ and the optimal discounted cost function $f_t(x_t,q_t)$ are differentiable, we write the following three possible cases:

Case 1: When $\hat{y}_t \leq x_t$,

$$\begin{aligned} f_t(x_t,q_t) &= E_{D_t}[b(x_t-d_t)^- + h(x_t-d_t)^+] + \alpha E_{D_t}f_{t+1}(x_t-d_t) \\ &= b\int_{x_t}^{\infty}(d_t-x_t)g_t(d_t)\mathrm{d}d_t + h\int_{0}^{x_t}(x_t-d_t)g_t(d_t)\mathrm{d}d_t + \alpha\int_{0}^{\infty}f_{t+1}(x_t-d_t)g_t(d_t)\mathrm{d}d_t. \end{aligned}$$

Case 2: When $\hat{y}_t - q_t \le x_t < \hat{y}_t$,

$$\begin{aligned}
f_t(x_t, q_t) &= \int_{\hat{y}_t - x_t}^{\infty} \left(E_{D_t}[b(\hat{y}_t - d_t)^- + h(\hat{y}_t - d_t)^+] + \alpha E_{D_t} f_{t+1}(\hat{y} - d_t)\right) r_t(q_t) \mathrm{d}q_t \\
&= (1 - R_t(\hat{z}_t)) \left[b \int_{x_t + \hat{z}_t}^{\infty} (d_t - x_t - \hat{z}_t) g_t(d_t) \mathrm{d}d_t + h \int_0^{x_t + \hat{z}_t} (x_t + \hat{z}_t - d_t) g_t(d_t) \mathrm{d}d_t \right] \\
&+ \alpha (1 - R_t(\hat{z}_t)) \int_0^{\infty} f_{t+1}(x_t + \hat{z}_t - d_t) g_t(d_t) \mathrm{d}d_t.
\end{aligned}$$

Case 3: When $x_t < \hat{y}_t - q_t$,

$$\begin{aligned}
f_t(x_t, q_t) &= \int_0^{\hat{y}_t - x_t} \left(E_{D_t}[b(x_t + q_t - d_t)^- + h(x_t + q_t - d_t)^+] + \alpha E_{D_t} f_{t+1}(x_t + q_t - d_t)\right) r_t(q_t) \mathrm{d}q_t \\
&= \int_0^{\hat{z}_t} \left[b \int_{x_t + q_t}^{\infty} (d_t - x_t - q_t) g_t(d_t) \mathrm{d}d_t + h \int_0^{x_t + q_t} (x_t + q_t - d_t) g_t(d_t) \mathrm{d}d_t \right] r_t(q_t) \mathrm{d}q_t \\
&+ \alpha \int_0^{\hat{z}_t} \int_0^{\infty} f_{t+1}(x_t + q_t - d_t) g_t(d_t) \mathrm{d}d_t.
\end{aligned}$$

The proof is by backward induction starting in time period T. Observe that for period T, $y_T^* = \hat{y}_T$ holds, since both optimal base-stock levels, since both optimal order sizes are solutions of the same single-period newsvendor problem (Ciarallo et al. (1994), Proposition 1). Based on this we conclude that for period T the equivalence of the models holds obviously for Case 1 and if we sum up the Cases 2 and 3 ($x_T < \hat{y}_T$), we see the sum is equivalent to the definition of $c_T(x_T, \hat{z}_T)$ given in (A.3). From here it holds $f_T(x_T) = H_T(x_T)$.

Assuming that $f_{t+1}(x_{t+1}) = H_{t+1}(x_{t+1})$. The optimal base-stock level $\hat{y}_t$ is a solution of the following equation:

$$(h + b)\left(G(\hat{y}_t) - \frac{b}{h + b}\right) + \alpha E_{D_t} f'_{t+1}(\hat{y}_t - d_t) = 0.$$

Thus the equivalence $\hat{y}_t = y_t^*$ holds from (A.5) and the induction assumption. Again we analyze the three possible cases given through (A.6). Adding up the Cases 2 and 3 ($x_t < \hat{y}_t$), leads to $f_t(x_t) = H_t(x_t)$ from the definition of $c_t(x_t, z_t)$ given in (A.3), and the induction argument is concluded. □

Lemma A.3 *Let $f(x)$ and $g(x)$ be convex, and x_f and x_g be their smallest minimizers. If $f'(x) \le g'(x)$ for all x, then $x_f \ge x_g$.*

Proof: Following Heyman and Sobel (1984), lets assume for a contradiction that $x_f < x_g$, then we can write $0 < \delta \le x_g - x_f$ and $f'(x_g - \delta) \ge 0$ holds. From $f'(x_g - \delta) \ge 0$ and initial assumption $f'(x) \leq g'(x)$ we have $0 \leq f'(x_g - \delta) \leq g'(x_g - \delta)$. Observe that $g'(x_g - \delta) \geq 0$ contradicts the definition of x_g being smallest minimizer of $g(\cdot)$, hence $x_f \geq x_g$. $\square$

Proof of Theorem 3.3: We first write the optimal cost function $f_t(x, q, \vec{q})$ using the definition given in (3.4) and following the results of Theorem 3.1, as

$$f_t(x, q, \vec{q}) = \begin{cases} J_t(x, \vec{q}), & \hat{y}_t(\vec{q}) \leq x, \\ J_t(\hat{y}_t(\vec{q}), \vec{q}), & \hat{y}_t(\vec{q}) - q \leq x < \hat{y}_t(\vec{q}), \\ J_t(x + q, \vec{q}), & x < \hat{y}_t(\vec{q}) - q. \end{cases} \tag{A.7}$$

From this, the definition of the first derivative and the convexity of J_t in y proven in Theorem 3.1, we can write $f_t'(x, q, \vec{q})$ in the following manner

$$f_t'(x, q, \vec{q}) = \begin{cases} \geq 0, & \hat{y}_t(\vec{q}) \leq x, \\ = 0, & \hat{y}_t(\vec{q}) - q \leq x < \hat{y}_t(\vec{q}), \\ < 0, & x < \hat{y}_t(\vec{q}) - q. \end{cases} \tag{A.8}$$

Starting the induction argument at time T we first observe that $\vec{q}_T$ has no components due to (3.1). Part 1 holds as an equality since $J_T'(y, \vec{q}) = J_T'(y) = C_T'(y)$, and using (A.7) and (A.8) it is easy to show that Part 2, $f_T'(x, q_2) \leq f_T'(x, q_1)$, holds. Part 3 holds as an equality since $\hat{y}_T$ and is independent of q. Assuming that $J_t'(y, \vec{q}_2) \leq J_t'(y, \vec{q}_1)$ also holds for t we have $\hat{y}_t(\vec{q}_2) \geq \hat{y}_t(\vec{q}_1)$ by using Lemma A.3. To prove that this implies $f_t'(x, q_2, \vec{q}_2) \leq f_t'(x, q_1, \vec{q}_1)$ for t, we need to analyze the following nine cases:

Case 1: If $x \geq \hat{y}_t(\vec{q}_2)$ and $x \geq \hat{y}_t(\vec{q}_1)$, then $f_t'(x, q_2, \vec{q}_2) = J_t'(x, \vec{q}_2) \leq J_t'(x, \vec{q}_1) = f_t'(x, q_1, \vec{q}_1)$, where the inequality is due to Part 1.

Case 2: If $\hat{y}_t(\vec{q}_2) - q_2 \leq x < \hat{y}_t(\vec{q}_2)$ and $x \geq \hat{y}_t(\vec{q}_1)$, then $f_t'(x, q_2, \vec{q}_2) = 0 \leq f_t'(x, q_1, \vec{q}_1)$.

Case 3: If $x < \hat{y}_t(\vec{q}_2) - q_2$ and $x \geq \hat{y}_t(\vec{q}_1)$, then $f_t'(x, q_2, \vec{q}_2) < 0 \leq f_t'(x, q_1, \vec{q}_1)$.

Case 4: $x \geq \hat{y}_t(\vec{q}_2)$ and $\hat{y}_t(\vec{q}_1) - q_1 \leq x < \hat{y}_t(\vec{q}_1)$ is not possible due to Part 3.

Case 5: If $\hat{y}_t(\vec{q}_2) - q_2 \leq x < \hat{y}_t(\vec{q}_2)$ and $\hat{y}_t(\vec{q}_1) - q_1 \leq x < \hat{y}_t(\vec{q}_1)$, then $f_t'(x, q_2, \vec{q}_2) = 0 = f_t'(x, q_1, \vec{q}_1)$.

Case 6: If $x < \hat{y}_t(\vec{q}_2) - q_2$ and $\hat{y}_t(\vec{q}_1) - q_1 \leq x < \hat{y}_t(\vec{q}_1)$, then $f_t'(x, q_2, \vec{q}_2) < 0 = f_t'(x, q_1, \vec{q}_1)$.

Case 7: $x \geq \hat{y}_t(\vec{q}_2)$ and $x < \hat{y}_t(\vec{q}_1) - q_1$ is not possible due to Part 3.

Case 8: $\hat{y}_t(\vec{q}_2) - q_2 \leq x < \hat{y}_t(\vec{q}_2)$ and $x < \hat{y}_t(\vec{q}_1) - q_1$ is not possible due to $\hat{y}_t(\vec{q}_2) - q_2 > \hat{y}_t(\vec{q}_1) - q_1$, which is due to Part 3 and $q_2 \leq q_1$.

Case 9: If $x < \hat{y}_t(\vec{q}_2) - q_2$ and $x < \hat{y}_t(\vec{q}_1) - q_1$, then $f_t'(x, q_2, \vec{q}_2) = J_t'(x + q_2, \vec{q}_2) \leq J_t'(x + q_1, \vec{q}_2) \leq J_t'(x+q_1, \vec{q}_1) = f_t'(x, q_1, \vec{q}_1)$, where the first inequality is due to $q_2 \leq q_1$ and convexity of J_t in y, and the second inequality is due to Part 1.

Going from t to $t-1$ we conclude the induction argument by showing that $f_t'(x, q_2, \vec{q}_2) \leq f_t'(x, q_1, \vec{q}_1)$ implies $J_{t-1}'(y, \vec{q}_2) \leq J_{t-1}'(y, \vec{q}_1)$. We write $J_{t-1}'(y_{t-1}, \vec{q}_{2,t-1}) = J_{t-1}'(x_{t-1}, q_{t-1}, \vec{q}_{2,t-1})$, where $(x_{t-1}, q_{t-1}, \vec{q}_{2,t-1})$ denotes the state space in $t-1$ and $(x_{2,t}, q_{2,t}, q_{2,t+1}, \ldots, q_{2,t+n-1}, Q_{t+n})$ refers to the resulting next period's state space, where the last element of ACI vector, Q_{t+n}, is random. Inventory position $x_{2,t}$ is updated through (3.2), where the order size z_{t-1} is limited by the available supply capacity q_{t-1} in period $t-1$. Using (3.5) we write:

$$\begin{aligned}
J_{t-1}'(y_{t-1}, \vec{q}_{2,t-1}) &= C_{t-1}'(y_{t-1}) + \alpha E f_t'(x_{2,t}, q_{2,t}, \vec{Q}_{2,t}) \\
&\leq C_{t-1}'(y_{t-1}) + \alpha E f_t'(x_{1,t}, q_{2,t}, q_{2,t+1}, \ldots, q_{2,t+n-1}, Q_{t+n}) \\
&\leq C_{t-1}'(y_{t-1}) + \alpha E f_t'(x_{1,t}, q_{1,t}, q_{1,t+1}, \ldots, q_{1,t+n-1}, Q_{t+n}) \\
&= J_{t-1}'(y_{t-1}, \vec{q}_{1,t-1}),
\end{aligned}$$

For $q_{2,t-1} \leq q_{1,t-1}$, it holds that $x_{2,t} \leq x_{1,t}$ from the state update (3.2). The first inequality holds directly from convexity of f_t in x. From $\vec{q}_{2,t-1} \leq \vec{q}_{1,t-1}$ and the state update (3.3), it follows that $(q_{2,t}, q_{2,t+1}, \ldots, q_{2,t+n-1}) \leq (q_{1,t}, q_{1,t+1}, \ldots, q_{1,t+n-1})$. The second inequality is due to induction argument and the fact that taking expectation over the same random variable Q_{t+n} preserves inequality. $\square$

Lemma A.4 *For any $\vec{q}$ and $\eta > 0$ and all t, we have:*

Q1. $J_t'(y - \eta, \vec{q}) \leq J_t'(y, \vec{q} - \eta e_1)$,

Q2. $\hat{y}_t(\vec{q} - \eta e_1) - \hat{y}_t(\vec{q}) \leq \eta$,

Q3. $f_t'(x - \eta, q, \vec{q}) \leq f_t'(x, q, \vec{q} - \eta e_1)$.

Proof: We first prove that $f_t'(x - \eta, q, \vec{q}) \leq f_t'(x, q - \eta, \vec{q})$ holds for all x and t. Observe that the smallest minimizer of $J_t(x - \eta, \vec{q})$ is $\hat{y}_t(\vec{q}) + \eta$, and we analyze 4 valid cases using (A.7) and (A.8):

Case 1: If $\hat{y}_t(\vec{q}) \leq x - \eta$ and $\hat{y}_t(\vec{q}) \leq x$, then $\hat{y}_t(\vec{q}) + \eta \leq x$ and $f_t'(x - \eta, q, \vec{q}) = J_t'(x - \eta, \vec{q}) \leq J_t'(x, \vec{q}) = f_t'(x, q - \eta, \vec{q})$, where the equality is due to convexity of J_t in y.

Case 2: If $\hat{y}_t(\vec{q}) - q \leq x - \eta < \hat{y}_t(\vec{q})$ and $\hat{y}_t(\vec{q}) \leq x$, then $\hat{y}_t(\vec{q}) \leq x < \hat{y}_t(\vec{q}) + \eta$ and $f_t'(x - \eta, q, \vec{q}) = 0 \leq f_t'(x, q - \eta, \vec{q})$.

Case 3: If $\hat{y}_t(\vec{q}) - q \leq x - \eta < \hat{y}_t(\vec{q})$ and $\hat{y}_t(\vec{q}) - q + \eta \leq x < \hat{y}_t(\vec{q})$, then $\hat{y}_t(\vec{q}) - q + \eta \leq x < \hat{y}_t(\vec{q})$ and $f_t'(x - \eta, q, \vec{q}) = 0 = f_t'(x, q - \eta, \vec{q})$.

Case 4: If $x < \hat{y}_t(\vec{q}) - q + \eta$, then $f_t'(x - \eta, q, \vec{q}) = J_t'(x - \eta + q, \vec{q}) = J_t'(x + q - \eta, \vec{q}) = f_t'(x, q - \eta, \vec{q})$.

To prove Part 1 we write:

$$\begin{aligned} J_t'(y-\eta,\vec{q}) &\leq C_t'(y)+\alpha E f_{t+1}'(y-\eta-D_t,q_{t+1},\vec{Q}_{t+1}) \\ &\leq C_t'(y)+\alpha E f_{t+1}'(y-D_t,q_{t+1}-\eta,\vec{Q}_{t+1}) \\ &= J_t'(y,\vec{q}-\eta e_1). \end{aligned} \tag{A.9}$$

The first inequality is due to convexity of $C_t(y)$, the second inequality is due to $f_t'(x-\eta,q,\vec{q}) \leq f_t'(x,q-\eta,\vec{q})$ and the equality is from definition of J_t in (3.5). Since the smallest minimizers of functions $J_t(y-\eta,\vec{q})$ and $J_t(y,\vec{q}-\eta e_1)$ are $\hat{y}_t(\vec{q})+\eta$ and $\hat{y}_t(\vec{q}-\eta e_1)$, Lemma A.3 implies $\hat{y}_t(\vec{q}-\eta e_1) \leq \hat{y}_t(\vec{q})+\eta$ and this proves Part 2.

We continue to prove Part 3 for all x and t. We use (A.7), (A.8) and the fact that $\hat{y}_t(\vec{q}-\eta e_1) \geq \hat{y}_t(\vec{q})$ holds from Part 3 of Theorem 3.3, and consider nine possible cases:

Case 1: If $\hat{y}_t(\vec{q}) \leq x-\eta$ and $\hat{y}_t(\vec{q}-\eta e_1) \leq x$, then $f_t'(x-\eta,q,\vec{q}) = J_t'(x-\eta,\vec{q}) \leq J_t'(x,\vec{q}-\eta e_1) = f_t'(x,q,\vec{q}-\eta e_1)$. The inequality follows from Part 1.

Case 2: If $\hat{y}_t(\vec{q})-q \leq x-\eta < \hat{y}_t(\vec{q})$ and $\hat{y}_t(\vec{q}-\eta e_1) \leq x$, then $f_t'(x-\eta,q,\vec{q}) = 0 \leq f_t'(x,q,\vec{q}-\eta e_1)$.

Case 3: If $x-\eta < \hat{y}_t(\vec{q})-q$ and $\hat{y}_t(\vec{q}-\eta e_1) \leq x$, then $f_t'(x-\eta,q,\vec{q}) < 0 \leq f_t'(x,q,\vec{q}-\eta e_1)$.

Case 4: $\hat{y}_t(\vec{q}) \leq x-\eta$ and $\hat{y}_t(\vec{q}-\eta e_1)-q \leq x < \hat{y}_t(\vec{q}-\eta e_1)$ is not possible due to Part 2.

Case 5: If $\hat{y}_t(\vec{q})-q \leq x-\eta < \hat{y}_t(\vec{q})$ and $\hat{y}_t(\vec{q}-\eta e_1)-q \leq x < \hat{y}_t(\vec{q}-\eta e_1)$, then $f_t'(x-\eta,q,\vec{q}) = 0 = f_t'(x,q,\vec{q}-\eta e_1)$.

Case 6: If $x-\eta < \hat{y}_t(\vec{q})-q$ and $\hat{y}_t(\vec{q}-\eta e_1)-q \leq x < \hat{y}_t(\vec{q}-\eta e_1)$, then $f_t'(x-\eta,q,\vec{q}) < 0 = f_t'(x,q,\vec{q}-\eta e_1)$.

Case 7: $x-\eta < \hat{y}_t(\vec{q})-q$ and $\hat{y}_t(\vec{q}-\eta e_1) \leq x$ is not possible due to Part 2.

Case 8: $\hat{y}_t(\vec{q})-q \leq x-\eta < \hat{y}_t(\vec{q})$ and $\hat{y}_t(\vec{q}-\eta e_1) \leq x$ is not possible due to Part 2.

Case 9: If $x-\eta < \hat{y}_t(\vec{q})-q$ and $x < \hat{y}_t(\vec{q}-\eta e_1)-q$, then $f_t'(x-\eta,q,\vec{q}) = J_t'(x-\eta+q,\vec{q}) \leq J_t'(x+q,\vec{q}-\eta e_1) = f_t'(x,q,\vec{q}-\eta e_1)$. The inequality follows from Part 1. With this we conclude the proof of Part 3. $\square$

Proof of Conjecture 3.2: The proof of follows directly from Conjecture 3.1 and Part 2 of Lemma A.4 for $i > 1$ because $\hat{y}_t(\vec{q}-\eta e_i)-\hat{y}_t(\vec{q}) \leq \hat{y}_t(\vec{q}-\eta e_{i-1})-\hat{y}_t(\vec{q}) \leq \cdots \leq \hat{y}_t(\vec{q}-\eta e_1)-\hat{y}_t(\vec{q}) \leq \eta$. This proof was inspired by Özer and Wei (2003). $\square$

Appendix B

Chapter 4 Proofs

Before giving the proofs, we first provide the needed notation and the definitions, which enables us to give the proofs in a concise manner. For clarity reasons, we elect to suppress the time subscripts in certain parts of the proofs. We also assume $\alpha = 1$ for the same reason.

For the m uncertain pipeline orders in $\vec{z}_t$, we know that any particular order z_i, where $i = t-m \ldots t-1$ can either be delivered in full or only partially depending on the available supply capacity revealed through ASI. Based on this we define the vector $\vec{z}_t^{\,-}$, which represents the set of orders z_i that will not be delivered in full, $z_i > Q_i$. The vector $\vec{z}_t^{\,+}$ represents the set of orders z_i that will be fully replenished, $z_i \leq Q_i$. Thus, $\vec{z}_t^{\,-} \cap \vec{z}_t^{\,+} = \vec{z}_t$ and $\vec{z}_t^{\,-} \cup \vec{z}_t^{\,+} = \varnothing$ holds. As it will be useful in some of the following derivations to include also the order z_t into the two vectors $\vec{z}_t^{\,-}$ and $\vec{z}_t^{\,+}$, we also define the extended vectors $\vec{Z}_t^{\,-}$ and $\vec{Z}_t^{\,+}$. The two corresponding supply capacity vectors are denoted as $\vec{Q}_t^{-}$ and $\vec{Q}_t^{+}$.

We denote the cumulative distribution function of the demand with $G(D_t)$, and the corresponding probability density function with $g(D_t)$, and the lead time demand counterparts with $G_t^L(D_t^L)$ and $g_t^L(D_t^L)$. The cumulative distribution function and the probability density function of supply capacity Q_t are denoted as $R_t(Q_t)$ and $r_t(Q_t)$. We assume that all the distributions are stationary.

In the following lemma, we provide the convexity results and the optimal solution to a single period cost function $C_t(y_t, \vec{z}_t, z_t)$.

Lemma B.1 *Let $\hat{y}_t^M$ be the smallest minimizer of $C_t(y_t, \vec{z}_t, z_t)$ to which the optimal order $\hat{z}_t^M$ is placed, where $\hat{z}_t^M = \hat{y}_t^M - x_t$:*

Q1. $C_t(y_t, \vec{z}_t, z_t)$ is convex in y_t.

Q2. $C_t(y_t, \vec{z}_t, z_t)$ is quasiconvex in z_t.

Q3. $\hat{y}_t^M(\vec{z}_t)$ is the state-dependent optimal myopic base-stock level.

Proof: $C(y, \vec{z}, z)$ is expressed in the following way:

$$C(y, \vec{z}, z) = b \int_0^{\vec{Z}^-} \int_{\vec{Z}^+}^{\infty} \int_{y-\sum(\vec{Z}^- - \vec{Q}^-)}^{\infty} \left(D^L - y + \sum(\vec{Z}^- - \vec{Q}^-)\right) r(\vec{Q}) \mathrm{d}\vec{Q} g^L(D^L) \mathrm{d}D^L$$
$$+h \int_0^{\vec{Z}^-} \int_{\vec{Z}^+}^{\infty} \int_0^{y-\sum(\vec{Z}^- - \vec{Q}^-)} \left(y - \sum(\vec{Z}^- - \vec{Q}^-) - D^L\right) r(\vec{Q}) \mathrm{d}\vec{Q} g^L(D^L) \mathrm{d}D^L. \quad (B.1)$$

To prove Part 1, we derive the first partial derivative of (B.1) with respect to y, where we take into account that $\prod \left((1 - R(\vec{Z}^+))R(\vec{Z}^-)\right) = 1$:

$$\frac{\partial}{\partial y} C(y, \vec{z}, z) = -b + (b+h) \prod (1 - R(\vec{Z}^+)) \int_0^{\vec{Z}^-} G^L \left(y - \sum(\vec{Z}^- - \vec{Q}^-)\right) r(\vec{Q}^-) \mathrm{d}\vec{Q}^- \quad (B.2)$$

and the second partial derivative:

$$\frac{\partial^2}{\partial y^2} C(y, \vec{z}, z) = (b+h) \prod (1 - R(\vec{Z}^+)) \int_0^{\vec{Z}^-} g^L \left(y - \sum(\vec{Z}^- - \vec{Q}^-)\right) r(\vec{Q}^-) \mathrm{d}\vec{Q}^-. \quad (B.3)$$

Since all terms in (B.3) are nonnegative, Part 1 holds. It is easy to see that the convexity also holds in x.

To show Part 2, we obtain the first two partial derivatives of $C(y, \vec{z}, z)$ with respect to z:

$$\frac{\partial}{\partial z} C(y, \vec{z}, z) = (b+h)(1 - R(z)) \left[\prod (1 - R(\vec{z}^+)) \int_0^{\vec{z}^-} G^L \left(y - \sum(\vec{z}^- - \vec{Q}^-)\right) r(\vec{Q}^-) \mathrm{d}\vec{Q}^- - \frac{b}{b+h}\right] \quad (B.4)$$

$$\frac{\partial^2}{\partial z^2} C(y, \vec{z}, z) = -r(z)(b+h) \left[\prod (1 - R(\vec{z}^+)) \int_0^{\vec{z}^-} G^L \left(y - \sum(\vec{z}^- - \vec{Q}^-)\right) r(\vec{Q}^-) \mathrm{d}\vec{Q}^- - \frac{b}{b+h}\right]$$
$$+ (b+h)(1 - R(z)) \left[\prod (1 - R(\vec{z}^+)) \int_0^{\vec{z}^-} g^L \left(y - \sum(\vec{z}^- - \vec{Q}^-)\right) r(\vec{Q}^-) \mathrm{d}\vec{Q}^-\right] \quad (B.5)$$

Setting (B.4) to 0 proves Part 3. Observe that $\hat{y}^M(\vec{z})$ only depends on $\vec{z}$, and not on z. Intuitively this makes sense, as due to the potential shortages in replenishment of any of uncertain pipeline orders $\vec{z}$ we increase $\hat{y}^M$ accordingly. However, when placing order z, it is not rational to adjust $\hat{y}^M$ to account for the potential shortage in replenishment of z. One merely has to hope that by ordering z up to $\hat{y}^M$, the available supply capacity will be sufficient.

For $z \leq \hat{z}^M$, the bracketed part in the first term of (B.5) is not positive, thus the first part is nonnegative as a whole. Since also the second term is always nonnegative, the function $C(y, \vec{z}, z)$ is convex on the respected interval. For $z > \hat{z}^M$ this does not hold, however we see that (B.4) is nonnegative, thus $C(y, \vec{z}, z)$ is nondecreasing on the respected interval, which proves Part 2. Due to this, the $C(y, \vec{z}, z)$ has a quasiconvex form, which is sufficient for $\hat{y}^M$ to be its global minimizer.

Note, that one can show that $C(y, \vec{z}, z)$ is quasiconvex in any of z_i, where $i = t-m \ldots t-1$, in the same way as presented above. The above derivation can be considered as a generalization of the derivations for the zero lead time model presented in Ciarallo et al. (1994). We have shown that the convexity properties of the single period function also holds for the positive lead time case. □

Proof of Lemma 4.1: The proof is by induction on t. For period T it holds $J_T(y_T, \vec{z}_T, z_T) = C_T(y_T, \vec{z}_T, z_T)$, which by using the result of Lemma B.1 proves the convexity of $J_T(y_T, \vec{z}_T, z_T)$ in y_T, and using (4.6) for T, also the convexity of $f_T(y_T, \vec{z}_T)$ in x_T.

Assuming that f_{t+1} is convex in x_{t+1}, we now want to show that this implies convexity of J_t in y_t and f_t in x_t. Using (4.5) we write J_t as:

$$\begin{aligned} J_t(y_t, \vec{z}_t, z_t) &= C_t(y_t, \vec{z}_t, z_t) \\ &+ \int_0^\infty \int_0^{z_{t-m}} f_{t+1}(y_t - (z_{t-m} - Q_{t-m}) - D_t, \vec{z}_{t+1}) r(Q_{t-m}) \mathrm{d}Q_{t-m} g(D_t) \mathrm{d}D_t \\ &+ (1 - R(z_{t-m})) \int_0^\infty f_{t+1}(y_t - D_t, \vec{z}_{t+1}) g(D_t) \mathrm{d}D_t. \end{aligned} \tag{B.6}$$

By taking the second partial derivative of (B.6) with respect to y_t we quickly see that the convexity of J_t in y_t is preserved due to the convexity of C_t in y_t coming from Lemma B.1, while the the remaining two terms are convex due to the induction argument.

To show that this also implies convexity of f_t in x_t, we first take the first partial derivative of J_t with respect to z_t[1]:

$$\begin{aligned} \frac{\partial}{\partial z} J(y_t, \vec{z}_t, z_t) &= \frac{\partial}{\partial z} C(y_t, \vec{z}_t, z_t) \\ &+ \int_0^\infty \int_0^{z_{t-m}} f'_{t+1}(y_t - (z_{t-m} - Q_{t-m}) - D_t, \vec{z}_{t+1}) r(Q_{t-m}) \mathrm{d}Q_{t-m} g(D_t) \mathrm{d}D_t \\ &+ (1 - R(z_{t-m})) \int_0^\infty f'_{t+1}(y_t - D_t, \vec{z}_{t+1}) g(D_t) \mathrm{d}D_t, \end{aligned} \tag{B.7}$$

Partially differentiating (4.6) with respect to x_t twice, using (B.2) and taking into account

[1]We define the first derivative of a function $f_t(x)$ with respect to x as $f'_t(x)$.

the first-order optimality condition by setting (B.7) to zero, yields the following:

$$\begin{aligned}\frac{\partial^2}{\partial x_t^2} f(x_t,\vec{z}_t) &= (b+h)\prod(1-R(\vec{Z_t}^{\,+}))\int_0^{\vec{Z_t}^{\,-}} g^L\left(x_t+\hat{z}_t-\sum(\vec{Z_t}^{\,-}-\vec{Q_t}^{\,-})\right)r(\vec{Q_t}^{\,-})\mathrm{d}\vec{Q_t}^{\,-} \\ &+ \int_0^\infty\int_0^{z_{t-m}} f''_{t+1}(x_t+\hat{z}_t-(z_{t-m}-Q_{t-m})-D_t,\vec{z}_{t+1})r(Q_{t-m})\mathrm{d}Q_{t-m}g(D_t)\mathrm{d}D_t \\ &+ (1-R(z_{t-m}))\int_0^\infty f''_{t+1}(x_t+\hat{z}_t-D_t,\vec{z}_{t+1})g(D_t)\mathrm{d}D_t. \qquad \text{(B.8)}\end{aligned}$$

While the expression does not get simplified as in the zero lead time case, we can easily conclude that the convexity of f_t in x_t is also preserved as all the terms above are nonnegative. $\square$

Proof of Theorem 4.1: The convexity results of Lemmas 4.1 and B.1 imply the proposed optimal policy structure. $\square$

Lemma B.2 *Let $f(x)$ and $g(x)$ be convex, and x_f and x_g be their smallest minimizers. If $f'(x) \le g'(x)$ for all x, then $x_f \ge x_g$.*

Proof: Following Heyman and Sobel (1984), lets assume for a contradiction that $x_f < x_g$, then we can write $0 < \delta \le x_g - x_f$ and $f'(x_g-\delta) \ge 0$ holds. From $f'(x_g-\delta)\ge 0$ and initial assumption $f'(x) \le g'(x)$ we have $0 \le f'(x_g-\delta) \le g'(x_g-\delta)$. Observe that $g'(x_g-\delta)\ge 0$ contradicts the definition of x_g being smallest minimizer of $g(\cdot)$, hence $x_f \ge x_g$. $\square$

Proof of Theorem 4.2: Using (4.4) and Part 2 of Theorem 4.1, the optimal cost function $f_t(x,\vec{z})$ can be written as:

$$f_t(x,\vec{z}) = \begin{cases} J_t(x,\vec{z},0), & \hat{y}_t(\vec{z}) \le x, \\ J_t(\hat{y}_t(\vec{z}),\vec{z},z), & x < \hat{y}_t(\vec{z}). \end{cases} \qquad \text{(B.9)}$$

Due to the convexity of J_t in y, we can write $f'_t(x,\vec{z})$ in the following manner

$$f'_t(x,\vec{z}) = \begin{cases} \ge 0, & \hat{y}_t(\vec{z}) \le x, \\ = 0, & x < \hat{y}_t(\vec{z}). \end{cases} \qquad \text{(B.10)}$$

We start the induction argument at time T, where $J_T(y,\vec{z},z) = C_T(y,\vec{z},z)$ holds. Using (B.2), we show that the following holds for any $\vec{z}_1 \le \vec{z}_2$ and $z_1 \le z_2$:

$$1 - R(\vec{Z}_1^+) \ge 1 - R(\vec{Z}_1^+),$$

$$\int_0^{\vec{Z}_1^-} G^L\left(y-\sum(\vec{Z}_1^- - \vec{Q}^-)\right)r(\vec{Q}^-)\mathrm{d}\vec{Q}^- \ge \int_0^{\vec{Z}_2^-} G^L\left(y-\sum(\vec{Z}_2^- - \vec{Q}^-)\right)r(\vec{Q}^-)\mathrm{d}\vec{Q}^-,$$

due to $y - \sum(\vec{Z}_1^- - \vec{Q}^-) \geq y - \sum(\vec{Z}_2^- - \vec{Q}^-)$. This proves $C'_T(y, \vec{z}_1, z_1) \geq C'_T(y, \vec{z}_2, z_2)$, and consequently $J'_T(y, \vec{z}_1, z_1) \geq J'_T(y, \vec{z}_2, z_2)$ holds. Using (B.9) and (B.10) one can quickly show that $f'_T(y, \vec{z}_1) \geq f'_T(y, \vec{z}_2)$ also holds, and due to Lemma B.2 also Part 3 holds for T.

Lets assume now that $J'_t(y, \vec{z}_1, z_1) \geq J'_t(y, \vec{z}_2, z_2)$ holds for t. From Lemma B.2, we have $\hat{y}_t(\vec{z}_1) \leq \hat{y}_t(\vec{z}_2)$, which proves Part 3. To prove Part 2, we need to show that this implies $f'_t(x, \vec{z}_1) \geq f'_t(x, \vec{z}_2)$. We need to analyze the following four cases:

Case 1: If $x \geq \hat{y}_t(\vec{z}_1)$ and $x \geq \hat{y}_t(\vec{z}_2)$, then $f'_t(x, \vec{z}_1) = J'_t(x, \vec{z}_1, 0) \geq J'_t(x, \vec{z}_2, 0) = f'_t(x, \vec{z}_2)$, where the inequality is due to Part 1.

Case 2: If $x \geq \hat{y}_t(\vec{z}_1)$ and $x < \hat{y}_t(\vec{z}_2)$, then $f'_t(x, \vec{z}_1) \geq f'_t(x, \vec{z}_2)$ comes directly from (B.10).

Case 3: $x < \hat{y}_t(\vec{z}_1)$ and $x \geq \hat{y}_t(\vec{z}_2)$ is not possible due to Part 3.

Case 4: If $x < \hat{y}_t(\vec{z}_1)$ and $x < \hat{y}_t(\vec{z}_2)$, then we have $f'_t(x, \vec{z}_1) = f'_t(x, \vec{z}_2) = 0$ from (B.10).

Going from t to $t-1$ we conclude the induction argument by showing that $f'_t(x, \vec{z}_1) \geq f'_t(x, \vec{z}_2)$ implies $J'_{t-1}(y, \vec{z}_1, z_1) \geq J'_{t-1}(y, \vec{z}_2, z_2)$. Using (4.5) we write:

$$
\begin{aligned}
J'_{t-1}(y_{t-1}, \vec{z}_{1,t-1}, z_{1,t-1}) &= C'_{t-1}(y_{t-1}, \vec{z}_{1,t-1}, z_{1,t-1}) + \alpha E f'_t(x_{1,t}, \vec{z}_{1,t}) \\
&\geq C'_{t-1}(y_{t-1}, \vec{z}_{2,t-1}, z_{2,t-1}) + \alpha E f'_t(x_{1,t}, \vec{z}_{2,t}) \\
&\geq C'_{t-1}(y_{t-1}, \vec{z}_{2,t-1}, z_{2,t-1}) + \alpha E f'_t(x_{2,t}, \vec{z}_{2,t}) \\
&= J'_{t-1}(y_{t-1}, \vec{z}_{2,t-1}, z_{2,t-1}),
\end{aligned}
$$

The first inequality is due to $C'_{t-1}(y_{t-1}, \vec{z}_{1,t-1}, z_{1,t-1}) \geq C'_{t-1}(y_{t-1}, \vec{z}_{2,t-1}, z_{2,t-1})$, and induction argument $f'_t(x, \vec{z}_1) \geq f'_t(x, \vec{z}_2)$ (where taking expectation over the random variables Q_{t-m-1} and D_{t-1} preserves the inequality). The inequality of C'_{t-1} holds from convexity of $\tilde{C}_{t+L-1}$ in $\tilde{x}_{t+L}$, where from the assumptions $\vec{z}_1 \leq \vec{z}_2$ and $z_1 \leq z_2$, we have $\tilde{x}_{1,t+L} \geq \tilde{x}_{2,t+L}$ due to the definition $\tilde{x}_{t+L} = y_{t-1} - \sum_{s=t-m-1}^{t-1}(z_s - Q_s)^+ - D^L_{t-1}$ in (4.3) (where again taking expectations over the random variables $Q_{t+m-1}, \ldots, Q_{t-1}$ and D^L_{t-1} preserves the inequality). The second inequality holds from convexity of f_t in x, and the fact that $x_{1,t} \geq x_{2,t}$ holds due to the state update $x_{1,t} = y_{t-1} - (z_{1,t-m-1} - q_{t-m-1})^+ - d_{t-1} \geq y_{t-1} - (z_{2,t-m-1} - q_{t-m-1})^+ - d_{t-1} = x_{2,t}$ in (4.2), and the assumption $z_{1,t-m-1} \leq z_{2,t-m-1}$. $\square$

Appendix C

Chapter 5 Proofs

In Lemma C.1 we provide the convexity results and the optimal solution to a single period cost function $C_t(y_t, z_t)$. We elect to suppress subscript t in the state variables for clarity reasons.

Lemma C.1 *Let $\hat{y}^M$ be the smallest minimizer of $C(y,z)$ to which the optimal order $\hat{z}^M$ is placed, where $\hat{z}^M = \hat{y}^M - x$:*

Q1. $C(y,z)$ is convex in y.

Q2. $C(y,z)$ is quasiconvex in z, where:

$$\frac{\partial^2}{\partial z^2}C(y,z) \geq 0, \text{ for } z \leq \hat{z}^M,$$
$$\frac{\partial}{\partial z}C(y,z) \geq 0, \text{ for } z > \hat{z}^M.$$

Q3. $\hat{y}^M$ is the optimal myopic base-stock level, where $\hat{y}^M = G^{-1}\left(\frac{c_b}{c_b+c_h}\right)$.

Proof: We first rewrite the single period cost function $C(y,z)$ as:

$$\begin{aligned} C(y,z) &= \alpha(1-R(z))\left[c_b\int_y^\infty (D-y)g(D)\mathrm{d}D + c_h\int_0^y (y-D)g(D)\mathrm{d}D\right] \\ &+ \alpha c_b \int_0^z \int_{y-z+Q}^\infty (D-y+z-Q)g(D)\mathrm{d}Dr(Q)\mathrm{d}Q \\ &+ \alpha c_h \int_0^z \int_0^{y-z+Q} (y-z+Q-D)g(D)\mathrm{d}Dr(Q)\mathrm{d}Q. \end{aligned} \tag{C.1}$$

To prove Part 1, we derive the first partial derivative of (C.1) with respect to y:

$$\begin{aligned} \frac{\partial}{\partial y}C(y,z) &= \alpha(c_b+c_h)(1-R(z))\left(G(y) - \frac{c_b}{c_b+c_h}\right) \\ &+ \alpha(c_b+c_h)\int_0^z G(y-z+Q)r(Q)\mathrm{d}Q - \alpha c_b R(z), \end{aligned} \tag{C.2}$$

and the second partial derivative:

$$\frac{\partial^2}{\partial y^2}C(y,z) = \alpha(c_b + c_h)\left[(1-R(z))g(y) + \int_0^z g(y-z+Q)r(Q)\mathrm{d}Q\right]. \tag{C.3}$$

Since all terms in (C.3) are nonnegative, Part 1 holds.

Similarly for Part 2, we obtain the first two partial derivatives of $C(y,z)$ with respect to z:

$$\frac{\partial}{\partial z}C(y,z) = \alpha(c_b + c_h)(1-R(z))\left(G(y) - \frac{c_b}{c_b + c_h}\right), \tag{C.4}$$

$$\frac{\partial^2}{\partial z^2}C(y,z) = \alpha(c_b + c_h)\left[(1-R(z))g(y) - r(z)\left(G(y) - \frac{c_b}{c_b + c_h}\right)\right]. \tag{C.5}$$

Observe first that setting (C.4) to 0 proves Part 3. For $z \le \hat{z}^M$, $G(y) \le \frac{c_b}{c_b+c_h}$ holds in the second term of (C.5). Since the first term is always nonnegative, the function $C(y,z)$ is convex on the respected interval. For $z > \hat{z}^M$ this does not hold, although we see from (C.4) that due to $G(y) > \frac{c_b}{c_b+c_h}$ and $1 - R(z) \to 0$ with z, the function $C(y,z)$ is nondecreasing, which proves Part 2. Due to this, the $C(y,z)$ has a quasiconvex form, which is sufficient for $\hat{y}^M$ to be its global minimizer. □

Proof of Proposition 5.1: We start by studying the convexity properties of the optimal cost functions (5.2)-(5.4), where we show that these are not convex. Note that the single-period cost function $C_t(y_t, z_t)$ is convex in y_t and quasiconvex in z_t for any t due to Lemma C.1. By differentiating (5.4) with respect to w_t and setting it to zero, we derive the first-order optimality condition, where the optimal inventory position after ordering with the slower supplier $\hat{w}_t(z_t)$ is the smallest minimizer of the function $H_t(w_t, z_t)$ in period t[1]:

$$\begin{aligned}\frac{\partial}{\partial w_t}H_t(w_t,z_t) &= \alpha\int_0^\infty\int_{z_t}^\infty f'_{t+1}(\hat{w}_t(z_t) - D_t)r_t(Q_t)\mathrm{d}Q_t g_t(D_t)\mathrm{d}D_t \\ &+ \alpha\int_0^\infty\int_0^{z_t} f'_{t+1}(\hat{w}_t(z_t) - (z_t - Q_t) - D_t)r_t(Q_t)\mathrm{d}Q_t g_t(D_t)\mathrm{d}D_t \equiv 0\end{aligned} \tag{C.6}$$

The second partial derivative of $H_t(w_t, z_t)$ with respect to w_t yields:

$$\begin{aligned}\frac{\partial^2}{\partial w_t^2}H_t(w_t,z_t) &= \alpha\int_0^\infty\int_{z_t}^\infty f''_{t+1}(\hat{w}_t(z_t) - D_t)r_t(Q_t)\mathrm{d}Q_t g_t(D_t)\mathrm{d}D_t \\ &+ \alpha\int_0^\infty\int_0^{z_t} f''_{t+1}(\hat{w}_t(z_t) - (z_t - Q_t) - D_t)r_t(Q_t)\mathrm{d}Q_t g_t(D_t)\mathrm{d}D_t.\end{aligned} \tag{C.7}$$

[1]We define the first derivative of a function $f_t(x)$ with respect to x as $f'_t(x)$.

In addition, we obtain the second partial derivative of $J_t(y_t, z_t)$ with respect to y_t, which will also be useful in the following discussions:

$$
\begin{aligned}
\frac{\partial^2}{\partial y_t^2} J_t(y_t, z_t) &= \alpha(c_b + c_h)\left((1 - R_t(z_t))g_t(y_t) + \int_0^{z_t} g_t(y_t - z_t + Q_t)r_t(Q_t)\mathrm{d}Q_t\right) \\
&+ \alpha \int_0^{\infty} \int_{z_t}^{\infty} f''_{t+1}(y_t + \hat{v}_t(z_t) - D_t)r_t(Q_t)\mathrm{d}Q_t g_t(D_t)\mathrm{d}D_t \\
&+ \alpha \int_0^{\infty} \int_0^{z_t} f''_{t+1}(y_t - (z_t - Q_t) + \hat{v}_t(z_t) - D_t)r_t(Q_t)\mathrm{d}Q_t g_t(D_t)\mathrm{d}D_t.
\end{aligned} \tag{C.8}
$$

Similarly as above, we derive the first-order optimality condition for $\hat{y}_t$ by differentiating (5.3) with respect to z_t, where $\hat{y}_t$ is the smallest minimizer of the function $J_t(y_t, z_t)$. Observe that the optimal order with the slower supplier $\hat{v}_t(\hat{z}_t)$ depends on the optimal order placed with the faster supplier $\hat{z}_t$.

$$
\begin{aligned}
\frac{\partial}{\partial z_t} J_t(x_t, z_t) &= \alpha(c_b + c_h)(1 - R_t(\hat{z}_t))\left(G(x_t + \hat{z}_t) - \frac{c_b}{c_b + c_h}\right) \\
&+ \alpha\left(1 + \frac{\partial \hat{v}_t(\hat{z}_t)}{\partial z_t}\right) \int_0^{\infty} \int_{\hat{z}_t}^{\infty} f'_{t+1}(x_t + \hat{z}_t + \hat{v}_t(\hat{z}_t) - D_t)r_t(Q_t)\mathrm{d}Q_t g_t(D_t)\mathrm{d}D_t D \\
&+ \alpha \frac{\partial \hat{v}_t(\hat{z}_t)}{\partial z_t} \int_0^{\infty} \int_0^{\hat{z}_t} f'_{t+1}(x_t + Q_t) + \hat{v}_t(\hat{z}_t) - D_t)r_t(Q_t)\mathrm{d}Q_t g_t(D_t)\mathrm{d}D_t \equiv 0
\end{aligned} \tag{C.9}
$$

Finally, evaluating (5.2) at $\hat{z}_t$, and partially differentiating it with respect to x_t twice, yields:

$$
\begin{aligned}
f''_t(x_t) &= \alpha(c_b + c_h) \int_0^{\hat{z}_t} g_t(x_t + Q_t)r_t(Q_t)\mathrm{d}Q_t \\
&- \alpha \frac{\partial \hat{v}_t(\hat{z}_t)}{\partial z_t} \int_0^{\infty} \int_{\hat{z}_t}^{\infty} f''_{t+1}(x_t + \hat{z}_t + \hat{v}_t(\hat{z}_t) - D_t)r_t(Q_t)\mathrm{d}Q_t g_t(D_t)\mathrm{d}D_t D \\
&+ \alpha\left(1 - \frac{\partial \hat{v}_t(\hat{z}_t)}{\partial z_t}\right) \int_0^{\infty} \int_0^{\hat{z}_t} f''_{t+1}(x_t + Q_t) + \hat{v}_t(\hat{z}_t) - D_t)r_t(Q_t)\mathrm{d}Q_t g_t(D_t)\mathrm{d}D_t
\end{aligned} \tag{C.10}
$$

If we would assume that $f_t(x_{t+1})$ is convex, using the regular inductive argument on t would require $f''_t(x_t) \geq 0$ to hold for the optimal cost to be convex. While the first term in (C.10) is nonnegative, we cannot say the same for the sum of the other two terms. Therefore, $f_t(x_t)$ is not a convex function in general. Observe that if $\partial \hat{v}_t(\hat{z}_t)/\partial z_t = 0$, the above condition for convexity would be satisfied. In this case, due to (C.7), (C.8) and $f_{T+1}(\cdot) \equiv 0$, H_t and J_t would also be convex in w_t and y_t, respectively. However, the dependency of $\hat{v}_t(\hat{z}_t)$ on $\hat{z}_t$ describes the core dynamics of this dual-sourcing model, where $\hat{v}_t$ is used to compensate for the potential shortage in replenishment of $\hat{z}_t$.

One can also quickly check that by differentiating (C.9) with respect to x_t, $d\hat{z}_t/dx_t = -1$ does not set the obtained expression to zero. Thus, $\hat{y}_t(x_t)$ is not independent of x_t, and therefore ordering with the faster supplier is not done in the manner of a base-stock policy, which proves Part 1.

By evaluating (C.6) at $\hat{z}_t$ and differentiating it with respect to y_t, we quickly see that $d\hat{w}_t(\hat{z}_t)/dy_t = 0$ needs to hold, which means that ordering with the slower supplier is done by placing order $\hat{v}_t(\hat{z}_t)$ up to the optimal base-stock level $\hat{w}_t(\hat{z}_t) = \hat{y}_t(x_t)+\hat{v}_t(\hat{z}_t)$, state-dependent on $\hat{z}_t$, which proves Part 2.

The above results directly imply the proposed structure of the optimal policy in Part 3. $\Box$

Proof of Proposition 5.2: We prove Part 1 by regular inductive argument on t. It is clear that Part 1 holds for $T+1$ due to $f_{T+1}(\cdot) \equiv 0$. Suppose that Part 1 also holds for $t+1$. We need to show that it also holds for t. We need to show that both parts of (5.6) are convex. Differentiating $C_t(y_t, z_t)$ with respect to x_t, evaluating it at $\hat{z}_t$ (using C.4), and differentiating again with respect to x_t yields:

$$\frac{\partial^2}{\partial x_t^2} C_t(x_t, \hat{z}_t) = \alpha(c_b + c_h) \int_0^{\hat{z}_t} g_t(x_t + Q_t) r_t(Q_t) \mathrm{d}Q_t. \tag{C.11}$$

The above expression is nonnegative, which proves that first part of (5.6) is convex. The second part is convex due to the fact that taking expectations over Q_t and D_t preserves the convexity of f_{t+1} (as in (C.7) evaluated at $\hat{z}$, presuming that f_{t+1} is now convex). This concludes the inductive argument and proves Part 1.

Part 2 holds directly from Part 1 and the definition of the myopic policy in (5.6). The optimality of the two base-stock levels $\hat{y}^M$ and $\hat{w}(\hat{z})$ can be shown by applying the steps of the analysis of Parts 1 and 2 of Proposition 5.1 to (5.6). $\Box$

Lemma C.2 *Let $f(x)$ and $g(x)$ be convex, and x_f and x_g be their smallest minimizers. If $f'(x) \leq g'(x)$ for all x, then $x_f \geq x_g$.*

Proof: Following Heyman and Sobel (1984), lets assume for a contradiction that $x_f < x_g$, then we can write $0 < \delta \leq x_g - x_f$ and $f'(x_g - \delta) \geq 0$ holds. From $f'(x_g - \delta) \geq 0$ and initial assumption $f'(x) \leq g'(x)$ we have $0 \leq f'(x_g - \delta) \leq g'(x_g - \delta)$. Observe that $g'(x_g - \delta) \geq 0$ contradicts the definition of x_g being smallest minimizer of $g(\cdot)$, hence $x_f \geq x_g$. $\Box$

Proof of Proposition 5.3: To prove Part 1, we need to show that for the first derivative of H_t with respect to w_t, $H_t'(w_t, \hat{z}_t)$, it holds that $H_t'(w_t, \hat{z}_{1,t}) \leq H_t'(w_t, \hat{z}_{2,t})$ for $\hat{z}_{1,t} \geq \hat{z}_{2,t}$ for all t:

$$
\begin{aligned}
H_t'(w_t, \hat{z}_{1,t}) &= \alpha E f_{t+1}'(x_{1,t+1}) \\
&= \alpha E f_{t+1}'(w_t - [\hat{z}_{1,t} - Q_t]^+ - D_t) \\
&\leq \alpha E f_{t+1}'(w_t - [\hat{z}_{2,t} - Q_t]^+ - D_t) \\
&= \alpha E f_{t+1}'(x_{2,t+1}) \\
&= H_t'(w_t, \hat{z}_{2,t}).
\end{aligned}
$$

The first equality is due to the definition of H_t in (5.4), and the second due to the state update $x_{t+1} = w_t - [\hat{z}_{1,t} - Q_t]^+ - D_t$. Due to $\hat{z}_{1,t} \geq \hat{z}_{2,t}$, it follows that $x_{1,t+1} \leq x_{2,t+1}$, and the inequality is due to convexity of f_t in x_t. As taking expectation over Q_t and D_t preserves inequality, we have shown that $H_t'(w_t, \hat{z}_{1,t}) \leq H_t'(w_t, \hat{z}_{2,t})$ holds. Induction on t, together with the result of Lemma C.2, proves Part 1.

To prove Part 2, we write:

$$
\begin{aligned}
H_t'(w_t, \widehat{z_t + \eta}) &= \alpha E f_{t+1}'(w_t - [\widehat{z_t + \eta} - Q_t]^+ - D_t) \\
&\geq \alpha E f_{t+1}'(w_t - \eta - [\hat{z}_t - Q_t]^+ - D_t) \\
&= H_t'(w_t - \eta, \hat{z}_t).
\end{aligned}
$$

The first equality is due to the definition of H_t in (5.4) and the state update $x_{t+1} = w_t - [\widehat{z_t + \eta} - Q_t]^+ - D_t$. The inequality is due to $[\widehat{z_t + \eta} - Q_t]^+ \leq \eta + [\hat{z}_t - Q_t]^+$ and the convexity of f_t in x_t. As taking expectation over Q_t and D_t preserves inequality, we have shown that $H_t'(w_t, \widehat{z_t + \eta}) \geq H_t'(w_t - \eta, \hat{z}_t)$ holds. Observe that the smallest minimizer of $H_t(w_t - \eta, \hat{z}_t)$ is $\hat{w}_t(\hat{z}_t) + \eta$, and together with Lemma C.2, this implies $\hat{w}_t(\widehat{z_t + \eta}) \leq \hat{w}_t(\hat{z}_t) + \eta$, which proves Part 2. □

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